COE 466 Quantum Architecture and Algorithms

Lecture 25

Variational Quantum Algorithms II

References:

Introduction to Variational Quantum Algorithms “https://arxiv.org/pdf/2402.15879

https://qiskit.org/textbook/ch-applications/vqe-molecules.html

https://qiskit.org/textbook/ch-applications/qaoa.html

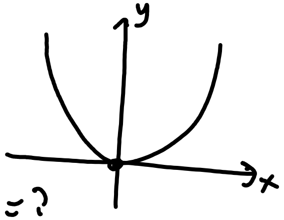
Combinatorial Optimization Problems

COP is a family of problems that involve finding an optimal set objects

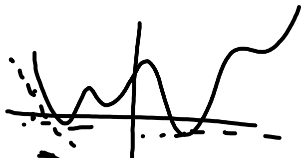
$$\max_{x \in S} \underline{C(x)}$$

x is the solution
$C(x)$ is the cost of solution
S is the set of all solutions

$$\min_x \; f(x) = x^2$$



$g(x) = ?$

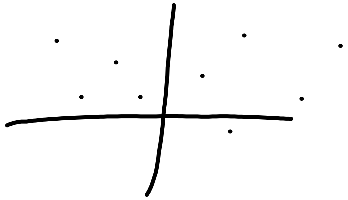


$\bar{g}(x)$

In general, $C(x)$ can look like

$$c(x) = \sum_{Q,\bar{Q} \in [n]} w_{Q,\bar{Q}} \prod_{i \in Q} x_i \prod_{i \in \bar{Q}} (1 - x_i)$$

Q is set of items assigned value 1
$\bar{Q}$ " " " " " " 0



min or max $C(x)$

$\min \equiv -\max$

find using QAOA

$$c(x_{min}) \leq c(x^*) \leq \alpha\, c(x_{min})$$

$\alpha = 1, 2, \ldots$

$$c(x_{max}) \geq c(x)$$

Combinatorial Optimization Problems

- Example: MAX-CUT problem

A graph $G=\{V,E\}$ V: set of vertices

E: set of edges

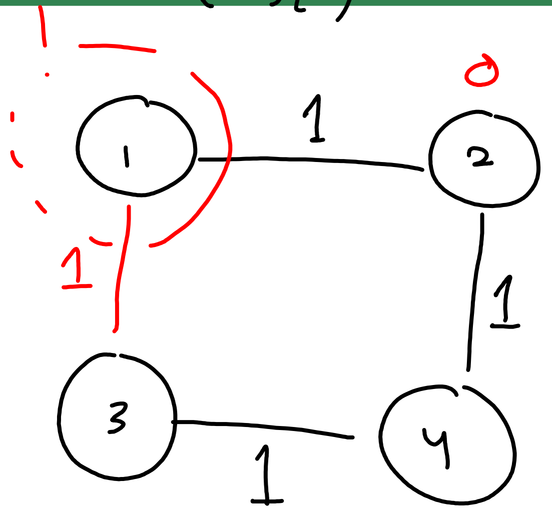
w_{ij}: weight on edge i,j

MAX-CUT

$$\max \quad C(X) = \sum_{j=1}^{n} \sum_{i=1}^{n} w_{ij} X_i (1 - X_j) \qquad X_i \in \{0,1\}$$

$= 4$

$G=\{V,E\}$



$w_{12} = 1$

$w_{13} = 1$

$w_{24} = 1$

$w_{34} = 1$

$\{1\}, \{2,3,4\}$

$\{2\}, \{1,3,4\}$

$\{3\}, \{$

8

$\cancel{4}$

$\cancel{4}$ 8

$\{1,4\}, \{2,3\}$

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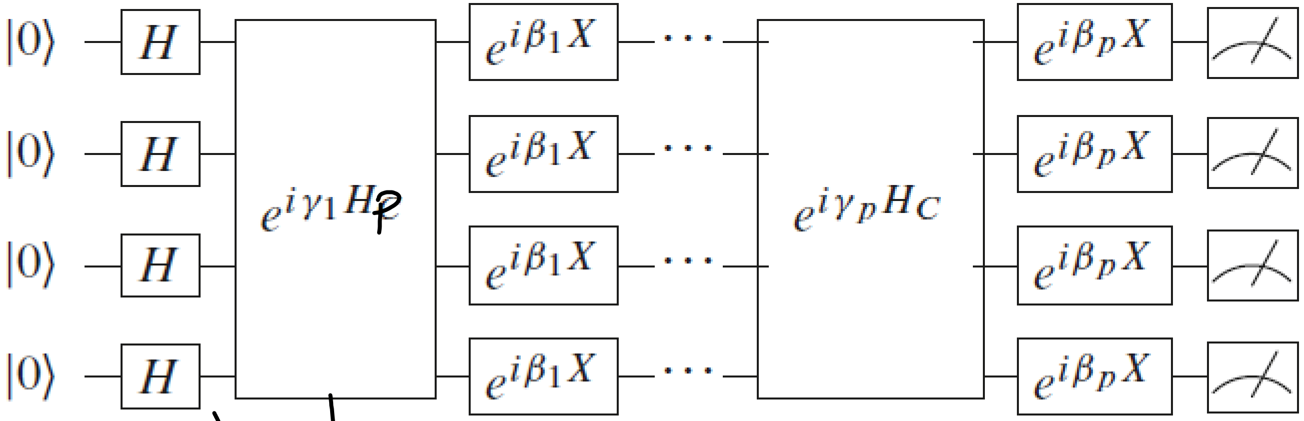
Quantum Approximation Optimization Algorithm (QAOA)

- QAOA is a VQA that find approximate solution to the COP by using a variational circuit parametrized by $U(\beta,\gamma)$ to prepare a quantum state $|\psi(\beta,\gamma)\rangle$
- The goal is to find β & γ s.t. $|\psi(\beta,\gamma)\rangle$ encodes the solution
- $U(\beta) = e^{-i\beta H_B}$, H_B is mix Hamiltonian
- $U(\gamma) = e^{-i\gamma H_P}$, H_P is the Problem Ham

- Algorithms

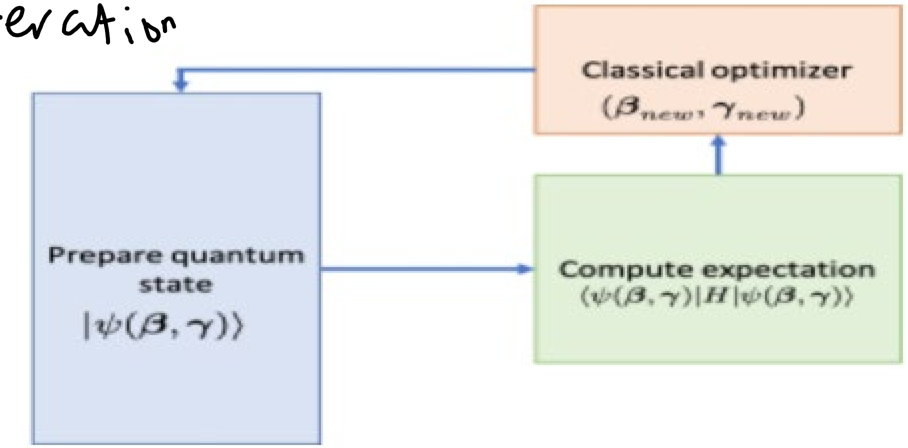


what is H_p?

Cost/prob H

mixer H

p iteration



$n=4$

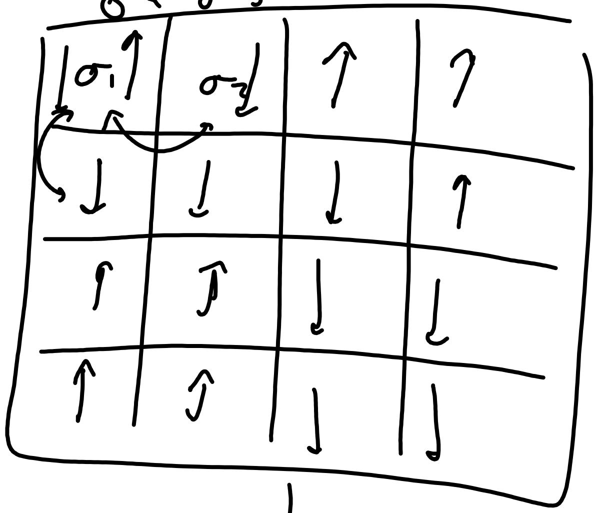
$$H_B = \sum_{i=1}^{n} \sigma_x^{(i)} = (X_0 \otimes I_1 \otimes I_2 \otimes I_3) + (X_1) + (X_2) + (X_3) \longrightarrow \text{standard}$$

$$H_P = \frac{1}{2} \sum_{i,j \in E} \left(I - \sigma_z^{(i)} \sigma_z^{(j)} \right)$$

Ising Model

$\sigma_i \in \{-1, +1\}$

$\sigma_i \quad \sigma_j$



G_0 ground state

$$\min \; H = -\sum_{\langle i,j \rangle} J \sigma_i \sigma_j - \sum_j h \sigma_j$$

J in the state-ste inter

h in external field

$$C(x) = \sum_{i,j \in E} x_i (1 - x_j)$$

$$x_i = \frac{1 - Z_i}{2}$$

$$C(Z) = \sum_{i,j \in E} \frac{1}{2} (1 - Z_i Z_j)$$

$Z_i = \pm 1$

$x_i \in \{0, 1\}$

$$X_i = \frac{1 - Z_i}{2}$$

$Z_i = -1, \; x_i = 1$

$Z_i = 1, \; x_i = 0$