3.6

Part 4 of table 3.1

This is the same signal as $x(t)$ of problem 3.4.

Method 1

$$X_n = \frac{1}{T_0}\left[\int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} A e^{-jn\omega_0 t}\,dt - \int_{\frac{T_0}{4}}^{\frac{3T_0}{4}} A e^{-jn\omega_0 t}\,dt\right]$$

$$= \frac{A}{-T_0 jn\omega_0}\left[e^{-jn\omega_0 t}\Big|_{-\frac{T_0}{4}}^{\frac{T_0}{4}} - e^{-jn\omega_0 t}\Big|_{\frac{T_0}{4}}^{\frac{3T_0}{4}}\right]$$

$$= \frac{jA}{n2\pi}\left[e^{-jn\pi/2} - e^{jn\pi/2} - e^{-jn3\pi/2} + e^{-jn\pi/2}\right]$$

$$= \frac{jA}{n2\pi}\left[-j2\sin\left(\frac{n\pi}{2}\right) + e^{-jn\pi/2}\left(1 - e^{-jn\pi}\right)\right]$$

$$= \begin{cases} 0 & , (n \text{ even}) \\ \dfrac{2A}{|n|\pi} & , (n = \pm1, \pm5, \pm9, \cdots) \\ \dfrac{-2A}{|n|\pi} & , (n = \pm3, \pm7, \pm11, \cdots \end{cases}$$

Method 2

$$X_n = 2\left[\frac{1}{T_0}\int_0^{T_0/2} x(t)\cos n\omega_0 t\,dt\right]$$, because $x(t)$ is even.

$$= \frac{2}{T_0}\left[\int_0^{T_0/4} A\cos n\omega_0 t\,dt - \int_{\frac{T_0}{4}}^{T_0/2} A\cos n\omega_0 t\,dt\right]$$

$$= \frac{2A}{n\omega_0 T_0}\left[\sin n\omega_0 t\Big|_0^{\frac{T_0}{4}} - \sin n\omega_0 t\Big|_{\frac{T_0}{4}}^{\frac{T_0}{2}}\right]$$

$$= \frac{2A}{n2\pi}\left[\sin n\frac{\pi}{2} - \sin 0 - \sin n\pi + \sin n\pi/2\right]$$

$$= \frac{2A}{n\pi}\sin\frac{n\pi}{2}$$

$$= \begin{cases} 0 & , (n \text{ even}) \\ \dfrac{2A}{|n|\pi} & , n = \pm 1, \pm 5, \pm 9, \cdots \\ \dfrac{-2A}{|n|\pi} & , n = \pm 3, \pm 7, \pm 11, \cdots \end{cases}$$

Method 3

since $x(t)$ is the same as that of problem 3.4 $\Longrightarrow$

$$X_n = \frac{a_n - j\,b_n}{2} \quad , n > 0$$

$$X_n = \frac{a_n}{2} = \begin{cases} 0 & , n = 0, 2, 4, 6, \cdots \\ \frac{2A}{n\pi} & , n = 1, 5, 9, \cdots \\ -\frac{2A}{n\pi} & , n = 3, 7, 11, \cdots \end{cases}$$

where $n = 0$ is included because $X_0 = a_0 = 0$.

but, it is known that if $x(t)$ is real $\Rightarrow$

$$X_{-n} = X_{-n}^* \Rightarrow$$

$$X_n = \begin{cases} 0 & , (n = \text{even}) \\ \frac{2A}{|n|\pi} & , n = \pm 1, \pm 5, \pm 9, \cdots \\ -\frac{2A}{|n|\pi} & , n = \pm 3, \pm 7, \pm 11, \cdots \end{cases}$$

<u>3-8</u>

a) $P_{avg} = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt = \frac{1}{T_0} \int_0^{T_0} 4 \sin^4(2500\pi t) \cos^2(2\times 10^4 \pi t) dt$

(3-55a)

integration procedure will be lengthy, but straight-forward.

Instead, use $x(t) = 2\sin^2(2500\pi t) \cos(2\times 10^4 \pi t)$

$$= 2\frac{(1 - \cos 5000\pi t)}{2} \cos(2\times 10^4 \pi t)$$

$$= \cos(2\times 10^4 \pi t) - \cos 5000\pi t \cos(2\times 10^4 \pi t)$$

$$= \cos\left(2\times10^4\pi t\right) - \frac{1}{2}\cos\left[\left(5000\pi t + 20000\pi t\right)\right] - \frac{1}{2}\cos\left[\left(5000\pi t - 20000\pi t\right)\right]$$

$$= \cos\left(20000\pi t\right) - 0.5\cos\left(25000\pi t\right) - 0.5\cos\left(15000\pi t\right)$$

$$= -0.5\cos\left(15000\pi t\right) + \cos\left(20000\pi t\right) - 0.5\cos\left(25000\pi t\right)$$

With $\omega_0 = 5000\pi \Rightarrow$

$a_3 = -0.5$, $a_4 = 1$, $a_5 = -0.5$

and all other other values of $a_n = 0$.

$b_n = 0$ for all n.

$$X_n = \frac{a_n - j b_n}{2} = \frac{a_n}{2} \quad , n > 0 \quad \Rightarrow$$

$X_{\pm 3} = -0.25$

$X_{\pm 4} = 0.5$

$X_{\pm 5} = -0.25$, using $(3-56) \Rightarrow$

$$\therefore P_{avg} = \left(\frac{1}{4}\right)^2 \times 2 + \left(\frac{1}{2}\right)^2 \times 2 + \left(\frac{1}{4}\right)^2 \times 2 = \frac{3}{4}\,W$$

b) Frequencies contained in $x(t)$

are $f = \frac{15000\pi}{2\pi} = 7.5\,kHz$

$f = \frac{20000\pi}{2\pi} = 10\,kHz$

$f = \frac{25000\pi}{2\pi} = 12.5\,kHz$

The 12.5kHz is blocked, 7.5kHz, 10kHz are passed.

$\therefore\ x_{out}(t) = -0.5\cos(15000\pi t) + \cos(20000\pi t)$.

$$(P_o)_{avg} = \left(\frac{1}{4}\right)^2 \times 2 + \left(\frac{1}{2}\right)^2 \times 2 = 0.625\,W$$

$$\frac{(P_o)_{avg}}{(P_i)_{avg}} = \frac{0.625}{0.75} = 0.833$$

3-9 a) $\frac{T_o}{\tau} = 2 \Rightarrow \tau f_o = \frac{1}{2}$ or $\frac{1}{\tau f_o} = 2$

which means $|nf_o| \le \tau^{-1} \Rightarrow |n| \le \frac{1}{\tau f_o}$ or $|n| \le 2$

$\therefore\ n = -2, -1, 0, 1, 2$.

$$X_n = \frac{A\tau}{T_o} e^{-j2\pi n f_o t_o} \operatorname{sinc}(nf_o\tau)\Big|_{\tau f_o = 1/2}$$

$$\therefore\ X_n = \frac{1}{2} A e^{-j2\pi n f_o t_o} \operatorname{sinc}\left(\frac{n}{2}\right)$$

$$\therefore\ P_{partial} = |X_{-2}|^2 + |X_{-1}|^2 + |X_0|^2 + |X_1|^2 + |X_2|^2$$

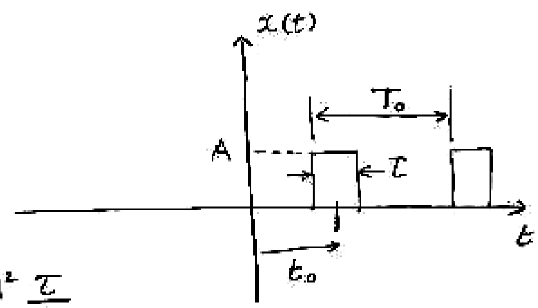
$$= 2\left[|X_2|^2 + |X_1|^2\right] + |X_0|^2$$

$$= 2\left[0^2 + \left|\tfrac{1}{2} A \operatorname{sinc}(\tfrac{1}{2})\right|^2\right] + \left|\tfrac{1}{2} A\right|^2$$

$$= 2 \times \tfrac{1}{4} A^2 \left|\frac{1}{\frac{\pi}{2}}\right|^2 + \tfrac{1}{4} A^2$$

$$= \frac{A^2}{4}\left[2\left(\frac{4}{\pi^2}\right) + 1\right] = \frac{A^2}{4}\left[1 + \frac{8}{\pi^2}\right]$$

$$P_{total} = \frac{1}{T_0}\int_{T_0} |x(t)|^2\, dt$$



$$= \frac{1}{T_0}\int_{\tau} A^2\, dt = A^2 \frac{\tau}{T_0}$$

For this case $P_{total} = A^2 \dfrac{\tau}{T_0} = \dfrac{1}{2} A^2$

$$\therefore\ \frac{P_{partial}}{P_{total}} = \frac{\frac{1}{4}\left(1 + \frac{8}{\pi^2}\right) A^2}{(1/2)\ A^2} = \frac{1}{2}\left(1 + \frac{8}{\pi^2}\right) = 0.9053$$

Similar procedure is done for parts b) and c).

The ratio $\frac{P_{partial}}{P_{total}} = \frac{\sum_{n=-n'}^{n'} \frac{A^2\tau^2}{T_o^2} \operatorname{sinc}^2(nf_o\tau)}{\left(A^2 \frac{\tau}{T_o}\right)}$

$$= \frac{\tau}{T_o} \sum_{n=-n'}^{n'} \operatorname{sinc}^2(nf_o\tau)$$

$$= \frac{\tau}{T_o} \sum_{n=-\frac{T_o}{\tau}}^{\frac{T_o}{\tau}} \operatorname{sinc}^2(nf_o\tau)$$

For b) $P_{fraction} = \frac{1}{4} \sum_{n=-4}^{4} \operatorname{sinc}^2(n\, 0.25)$

c) $P_{fraction} = \frac{1}{10} \sum_{n=-10}^{10} \operatorname{sinc}^2(n\, 0.1)$

3-12

a) $X_n = \frac{1}{2} \int_{-1}^{1} e^{-|t|} e^{-j\frac{2\pi}{2}nt}\, dt = \int_0^1 e^{-t} \cos n\pi t\, dt$

$$= \frac{1-(-1)^n e^{-1}}{1+(\pi n)^2}$$

b) $a_n = 2 \operatorname{Re} X_n = 2\left[\frac{1-(-1)^n e^{-1}}{1+(\pi n)^2}\right]$

$b_n = -2 \operatorname{Im} X_n = 0$

3.17

Property	a	b	c	d	e	f
Real Coefficients		X			X	
Imag. "	X			X		
Complex "			X			X
Even - Indexed " $= 0$	X	X	X	X	X	X
$X_0 = 0$	X	X	X	X	X	X

All the signals are half-wave (odd) symmetrical, i.e. $x(t) = -x(t - \frac{T_0}{2})$.

3.22

a) Same as 3-11 a) with null spacing $= \frac{1}{\tau} = 500 Hz$, and line spacing $= \frac{1}{T_0} = 125 Hz$.

b) Same as 3-11 b) with null spacing $= \frac{1}{\tau} = 1000 Hz$ and line spacing $= \frac{1}{T_0} = 125 Hz$.

c) Same as 3-11 c) with null spacing $= \frac{1}{\tau} = 500 Hz$ and line spacing $= \frac{1}{T_0} = 62.5 Hz$.