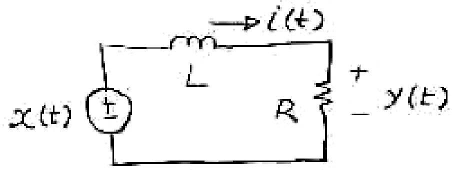
2-10



a) $-x + L\dfrac{di}{dt} + y = 0 \qquad (1)$

but $\quad i = \dfrac{y}{R} \qquad (2)$

$\therefore \quad -x + \dfrac{L}{R}\dfrac{dy}{dt} + y = 0$

$\therefore \quad \dfrac{d\,y(t)}{dt} + \dfrac{R}{L}\,y(t) = \dfrac{R}{L}\,x(t) \qquad (3)$

b) Proof of linearity is similar to 2-4 a).

c) $$\frac{d\,y(t-\tau)}{dt} + \frac{R}{L}\,y(t-\tau) = \frac{d\,y(t-\tau)}{d(t-\tau)} + \frac{R}{L}\,y(t-\tau)$$

$$= \frac{R}{L}\,x(t-\tau)$$

$$\therefore \quad \frac{d\,y(t-\tau)}{dt} + \frac{R}{L}\,y(t-\tau) = \frac{R}{L}\,x(t-\tau)$$

$\therefore$ system is fixed.

d) The solution to the first order differential equation:

$$\frac{dy}{dt} + \frac{R}{L}y = \frac{R}{L}x \quad \text{is given by:}$$

$$y(t) = e^{-\frac{R}{L}t}\int_{t_0}^{t} e^{\frac{R}{L}\lambda}\,\frac{R}{L}\,x(\lambda)\,d\lambda + A\,e^{-\frac{R}{L}t}, \quad t \geq t_0$$

$$\text{since} \quad i(0) = 0 \Rightarrow \left.\frac{y}{R}\right|_{t=0} = 0 \Rightarrow y(0) = 0$$

$\therefore$ $t_0 = 0$ and

$$y(0) = e^{-\frac{R}{L}t}\int_0^0 e^{\frac{R}{L}\lambda}\,\frac{R}{L}\,x(\lambda)\,d\lambda + A = 0 + A \Rightarrow A = 0$$

$$\therefore \quad y(t) = e^{-\frac{R}{L}t}\int_0^{t} e^{\frac{R}{L}\lambda}\,\frac{R}{L}\,x(\lambda)\,d\lambda$$

$$= \int_0^{t} e^{-\frac{R}{L}(t-\lambda)}\,\frac{R}{L}\,x(\lambda)\,d\lambda$$

$$= \int_{-\infty}^{t} e^{-\frac{R}{L}(t-\lambda)} \frac{R}{L} x(\lambda)\, d\lambda \qquad (4)$$

because $x(t) = 0$ for $t < 0$. (i.e. $x(\lambda) = 0$ for $\lambda < 0$).

Equation (4) can be rewritten as:

$$y(t) = \int_{-\infty}^{\infty} e^{-\frac{R}{L}(t-\lambda)} u(t-\lambda) \frac{R}{L} x(\lambda)\, d\lambda$$

because $u(t-\lambda) = 0$ for $\lambda > t$ and
$u(t-\lambda) = 1$ for $\lambda < t$.

$$\therefore \quad y(t) = \int_{-\infty}^{\infty} x(\lambda) \left[\frac{R}{L} e^{-\frac{R}{L}(t-\lambda)} u(t-\lambda) \right] d\lambda$$

$$= \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$

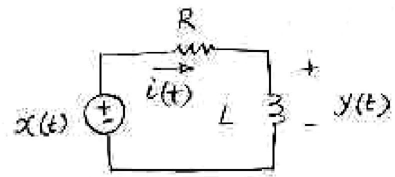
which is the same form as 2-56.

2-11

Property	a	b	c	d	e	f
Linear	X		X		X	
Causal	X	X	X	X		
Fixed	X	X			X	
Dynamic	X	X	X	X	X	X
Order	2	3	2	2	0	2

2-22

KVL $\Rightarrow$ $-x + Ri + y = 0$



$$i = \frac{1}{L}\int_{-\infty}^{t} y(t)\,dt$$

$$\therefore \quad -x(t) + \frac{R}{L}\int_{-\infty}^{t} y(t)\,dt + y(t) = 0$$

$$\therefore \quad \frac{dy(t)}{dt} + \frac{R}{L}\,y(t) = \frac{dx(t)}{dt} \qquad (1)$$

since $\frac{dx(t)}{dt}$ appears on the right-hand side, it is easier to find the step response $a(t)$ first.

$$\therefore \quad \frac{d\,a(t)}{dt} + \frac{R}{L}\,a(t) = \frac{du(t)}{dt} = \delta(t) \qquad (2)$$

For $t < 0 \Rightarrow a(t) = 0$

For $t > 0 \Rightarrow \frac{d\,a(t)}{dt} + \frac{R}{L}\,a(t) = 0 \qquad (3)$

The solution to equation ③ $\Rightarrow$

$$a(t) = A e^{-\frac{R}{L}t} \quad , t > 0 .$$

$$\therefore \; a(t) = A e^{-\frac{R}{L}t} \, u(t) .$$

To determine A , we integrate ② $\Rightarrow$

$$\int_{0^-}^{0^+} \frac{d\,a(t)}{dt}\,dt + \frac{R}{L}\int_{0^-}^{0^+} a(t)dt = \int_{0^-}^{0^+} \delta(t)\,dt$$

$$a(t)\Big|_{0^-}^{0^+} + 0 = 1$$

$$a(0^+) - a(0^-) + 0 = 1$$

$$A - 0 + 0 = 1 \Rightarrow A = 1$$

$$\therefore \; a(t) = e^{-\frac{R}{L}t} \, u(t)$$

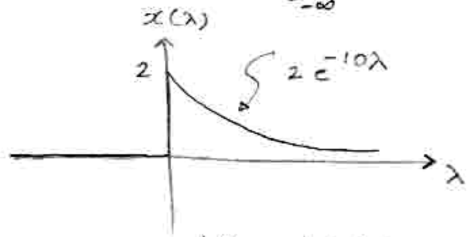
$$\therefore \; h(t) = \frac{d\,a(t)}{dt} = -\frac{R}{L} e^{-\frac{R}{L}t} u(t) + e^{-\frac{R}{L}t} \delta(t)$$

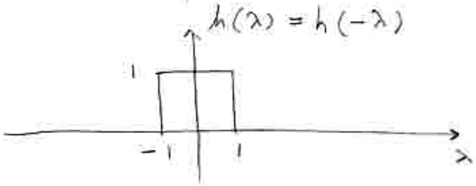
$$h(t) = -\frac{R}{L} e^{-\frac{R}{L}t} u(t) + \delta(t)$$

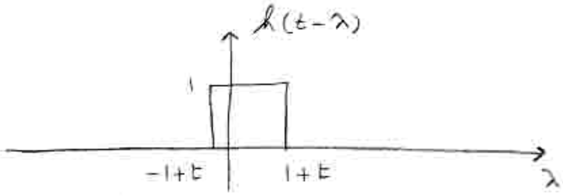
1.17

a)

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$

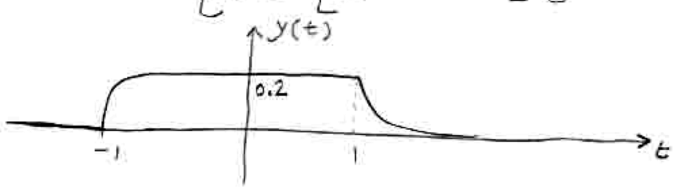




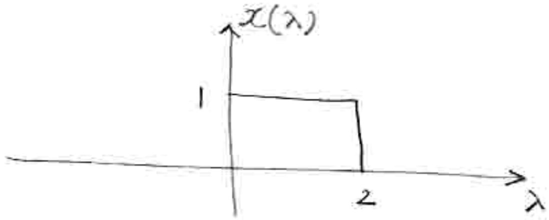


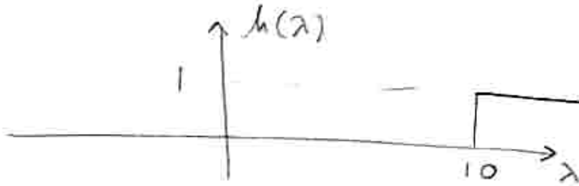
$$y(t) = \begin{cases} 0 & , \; 1+t < 0 \Rightarrow t < -1 \\ \int_0^{1+t} 2e^{-10\lambda}\, d\lambda & , \; -1 < t < 1 \\ \int_{-1+t}^{1+t} 2e^{-10\lambda}\, d\lambda & , \; t > 1 \end{cases}$$

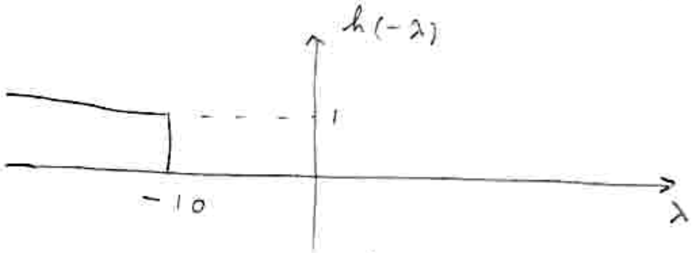
$$= \begin{cases} 0 & , \; t < -1 \\ 0.2\left[1 - e^{-10(t+1)}\right] & , \; -1 < t < 1 \\ 0.2\left[e^{-10(t-1)} - e^{-10(t+1)}\right] & , \; t > 1 \end{cases}$$

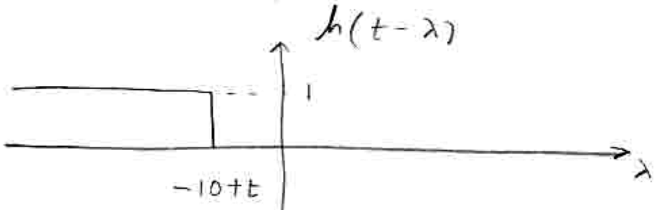


b) $y(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda = x(t) * h(t)$









$$\therefore\ y(t) = \begin{cases} 0 & , \quad -10+t < 0 \Rightarrow t < 10 \\ (-10+t)(1) & , \quad 0 < -10+t < 2 \Rightarrow 10 < t < 12 \\ 2(1) & , \quad -10+t > 2 \Rightarrow t > 12 \end{cases}$$

$$= \begin{cases} 0 & , \quad t < 10 \\ t-10 & , \quad 10 < t < 12 \\ 2 & , \quad t > 12 \end{cases}$$