5-13

a) $X(S) = \dfrac{S+10}{S^2+3S+2}$

$$x(0) = \lim_{S \to \infty} \frac{S(S+10)}{S^2+3S+2} = 1$$

$$x(\infty) = \lim_{S \to 0} \frac{S(S+10)}{S^2+3S+2} = 0$$

b) $X(S) = \dfrac{5}{S^3+S^2+9S+9}$

$$x(0) = \lim_{S \to \infty} \frac{5S}{S^3+S^2+9S+9} = 0$$

$$x(\infty) = \lim_{S \to 0} \frac{5S}{S^3+S^2+9S+9} = 0$$

c) $x(0) = \lim\limits_{S \to \infty} \dfrac{S(S^2+5S+7)}{S^2+3S+2} = \infty$

$$x(\infty) = \lim_{S \to 0} \frac{S(S^2+5S+7)}{S^2+3S+2} = 0$$

d) $x(0) = 1$

$$x(\infty) = \lim_{S \to 0} \frac{S^2+3S}{S^2+2S} = \lim_{S \to 0} \frac{2S+3}{2S+2} = \frac{3}{2}$$

5.14

a) $y_1(t) = x(2t-1)\,u(2t-1) = x\left[2(t-\tfrac{1}{2})\right]u\left[2(t-\tfrac{1}{2})\right]$

Using $x(at) \Rightarrow \frac{1}{a}X\left(\frac{s}{a}\right)$, $a>0$

& $x(t-t_0)\,u(t-t_0) \Rightarrow X(s)e^{-st_0}$

$$\therefore Y_1(s) = \frac{1}{2}\,\frac{\frac{s}{2}+2}{\left(\frac{s}{2}\right)^2+4\left(\frac{s}{2}\right)+5}\,e^{-\frac{1}{2}s}$$

$$= \frac{s+4}{s^2+8s+20}\,e^{-s/2}$$

d) $y_4(t) = x(t) * x(t)$

Using $x_1(t) * x_2(t) \Rightarrow X_1(s)X_2(s)$

$$\therefore Y_4(s) = \frac{(s+2)^2}{(s^2+4s+5)^2}$$

e) $y_5(t) = \frac{dx(t)}{dt}$

Using $\frac{dx(t)}{dt} \Rightarrow sX(s) - x(0^-)$

& $x(0^+) = \lim_{s\to\infty} sX(s)$

$$x(0^+) = \lim_{s\to\infty} s\left(\frac{s+2}{s^2+4s+5}\right) = \lim_{s\to\infty}\frac{s^2+2s}{s^2+4s+5} = \underline{1}$$

Assuming $x(t)$ is continuous at $t=0 \Rightarrow$

$x(0^-) = x(0^+) = 1$.

$$\therefore Y_5(s) = \frac{s^2+2s}{s^2+4s+5} - 1$$

5-15

b) $x_1(t) = (\cos 5t)\,u(t)$, $x_2(t) = (\sin 3t)\,u(t)$

$$Y(S) = X_1(S)\,X_2(S) = \frac{S}{S^2+5^2} \cdot \frac{3}{S^2+3^2}$$

$$= \frac{3S}{(S^2+5^2)(S^2+3^2)} = \frac{A_1 S + B_1}{S^2+5^2} + \frac{A_2 S + B_2}{S^2+3^2}$$

$$\therefore\ 3S = (A_1 S + B_1)(S^2+9) + (A_2 S + B_2)(S^2+25)$$

$$B_1 = 0\ ,\quad B_2 = 0\ ,\quad A_1 = -\frac{3}{16}\ ,\quad A_2 = \frac{3}{16}$$

$$\therefore\ Y(s) = \frac{-\frac{3}{16}s}{s^2+5^2} + \frac{\frac{3}{16}s}{s^2+3^2}$$

$$y(t) = -\frac{3}{16}\cos 5t + \frac{3}{16}\cos 3t$$

d) $Y(s) = \dfrac{3}{s^2+3^2} \cdot \dfrac{e^{-5s}}{s}$

$$= e^{-5s}\left[\frac{A_1}{s} + \frac{A_2 s + B_2}{s^2+3^2}\right]$$

$A_1 = \frac{1}{3}$, $A_2 = -\frac{1}{3}$, $B_2 = 0$

$$Y(s) = e^{-5s}\left[\frac{1/3}{s} - \frac{1}{3}\,\frac{s}{s^2+3^2}\right]$$

$$y(t) = \frac{1}{3}\,u(t-5) - \frac{1}{3}\cos\left[3(t-5)\right]u(t-5)$$

5-18

a) $X(s) = \dfrac{7s^3+20s^2+33s+82}{(s^2+4)(s+2)(s+3)}$

$$= \frac{A_1 s + B_1}{s^2+4} + \frac{A_2}{s+2} + \frac{A_3}{s+3}$$

$A_2 = 5$, $A_3 = 2$, $A_1 = 0$, $B_1 = 1$

$$X(s) = \frac{1}{s^2+4} + \frac{5}{s+2} + \frac{2}{s+3}$$

$$x(t) = \frac{1}{2}\sin 2t + 5e^{-2t} + 2e^{-3t}$$

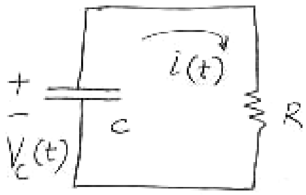
b) $X(s) = \dfrac{2s^3+9s^2+22s+23}{[(s+1)^2+4](s+1)(s+3)} = \dfrac{A_1 s + B_1}{(s+1)^2+4} + \dfrac{A_2}{s+1} + \dfrac{A_3}{s+3}$

$A_2 = 1 \quad , \quad A_3 = 1 \quad , \quad A_1 = 0 \quad , \quad B_1 = 1$

$x(t) = \frac{1}{2} e^{-t} \sin 2t + e^{-t} + e^{-3t}$

<u>5-27</u>

a)



$t \geq 0$

$-V_c(t) + R\,i(t) = 0$

$V_c(t) = -\frac{1}{C}\int_0^t i(t)\,dt + V_c(0)$

$= -\frac{1}{C}\int_0^t i(t)\,dt + V$

$\therefore \frac{1}{C}\int_0^t i(t)\,dt - V + R\,i(t) = 0$

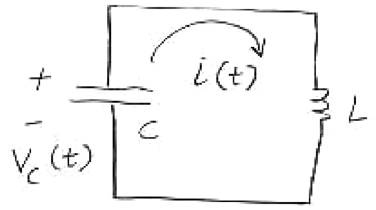
$\frac{1}{C}\,\frac{I(s)}{s} + \frac{1}{C}\,\frac{\cancelto{0}{\int_0^0 i(t)\,dt}}{s} - \frac{V}{s} + R\,I(s) = 0$

$I(s)\left[\frac{1}{Cs} + R\right] = V/s$

$I(s) = \dfrac{V}{Rs + \frac{1}{C}} = \dfrac{VC}{RCs+1} = \dfrac{V/R}{s + 1/RC}$

$i(t) = \frac{V}{R} e^{-t/RC} \quad , \quad t \geq 0$

b)



$t \geqslant 0$

$$-V_c(t) + L\,\frac{di(t)}{dt} = 0$$

$$\frac{1}{C}\int_0^t i(t)\,dt - V + L\,\frac{di(t)}{dt} = 0$$

$$\frac{1}{C}\,\frac{I(s)}{s} + \frac{1}{C}\,\frac{\cancelto{0}{\int_0^0 i(t)\,dt}}{s} - \frac{V}{s} + L\left[sI(s) - \cancelto{0}{i(0)}\right] = 0$$

$$I(s)\left[\frac{1}{Cs} + Ls\right] = \frac{V}{s}$$

$$I(s) = \frac{V}{Ls^2 + \frac{1}{C}} = \frac{VC}{LCs^2+1} = \frac{V/L}{s^2 + \frac{1}{LC}} = \frac{V}{L}\sqrt{LC}\,\frac{\frac{1}{\sqrt{LC}}}{s^2+\frac{1}{LC}}$$

$$i(t) = V\sqrt{\frac{C}{L}}\,\sin\frac{1}{\sqrt{LC}}t\ , \quad t \geqslant 0\,.$$