EE 200- Digital Logic Circuit Design
2.6 Canonical and Standard Forms.

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Entry Questions

- Can we represent a Boolean function in more than one form?
- What for look for to get a better Boolean function form?

Objectives

1. Canonical and Standard Forms
   - Canonical Forms
   - Standard Forms

Canonical Forms

- Canonical forms: each term of the Boolean function must contain all the variables.

x	y	z	Function f_1
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

- $f_1 = x'y'z + xy'z' + xyz$

Minterms and Maxterms

Expressing combinations of 1's and 0's with binary variables (normal form x or complement form x')

- For n-variables, we have 2^n compilations.
- Example: for variables x and y, we have $x'y'$, xy', $x'y$, and xy.
- Each variable is primed " ' " if it represent a "0", otherwise it is unprimed.

Sum of Minterms

- Each of these AND terms is called *minterm* or *standard product*.

Minterms for Three Binary Variables

x	y	z	Minterms Term	Minterms Designation
0	0	0	$x'y'z'$	m_0
0	0	1	$x'y'z$	m_1
0	1	0	$x'yz'$	m_2
0	1	1	$x'yz$	m_3
1	0	0	$xy'z'$	m_4
1	0	1	$xy'z$	m_5
1	1	0	xyz'	m_6
1	1	1	xyz	m_7

x	y	z	Function f_1
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

- $f_1 = x'y'z + xy'z' + xyz = m_1 + m_4 + m_7$

Product of Maxterms

x	y	z	Function f_1
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Maxterms for Three Binary Variables

			Maxterms	
x	**y**	**z**	**Term**	**Designation**
0	0	0	$x + y + z$	M_0
0	0	1	$x + y + z'$	M_1
0	1	0	$x + y' + z$	M_2
0	1	1	$x + y' + z'$	M_3
1	0	0	$x' + y + z$	M_4
1	0	1	$x' + y + z'$	M_5
1	1	0	$x' + y' + z$	M_6
1	1	1	$x' + y' + z'$	M_7

- if $f_1' = x'y'z' + x'yz' + x'yz + xy'z + xyz'$
  $f_1 = (x+y+z)(x+y'+z)(x+y'+z')(x'+y+z')(x'+y'+z)$
  $f_1 = M_0 M_2 M_3 M_5 M_6$
- Each grouped OR term is called *maxterm* or *standard sum*.

Minterms and Maxterms Conversions

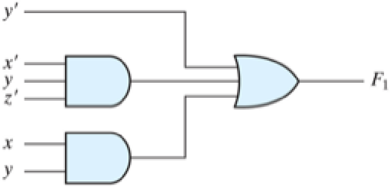
- Canonical form: expressing a Boolean function using sum of minterms or product of maxterms.
- Minterms whose sum defines the Boolean function are those which give 1's in the truth table.
- Maxterms whose product defines the Boolean function are those which give 0's in the truth table.
- Maxterm M_j is the complement of minterm m_j.
- $f_1 = x'y'z + xy'z' + xyz = m_1 + m_4 + m_7 = \sum(1, 4, 7)$
- $f_1 = M_0 M_1 M_2 M_5 M_6 = \prod(0, 1, 2, 5, 6)$

Standard Forms

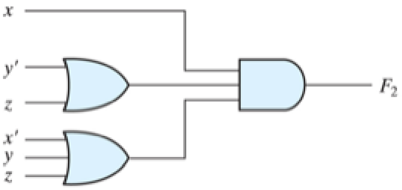
- Standard forms: the terms that form the function may contain one, two, or any number of variables.

- Sum of products
  $F_1 = y' + xy + x'yz'$

- Product of sums
  $F_2 = x(y' + z)(x' + y + z')$



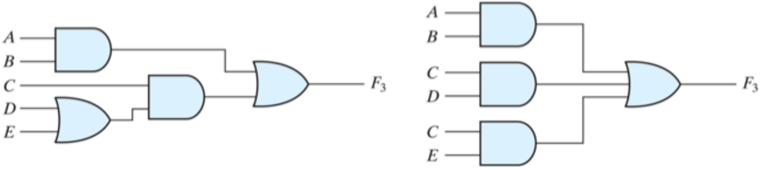
(a) Sum of Products



(b) Product of Sums

Minterms and Maxterms Conversions

- A nonstandard form Boolean function, $F_3 = AB + C(D + E)$ can be written in standard form as, $F_3 = AB + CD + CE$.



(a) $AB + C(D + E)$ (b) $AB + CD + CE$

- A two-level implementation is preferred: produces the least amount of delay through the gates when the signal propagates from the inputs to the output.

Conversion to Canonical Forms

Express $F = A + B'C$ as a sum of minterms

- each term should have all variables.
- 1st term missing B & C.

  $= A(B + B') = AB + AB'$

  $= AB(C + C') + AB'(C + C') = ABC + ABC' + AB'C + AB'C'$
- 2nd term missing A.

  $= B'C(A + A') = AB'C + A'B'C$
- $F = ABC + ABC' + AB'C + AB'C' + AB'C + A'B'C$

  $= m_1 + m_4 + m_5 + m_6 + m_7 = \sum(1, 4, 5, 6, 7)$

Conversion to Canonical Forms

Express $F = A + B'C$ as a product of maxterms

- convert to OR terms $(A + B')(A + C)$.
- 1st term missing C, add CC'.

  $A + B' = A + B' + CC'$

  $= (A + B' + C)(A + B' + C')$
- 2nd term missing B, add BB'.

  $A + C = A + C + BB' = (A + B + C)(A + B' + C)$
- $F = (A + B' + C)(A + B' + C')(A + B + C)(A + B' + C)$

  $= m_2 + m_3 + m_0 = \prod(0, 2, 3)$

Summary

1. Canonical and Standard Forms
   - Canonical Forms
   - Standard Forms

Next Lecture

- Gate-Level Minimization.