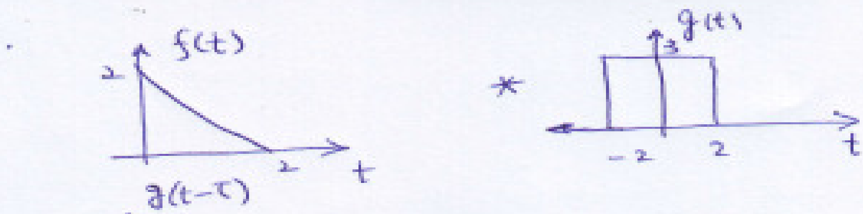
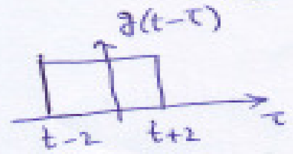
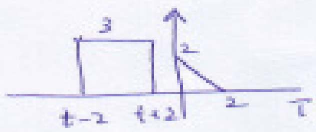
Problem 2.1





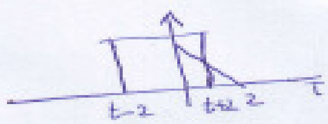
Case I



for $t \leq -2$, Area $= 0$

$y(t) = 0$

Case II

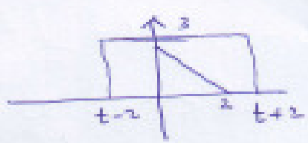


for $-2 \leq t < 0$

$$y(t) = \int_0^{t+2} 3(-\tau+2)\,d\tau$$

$$y(t) = -\frac{3}{2}t^2 + 6$$

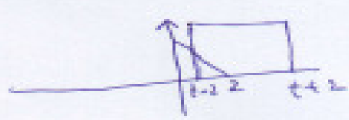
Case III



for $0 \leq t < 2$

$$y(t) = \int_0^{2} 3(-\tau+2)\,d\tau = 6$$

Case IV

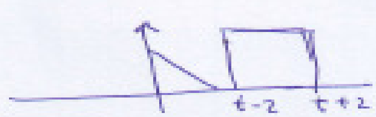


for $2 \leq t < 4$

$$y(t) = \int_{t-2}^{2} 3(-\tau+2)\,d\tau$$

$$= \frac{3}{2}t^2 - 12t + 24$$

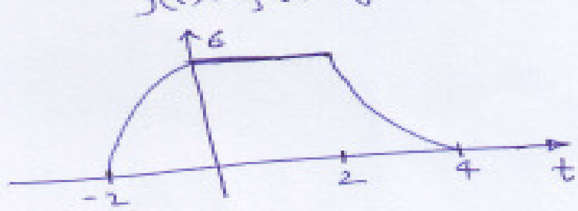
Case V



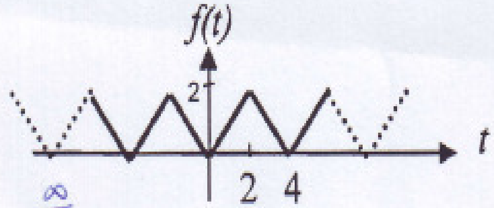
for $t \geq 4$

$y(t) = 0$.

$y(t) = f(t) * g(t)$



Problem 2.2



$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n w_0 t) + \sum_{n=1}^{\infty} b_n \cos(n w_0 t)$$

Since the signal is even $\Rightarrow b_n = 0$

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt$$

$T_0 = 4$ sec

$$a_0 = \frac{1}{4}\left[\int_{-2}^{0} -t\,dt + \int_0^2 t\,dt\right]$$

$a_0 = 1$ OR Since it is an even function (signal)

$$a_0 = \frac{2}{T}\int_0^2 t\,dt = 1$$

$$a_n = \frac{2}{T_0}\int^{T_0} x(t) \cos n w_0 t\,dt$$

$$= \frac{\cancel{2}}{\cancel{4}\,2}\int_{-2}^{2} x(t) \cos n w_0 t\,dt.$$

OR

$$= 2\cdot\left[\frac{1}{\cancel{2}}\int_0^2 x(t) \cos n w_0 t\,dt\right]$$

$$= \int_0^2 t \cos n w_0 t\,dt$$

using integration by parts.

$$= \frac{t \cdot \sin(n w_0 t)}{n w_0} + \frac{\cos(n w_0 t)}{(n w_0)^2}\Bigg]_0^2$$

$$= 2\,\frac{\sin(2 n w_0)}{n w_0} + \frac{\cos 2 n w_0}{(n w_0)^2} - \frac{1}{(n w_0)^2}$$

Since $T_0 = 4$, $w_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$$a_n = \frac{2\sin(n\pi)}{n\,\pi/2} + \frac{\cos(n\pi)}{(n\cdot\pi/2)^2} - \frac{1}{(\frac{n\pi}{2})^2}$$

Since $\sin n\pi = 0$.

also $\cos n\pi = \begin{cases} -1, & \text{odd } n \\ +1, & \text{even } n \end{cases}$

$$a_n = \begin{cases} \dfrac{-8}{(n\pi)^2}, & n \text{ odd} \\ 0, & n \text{ even (otherwise).} \end{cases}$$

so.

The trignometric Fourier Series is

$$\boxed{x(t) = 1 - \frac{8}{\pi^2}\sum^{\infty} \frac{1}{n^2}\cos\left(\frac{\pi n t}{2}\right)}$$

Problem 2.3

Sol.

$T_0 = 2\,\text{sec.}, \quad f_0 = \frac{1}{2}\,\text{Hz}$

$$X_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt = \frac{1}{2}\int_0^1 e^{-t}\,dt$$

$$= -\frac{1}{2}\left[e^{-t}\right]_0^1 = \frac{1}{2}\left[1 - e^{-1}\right]$$

$$\cong 0.31$$

$$X_n = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-jn\omega_0 t}\,dt \qquad , \omega_0 = 2\pi f_0$$

$$= \frac{1}{2}\int_0^1 e^{-t}\, e^{-jn(2\pi\frac{1}{2})t}\,dt$$

$$= \frac{1}{2}\int_0^1 e^{-(1+jn\pi)t}\,dt$$

$$= -\frac{1}{2}\left.\frac{e^{-(1+jn\pi)t}}{(1+jn\pi)}\right]_0^1$$

$$= -\frac{1}{2}\cdot\frac{1}{(1+j\pi n)}\,e^{-(1+jn\pi)} + \frac{1}{2}\left(\frac{1}{1+j\pi n}\right)$$

$$\boxed{X_n = \frac{1}{2}\cdot\frac{1}{(1+j\pi n)}\left[1 - e^{-(1+jn\pi)}\right]}$$

Problem 2.4

(1) Neither Even nor Odd.

(2) $T_0 = 2$ Sec.

$w_0 = \frac{2\pi}{T_0} = \pi$ rad/sec.

(3) $$X_n = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-jnw_0 t}\, dt$$

$$= \frac{1}{2}\int_0^{2} 5t \cdot e^{-jnw_0 t}\, dt$$

$$= \frac{5}{2}\int_0^{2} t \cdot e^{-jnw_0 t}\, dt$$

using integration by parts.

$$X_n = \frac{5}{2}\left(t - \frac{1}{-jnw_0}\right)\frac{e^{-jnw_0 t}}{-jnw_0}\Bigg]^2$$

$$= \frac{5}{2}\left[\frac{t\, e^{-jnw_0 t}}{-jnw_0} + \frac{e^{-jnw_0 t}}{(nw_0)^2}\right]_0^2$$

$\because w_0 = \pi$

$$= \frac{5}{2}\left[\frac{2 \cdot e^{-2jn\pi}}{-jn\pi} + \frac{e^{-2jn\pi}}{(n\pi)^2} - e^0 - \frac{e^0}{(n\pi)^2}\right]$$

$$X_n = \frac{5}{2}\left[\frac{2\, e^{-j2n\pi}}{-jn\pi} + \frac{e^{-j2n\pi}}{(n\pi)^2} - 1 - \frac{1}{(n\pi)^2}\right]$$

$$\boxed{X_1 = j\frac{5}{\pi}}$$

$$X_0 = a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\, dt$$

$$= \frac{1}{2}\int_0^2 5t\, dt$$

$$= \frac{1}{2}\left[\frac{5t^2}{2}\right]_0^2$$

$$= \frac{1}{4}\left[5 \times 4 - 0\right]$$

$$\boxed{X_0 = 5}$$