8-45

a) $y(nT) - y(nT - T) = x(nT)$

$$\Downarrow$$

$$Y(z) - z^{-1} Y(z) = X(z)$$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - z^{-1}}$$

b) $y(nT) - 2y(nT - T) + y(nT - 2T) = x(nT) + 3x(nT - 3T)$

$$\Downarrow$$

$$Y(z) - 2z^{-1}Y(z) + z^{-2}Y(z) = X(z) + 3z^{-3}X(z)$$

$$\therefore H(z) = \frac{1 + 3z^{-3}}{1 - 2z^{-1} + z^{-2}}$$

8-55

$$y(nT) = x(nT) * h(nT) = \sum_{K=-\infty}^{\infty} x(KT)\, h(KT - nT)$$

$$= \sum_{k=-\infty}^{\infty} \left(\tfrac{1}{4}\right)^{k} u(k) \left(\tfrac{1}{3}\right)^{n-k} u(n-k)$$

$$= \sum_{k=0}^{\infty} \left(\tfrac{1}{4}\right)^{k} (1) \left(\tfrac{1}{3}\right)^{n-k} u(n-k)$$

$n \geq 0$ $\qquad\qquad$ $n < 0$

$$= \sum_{k=0}^{n} \left(\tfrac{1}{4}\right)^{k} \left(\tfrac{1}{3}\right)^{n-k} \qquad\qquad y(nT) = 0$$

$$= \left(\tfrac{1}{3}\right)^{n} \sum_{k=0}^{n} \left(\tfrac{3}{4}\right)^{k} = \left(\tfrac{1}{3}\right)^{n} \left[\frac{1 - \left(\tfrac{3}{4}\right)^{n+1}}{1 - \tfrac{3}{4}} \right]$$

$$= 4\left(\tfrac{1}{3}\right)^{n} \left[1 - \left(\tfrac{3}{4}\right)^{n+1}\right] = 4\left(\tfrac{1}{3}\right)^{n} - 3\left(\tfrac{1}{4}\right)^{n}, \quad n \geq 0$$

$$\therefore y(nT) = \left[4\left(\tfrac{1}{3}\right)^{n} - 3\left(\tfrac{1}{4}\right)^{n}\right] u(n) \quad \text{for all } n.$$

8-68

$$y(nT) = x(nT) + 0.2y(nT-T) - 0.5y(nT-2T)$$

$$Y(z) = X(z) + 0.2z^{-1}Y(z) - 0.5Y(z)z^{-2}$$

$$\therefore H(z) = \frac{1}{1-0.2z^{-1}+0.5z^{-2}}$$

$$z = e^{sT_s} \Rightarrow z = e^{j\omega T_s} = e^{j2\pi r}, \quad r = \frac{\omega}{\omega_s} = \frac{f}{f_s}$$

a) For $f=0 \Rightarrow r=0 \Rightarrow z = e^{j(0)} = 1$

$$\therefore H(e^{j0}) = H(1) = \frac{1}{1-0.2+0.5} = \frac{1}{1.3} = 0.77$$

$\therefore |H| = 0.77, \quad \angle H = 0°.$

b) For $f = 500\,Hz$, $r = \frac{500}{2000} = \frac{1}{4} \Rightarrow z = e^{j2\pi(\frac{1}{4})}$

$$= e^{j\pi/2} = j$$

$$\therefore H(e^{j\pi/2}) = \frac{1}{1-0.2j^{-1}+0.5(j)^{-2}} = \frac{1}{1+j0.2-0.5}$$

$$= \frac{1}{0.5+j0.2}$$

$$\therefore |H| = \frac{1}{\sqrt{0.5^2+0.2^2}} = 1.857, \quad \angle H = -\tan^{-1}\frac{0.2}{0.5}$$

$$= -21.8°$$

c) $r = \frac{1000}{2000} = \frac{1}{2} \Rightarrow z = e^{j\pi} = -1$

$$\therefore H(e^{j\pi}) = \frac{1}{1+0.2+0.5} = \frac{1}{1.7} = 0.588$$

$\therefore |H| = 0.588, \quad \angle H = 0°.$