

Homomorphic Encryption (HE)Defⁿ

An HE solution consists of four components: KeyGen, Enc, Dec, and Evaluation

- $\text{KeyGen}(\lambda) \rightarrow (Pk, Sk)$: Given encryption parameter λ , it generates pair Pk, Sk
- $\text{Enc}(Pt, Pk) \rightarrow Ct$: Given Pk , it encrypts a plaintext Pt into ciphertext Ct
- $\text{Dec}(Sk, Ct) \rightarrow Pt$: Given Sk , it decrypts the ciphertext Ct into Pt
- * - $\text{Evaluate}(Pk, \Pi, Ct_1, Ct_2, \dots) \rightarrow (Ct'_1, Ct'_2, \dots)$: Given the Pk , the input ciphertexts (Ct_1, Ct_2, \dots) and the computing function Π , it will perform Π on Ct_1, Ct_2, \dots and produces Ct'_1, Ct'_2, \dots

Defⁿ (Correctness)

Generally HE is correct for operation Π if it correctly decrypts ciphertexts with the following prop.

1- HE is correct if it always retrieves a pt that has not been evaluated

$$\rightarrow \Pr[\text{Dec}(sk, \text{Enc}(Pk, Pt)) = Pt] = 1$$

2- HE correctly decrypts ciphertexts evaluated on Π

$$\Pr[\text{Dec}(sk, \text{Evaluate}(Pk, \Pi, Ct_1, Ct_2, \dots)) = \Pi(Pt_1, Pt_2, \dots)] = 1$$

ExRSA

Recall $Pk(e, n)$ and $Sk(d)$

$$\text{Enc}(x) = x^e \bmod n = Ct_1$$

$$\text{Enc}(y) = y^e \bmod n = Ct_2$$

$$\text{Evaluate}(Pk, x, Ct_1, Ct_2) = x^e \bmod n \times y^e \bmod n = (x \cdot y)^e \bmod n = \text{Enc}(x \cdot y)$$

$$\text{dec}(\text{Enc}(x \cdot y)) = \text{X} \cdot \text{Y}$$

