Lecture 4

Thursday, September 5, 2024 7:45 AM

- $|0\rangle$ state 0
- $|1\rangle$ state 1

} z axis

- $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
- $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

} X axis

- $|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$
- $|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$

} y axis

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}|0\rangle + \cancel{\frac{1}{\sqrt{2}}|1\rangle} \quad ①$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}|0\rangle - \cancel{\frac{1}{\sqrt{2}}|1\rangle} \quad ②$$

$\oplus$

$$|+\rangle + |-\rangle = \frac{2}{\sqrt{2}}|0\rangle$$

$$|0\rangle = \frac{\sqrt{2}}{2}(|+\rangle + |-\rangle)$$

$$= \frac{2}{2\sqrt{2}}(|+\rangle + |-\rangle)$$

$$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

$$|1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\pi/6}|1\rangle\right)$$

$$P(|0\rangle) = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

$$P(|1\rangle) = \left|\frac{e^{i\pi/6}}{\sqrt{2}}\right|^2 = \frac{1}{\sqrt{2}}^{2} \left|e^{i\pi/6} \cdot e^{-i\pi/6}\right|$$

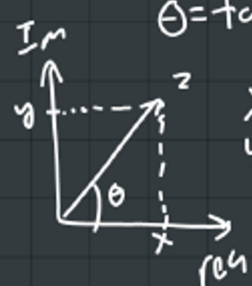
$$= \left|\frac{1}{\sqrt{2}}\right|(1) = \frac{1}{2}$$

Review of complex numbers

$z = x + iy$ $\quad$ $z = re^{i\theta}$

$\Re(z) = x$ $\quad$ $r = \sqrt{x^2 + y^2}$

$\Im(z) = y$ $\quad$ $\theta = \tan^{-1}\frac{y}{x}$



$x = r\cos\theta$

$y = r\sin\theta$

$e^{i\theta} = \cos\theta + i\sin\theta$

- Complex conjugate

$z = x + iy$

$z^* = x - iy$

$|z| = r$

$|z|^2 = r^2 = zz^*$

$|0\rangle \rightsquigarrow \dfrac{\sqrt{3}}{2}|0\rangle + \dfrac{1}{2}|1\rangle$

measurement

$\to P(|0\rangle) = \frac{3}{4}$

$\to P(|1\rangle) = \frac{1}{4}$

$A(\sqrt{2}\,|0\rangle + i|1\rangle)$

$P(|0\rangle) = |A\sqrt{2}|^2 = 2A^2$

$P(|1\rangle) = |Ai|^2 = (Ai)(-Ai) = A^2$

$1 = 2A^2 + A^2 \Rightarrow A^2 = \frac{1}{3}$

$A = \pm\frac{1}{\sqrt{3}}$

$A = \frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}}(\sqrt{2}\,|0\rangle + i|1\rangle)$

$P(|0\rangle) = \frac{2}{3}$ $P(|1\rangle) = \frac{1}{3}$

Measurement of $\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$

1) Z-axis (computational basis)

$P(|0\rangle) = \frac{3}{4} \quad P(|1\rangle) = \frac{1}{4}$

??

2) X-axis (<u>Fourier</u> basis)

$P(|+\rangle) \quad P(|-\rangle)$

$|\psi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$ as $|+\rangle$ & $|-\rangle$

$$|\psi\rangle = \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)\right) + \frac{1}{2\sqrt{2}}(|+\rangle - |-\rangle)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}}|+\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|-\rangle + \frac{1}{2\sqrt{2}}|+\rangle - \frac{1}{2\sqrt{2}}|-\rangle$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}|+\rangle + \frac{\sqrt{3}-1}{2\sqrt{2}}|-\rangle$$

$$|\psi\rangle = \frac{\sqrt{3}+1}{2\sqrt{2}}|+\rangle + \frac{\sqrt{3}-1}{2\sqrt{2}}|-\rangle$$

$$P(|+\rangle) = \frac{3+2\sqrt{3}+1}{8}$$

$$P(|-\rangle) = \frac{3-2\sqrt{3}+1}{8}$$

3) y-axis

$|0\rangle = \frac{1}{\sqrt{2}}(|i\rangle + |-i\rangle) \quad |1\rangle = \frac{-i}{\sqrt{2}}(|i\rangle - |-i\rangle)$

$$|\psi\rangle = \frac{\sqrt{3}-i}{2\sqrt{2}}|i\rangle + \frac{\sqrt{3}+i}{2\sqrt{2}}|-i\rangle$$

4) any basis

$$|a\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{i}{2}|1\rangle$$

$$|b\rangle = \frac{i}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

$$|0\rangle = \left(|a\rangle - \frac{i}{2}|1\rangle\right)\frac{2}{\sqrt{3}}$$

$$|1\rangle = \frac{2}{\sqrt{3}}\left(|b\rangle - \frac{i}{2}|0\rangle\right)$$

$$|1\rangle = \frac{2}{\sqrt{3}}\left(|b\rangle - \frac{i}{2}\left(\frac{2}{\sqrt{3}}\left(|a\rangle - \frac{i}{2}|1\rangle\right)\right)\right)$$

$$= \frac{2}{\sqrt{3}}\left(|b\rangle - \frac{2}{\sqrt{3}}\left(\frac{i}{2}|a\rangle + \frac{1}{4}|1\rangle\right)\right)$$

$$|0\rangle = \frac{3-i}{4}|a\rangle + \frac{\sqrt{3}(1-i)}{4}|b\rangle$$

$$|\psi\rangle = |0\rangle$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|0\rangle \quad P = \frac{1}{2}$$