Spacecraft Attitude Control using Internal Model Principle

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***Abstract* — In this paper, we propose a design methodology for the tracking of Spacecraft attitude. The plant (spacecraft model) is a sixth order nonlinear multi-input-multi-output system. The attitude tracking objective is accomplished using Internal Model principle. Internal model of reference and disturbance signal driven by the tracking error is included in the controller. The augmented system formed of the nonlinear plant and the internal model is stabilized by output feedback, where the derivative of the attitude is obtained from the attitude measurement using a differentiator. Small gain theorem is employed to determine the feedback gain of the nonlinear plant. Computer simulation results are given to illustrate the effectiveness of the proposed controller.**

***Index Terms* — Attitude control, Internal Model principle, Output feedback, Small gain theorem, Spacecraft model.**

I. INTRODUCTION

Attitude control is an important subsystem in a spacecraft [1]. The attitude of a spacecraft is its orientation in space. The orientation is with respect to a particular reference like the earth and sun [1]. Stable control of a rigid body orientation is required in pointing and slewing of spacecrafts. The solar arrays in a spacecraft are required to point normal to the sun for the purpose of solar power generation. The transceiver in a communications satellite may be required to point in a specific region of earth for the purpose of transmitting and receiving of communication signals [2]. An earth observing remote sensing satellite is required to selectively point and track specific ground targets. A space observing satellite would have to point away from the earth with an additional requirement to avoid pointing at the sun. All these operations are controlled by the attitude determination and control subsystem (ADCS) of a spacecraft [2]. Hence the motion of a spacecraft has to be in predetermined fashion such that the spacecraft follows a specific trajectory. The Spacecraft is considered to be a rigid body whose attitude can be described by two sets of equations, namely, kinematic equation, which relates the time derivatives of the orientation angles to the angular velocity vector and dynamic equation, which describes the time evolution of the angular velocity vector [3]. Various parameterizations of attitude exist to represent the orientation angles. A comprehensive survey of attitude representations is given in [4]. Attitude representation can be done using three parameters as well as four parameters. The three parameter attitude representation techniques are Euler angles, Gibbs vector, Cayley Rodrigues vector and Modified Rodrigues parameter [4]. They exhibit singular orientations due to the Jacobian matrix in the kinematic equation, which is singular for some orientations. The four parameter representation of attitude is given by the unit quaternion (Euler parameters). The unit quaternion globally represents the spacecraft attitude without singularities. However, an additional constraint equation is introduced with the four parameters as stated in [4]. The attitude control problem was first presented in the literature by [5]. A general procedure for the design and analysis of three axis, large-angle attitude control system was developed based on properties common to all attitude control systems. [6] presented a general framework for the analysis of attitude tracking control of a rigid body using the nonsingular unit quaternion representation. The controllers in [6] share the common structure of a proportional-derivative feedback. An adaptive tracking control scheme wherein the unknown spacecraft inertia matrix is compensated using linear parameterization is discussed in [7]. [8] proposed an adaptive attitude tracking controller that identifies the inertia matrix via periodic command signals. [9] discussed the adaptive attitude tracking control using synthesized velocity from attitude measurements by incorporating a velocity filter formulation.

The problem of designing an attitude controller to track a given reference signal and reject disturbance has not been given adequate attention using a fixed controller. The controllers which do not include internal model of the reference and disturbance signal [6-9] will not ensure that the tracking error is asymptotically zero, especially when periodic reference and disturbance signals are present [10]. In this paper we address this problem using Internal Model principle. Internal Model principle was first proposed in [11] for linear systems. Its extension to a class of nonlinear systems is presented in [12]. An internal model of the reference and disturbance signal for constant and periodic signals driven by the tracking error is included in the controller. The augmented system comprising of the nonlinear plant and the internal model is stabilized using output feedback. Initially an output feedback stabilizer gain is obtained from the linearized model of the augmented system. To ensure the stability of the closed loop nonlinear system the closed loop poles of the linearized model are varied so that the loop gain is less than one (small gain theorem) [13]. The paper is organized as follows. The spacecraft model is discussed in section II. Section III is devoted to the problem statement. Controller design methodology is discussed in section IV. Simulation results are discussed in section V. The paper concludes in section VI with some remarks.

II. SPACECRAFT MODEL

The mathematical model of a rigid spacecraft is given by the following kinematic and dynamic equation discussed in [3].

$$\dot{q} = T(q)\omega \tag{1}$$

$$J\dot{\omega} = -\omega \times J\omega + \tau = -S(\omega)J\omega + \tau \tag{2}$$

Where (1) represents the kinematic equation and (2) represents the rigid body dynamics. In (1), $q \in R^3$ represents the Modified Rodrigues parameter describing the spacecraft attitude with respect to an inertial frame, given by

$$q(t) = \hat{k}\tan\left(\frac{\varphi(t)}{4}\right), (0 \le \varphi \le 2\pi)\,\text{rad} \tag{3}$$

Where $\hat{k}(t)$ and $\varphi(t)$ are the Euler eigenaxis and eigenangle. The Jacobian matrix $T \in R^{3x3}$ for the Modified Rodrigues parameter is given by

$$T(q) = \frac{1}{2}\left[\left(\frac{1-q^T q}{2}\right)I_{3x3} + S(q) + qq^T\right] \tag{4}$$

In (2) $\omega \in R^3$ is the angular velocity of the spacecraft in a body-fixed frame, $J \in R^{3x3}$ is the spacecraft's constant inertia matrix, $\tau \in R^3$ is the control torque input, $S \in R^{3x3}$ is the skew symmetric matrix representing the cross product operation.

$$S(\omega) = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix} \tag{5}$$

(1) and (2) are combined to form the nonlinear state space model of the spacecraft given by

$$\underbrace{\begin{pmatrix} \dot{q} \\ \dot{\omega} \end{pmatrix}}_{\dot{X}_P} = \underbrace{\begin{pmatrix} 0 & f_1(q) \\ 0 & f_2(\omega) \end{pmatrix}}_{f_P(X_P)} \begin{matrix} q \\ \omega \end{matrix} + \underbrace{\begin{pmatrix} 0 \\ J^{-1} \end{pmatrix}}_{B_P} \tau \tag{6}$$

$$q = C_P X_P$$

Where $f_1(q) = T(q)$ and $f_2(\omega) = -J^{-1}S(\omega)J$.

The nonlinear kinematic and dynamic equation given by (1) and (2) are linearized using small disturbance theory [14] given by

$$\underbrace{\begin{pmatrix} \Delta\dot{q} \\ \Delta\dot{\omega} \end{pmatrix}}_{\Delta\dot{X}_P} = \underbrace{\begin{pmatrix} f_{\Delta 1} & f_{\Delta 2} \\ 0 & J^{-1}f_{\Delta 3} \end{pmatrix}}_{A_P} \underbrace{\begin{pmatrix} \Delta q \\ \Delta\omega \end{pmatrix}}_{\Delta X_P} + \underbrace{\begin{pmatrix} 0 \\ J^{-1} \end{pmatrix}}_{B_P} \tau \tag{7}$$

Where $f_{\Delta 1} = f(\omega, q)$, $f_{\Delta 2} = f(q)$ and $f_{\Delta 3} = f(\omega, I)$

(2)

III. PROBLEM STATEMENT

The objective of this paper is to design an attitude controller such that the spacecraft follows a desired trajectory $q_d(t)$. Let $\tilde{q}(t)$ represent the error i.e. $\tilde{q}(t) = q_d(t) - q(t)$. The tracking objective is accomplished if

$$\lim_{t\to\infty} \tilde{q}(t) = 0 \tag{8}$$

in the presence of reference and disturbance signals and plant parameter variations as long as the closed-loop system is stable.

IV. CONTROLLER DESIGN

The controller consists of an Internal Model and an output feedback stabilizer as shown in Fig.1.

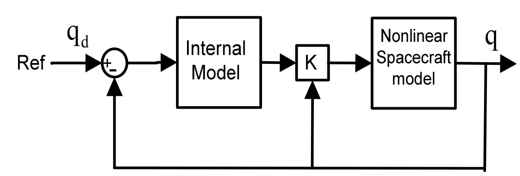

Fig. 1. Internal Model principle based spacecraft attitude control

A. Internal Model

The Internal Model is a linear time invariant system which generates copy of the reference and disturbance signal. Its state space model is given by

$$\begin{aligned}\dot{X}_{IM} &= A_{IM}X_{IM} + B_{IM}\tilde{q}\\ Y_{IM} &= C_{IM}X_{IM} + D_{IM}\tilde{q}\end{aligned} \tag{9}$$

In the steady state (9) generates a copy of the reference and disturbance signal such that the disturbance affecting the plant is cancelled at the output and the plant output tracks the reference input. The controllers which do not include an Internal Model will not have zero tracking error.

B. Augmented System

The augmented system comprising of the nonlinear plant and the internal model from (6) and (9) is given by the following state space model.

$$\begin{pmatrix}\dot{X}_P\\ \dot{X}_{IM}\end{pmatrix} = \begin{pmatrix}A_P & 0\\ -B_{IM}C_P & A_{IM}\end{pmatrix}\begin{pmatrix}X_P\\ X_{IM}\end{pmatrix} + \begin{pmatrix}B_P & 0\\ 0 & B_{IM}\end{pmatrix}\begin{pmatrix}\tau\\ q_d\end{pmatrix} \tag{10}$$

C. Stabilization of Augmented System

The stabilization of the augmented system is achieved in two steps. Firstly the linearized augmented system is stabilized using pole placement technique. Since the pole placement technique gives state feedback controller, an output feedback strategy was derived by estimating unmeasured states from the output using a 'differentiator' [15-16]. Secondly the gains of output feedback are varied to ensure stability of the nonlinear augmented system.

The output feedback controller which is stabilizing the augmented linearized model is given by

$$\tau = K\begin{bmatrix} q\\ X_{IM}\end{bmatrix} \tag{11}$$

D. Small Gain Theorem

The nonlinear system given in (6) may be expressed as a linear system with nonlinear perturbation as

$$\dot{X}_P = A_P X_P + \psi_P\left(X_P\right) + B_P\tau \tag{12}$$

where $\psi_P\left(X_P\right) = f_P\left(X_P\right) - A_P X_P$ is the nonlinear perturbation. This perturbed linear system is represented in Fig. 2. The output feedback gain is determined using pole placement. The location of the pole is varied so that the loop gain of the closed-loop system in Fig.2 is less than one. Let $\gamma_1 = \left\|G\right\|$ where $G = \left(sI - A_P + B_P K\right)^{-1}$.

Let $\gamma_2 = \dfrac{\left\|\psi_P\left(X_P\right)\right\|}{\left\|X_P\right\|}$ then the nonlinear closed-loop system is stable if $\gamma_1\gamma_2 < 1$

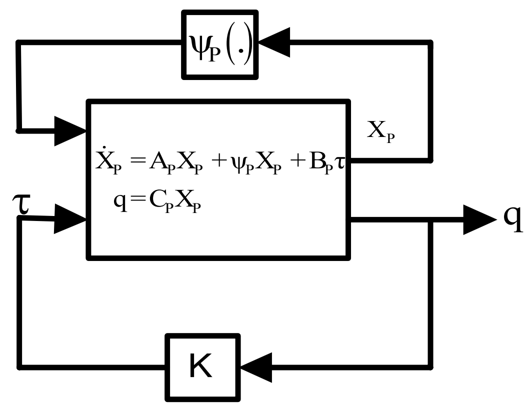

Fig. 2. Small gain theorem implementation.

V. SIMULATION RESULTS

To demonstrate the application of the proposed scheme, we illustrate the results obtained in this section. The nonlinear spacecraft model given by (1 & 2) is simulated with the control signal given by (11). The inertia matrix is selected to be

$$J = \begin{pmatrix}20 & 1.2 & 0.9\\ 1.2 & 17 & 1.4\\ 0.9 & 1.4 & 15\end{pmatrix} \tag{13}$$

The desired input (attitude trajectory) is selected as

$$q_d\left(t\right) = \hat{k}_d \tan\left(\frac{\varphi_d\left(t\right)}{4}\right) \tag{14}$$

with $\hat{k}_d = \frac{1}{2}\left(\cos\left(0.2t\right) \quad \sin\left(0.2t\right) \quad \sqrt{3}\right)^T$ and $\varphi_d\left(t\right) = \pi$ rad. The periodic disturbance is selected as $\left[\cos\left(0.2t\right) \quad \cos\left(0.2t\right) \quad \sin\left(0.2t\right)\right]^T$.

The Internal model for the reference and disturbance signal is computed using (9).The internal model principle based attitude tracking for first reference input is shown in Fig. 3. and the error in this case is shown in Fig. 4. Similarly the attitude tracking for second and third reference input is shown in Fig. 5 and Fig. 7 and the error in their respective cases is shown in Fig. 6 and Fig. 8.

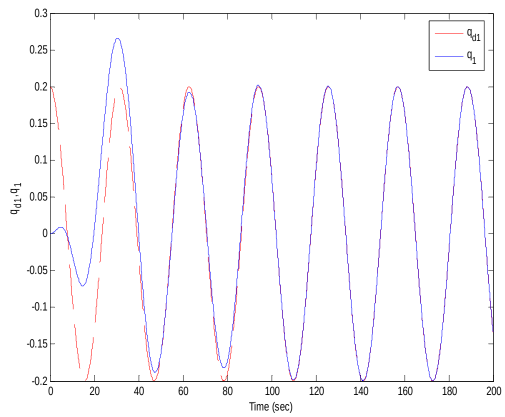

Fig. 3. Internal Model principle based attitude tracking $q_{d1}(t)$, $q_1(t)$

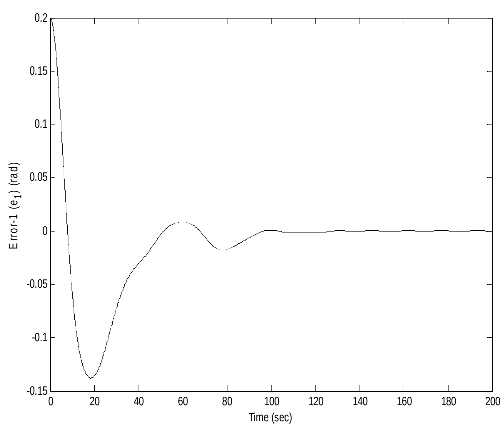

Fig. 4. Attitude tracking error $e_1(t)$

It is evident from the plots that the internal model principle based attitude controller is able to track the attitude parameters asymptotically. Initially, while the system is improving the model, the error is high, however, it drops rapidly soon after and the tracking is stable. The tracking is ensured even when the plant is subjected to large perturbation as long as the closed-loop system is stable. However, when the internal model is not matched to the reference and disturbance signal it doesn't track. It is interesting to note that the nonlinear system is able to track the reference input, that is internal model principle applies to a class of nonlinear system. However, unlike the linearized model the nonlinear system is more sensitive to model perturbations in the internal model.

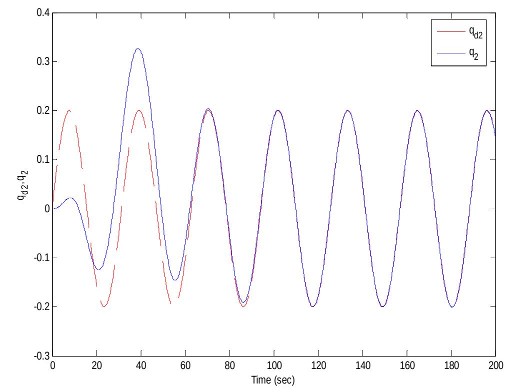

Fig. 5. Internal Model principle based attitude tracking $q_{d2}(t)$, $q_2(t)$

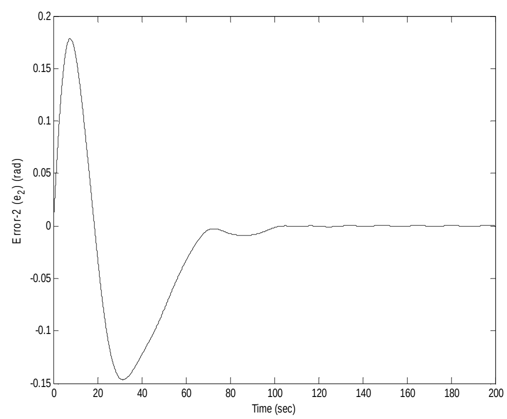

Fig. 6. Attitude tracking error $e_2(t)$

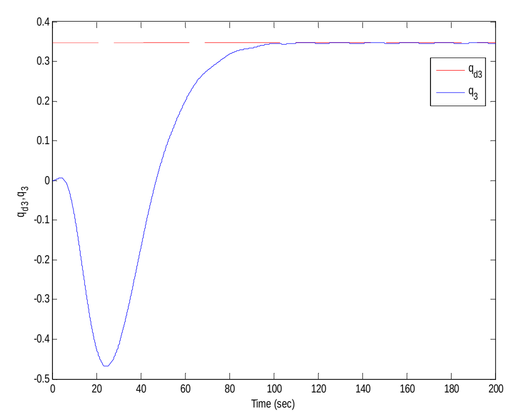

Fig. 7. Internal Model principle based attitude tracking $q_{d3}(t)$, $q_3(t)$

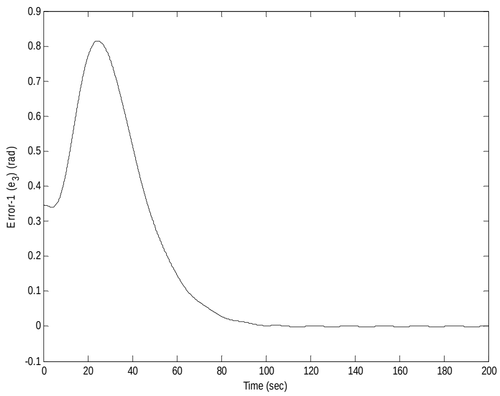

Fig. 8. Attitude tracking error $e_3(t)$

VI. Conclusion

In this paper an internal model principle based design methodology for tracking of spacecraft attitude is shown to achieve zero tracking error asymptotically. The internal model principle applies to a class of nonlinear system including spacecraft attitude control system. An internal model of reference and disturbance signal for constant and periodic signals driven by the tracking error is included in the controller. The augmented system comprising of the nonlinear plant and the internal model is stabilized using output feedback. Initially an output feedback stabilizer gain is obtained from the linearized model of the augmented system. To ensure the stability of the closed loop nonlinear system the closed loop poles of the linearized model are varied so that the loop gain is less than one thereby utilizing small gain theorem. A fixed controller can achieve tracking by including an internal model without the need for computationally burdensome adaptation scheme.

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