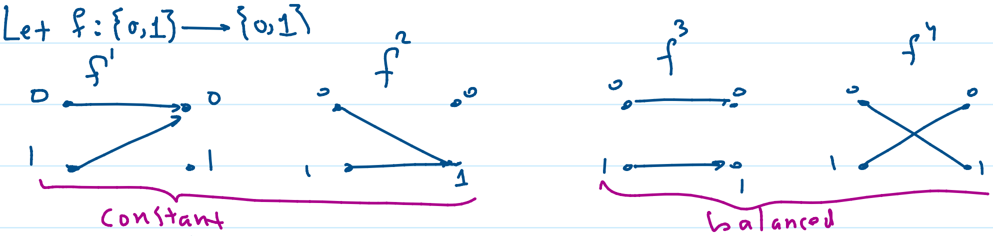
Lecture 15

Thursday, October 17, 2024 9:00 AM

Parity Problem

- Let $f : \{0,1\} \longrightarrow \{0,1\}$



or $f(x) = b_x$

The parity problem is to determine whether $b_0 \oplus b_1 =$ 0 or 1

even parity bit
1011 1
1001 0

odd parity
1011 0
1001 1

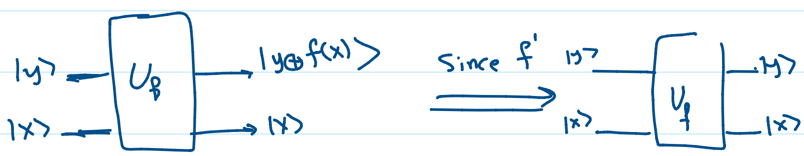
Classically, we need 2 evaluations of $f(x)$ to determine the parity (constant or balanced)

1^{st} time evaluate $f(0) = 0$

2^{nd} " " $f(1) = 0$

The "query complexity" is 2

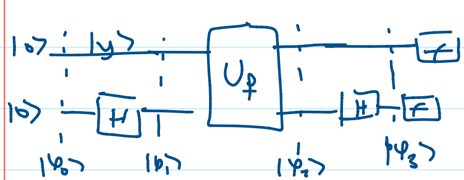
1^{st} trial



X we need to evaluate twice

2^{nd} trial

- We want to use superposition



$|\varphi_0\rangle = |x\rangle|y\rangle$

$|\varphi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|y\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|y\rangle + |1\rangle|y\rangle\right)$

$|\varphi_2\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|y \oplus f(0)\rangle + |1\rangle|y \oplus f(1)\rangle\right)$

$|\varphi_3\rangle = |0\rangle \quad \frac{1}{\sqrt{2}}\left(|y \oplus f(0)\rangle + |y \oplus f(1)\rangle\right)$

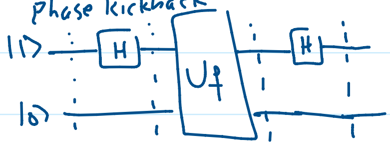
$|0\rangle \quad \frac{1}{\sqrt{2}}\left(|f(0)\rangle + |f(1)\rangle\right)$

We need to run "at least" 2

$|0\rangle \quad \frac{1}{\sqrt{2}}(|1(0)\rangle + |1\dots\rangle)$ we need to run at least 2

3rd trial

Phase kickback



$|\varphi_0\rangle = |0\rangle|1\rangle$

$|\varphi_1\rangle = |0\rangle|-\rangle = |0\rangle\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle - |0\rangle|1\rangle)$

$|\varphi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\oplus f(0)\rangle - |0\rangle|1\oplus f(0)\rangle)$

$= \frac{1}{\sqrt{2}}(|0\rangle|f(0)\rangle - |0\rangle|\overline{f(0)}\rangle)$

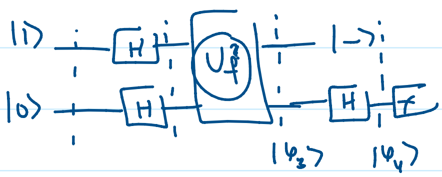
$= |0\rangle\frac{1}{\sqrt{2}}(|f(0)\rangle - |\overline{f(0)}\rangle)$ assume f^1

$= |0\rangle\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$= |0\rangle|-\rangle$

$|\varphi_3\rangle = |0\rangle|1\rangle \longrightarrow$ This will be the results for all f^1, f^2, f^3, f^4

4th trial



$|\varphi_0\rangle = |0\rangle|1\rangle$

$|\varphi_2\rangle = |+\rangle|-\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$

$|\varphi_3\rangle = \frac{1}{2}(|0\rangle|0\oplus f(0)\rangle - |0\rangle|1\oplus f(0)\rangle + |1\rangle|0\oplus f(1)\rangle - |1\rangle|1\oplus f(1)\rangle)$

$= \frac{1}{2}(|0\rangle|f(0)\rangle - |0\rangle|\overline{f(0)}\rangle + |1\rangle|f(1)\rangle - |1\rangle|\overline{f(1)}\rangle)$ assume f^1

$= \frac{1}{2}(|0\rangle|0\rangle - |0\rangle|1\rangle + |1\rangle|0\rangle - |1\rangle|1\rangle)$

$= \frac{1}{\sqrt{2}}((|0\rangle|-\rangle) + (|1\rangle|-\rangle))$

$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|-\rangle$

$|\varphi_4\rangle = |0\rangle|-\rangle$

How about f^2

$|\varphi_3\rangle = \frac{1}{2}(|0\rangle|1\rangle - |0\rangle|0\rangle + |1\rangle|1\rangle - |1\rangle|0\rangle)$

$= \frac{1}{\sqrt{2}}((|0\rangle - |-\rangle) + (|1\rangle - |-\rangle))$

$= \frac{-1}{\sqrt{2}}(|0\rangle + |1\rangle)|-\rangle$

$= \frac{1}{\sqrt{2}}\left((|0\rangle - |-\rangle) + (|1\rangle - |-\rangle)\right)$

$= \frac{-1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)|-\rangle$

$|\varphi_4\rangle = -|0\rangle|-\rangle$

How about f^3

$|\varphi_3\rangle = \frac{1}{2}\left(|0\rangle|0\rangle - |0\rangle|1\rangle + |1\rangle|1\rangle - |1\rangle|0\rangle\right)$

$= \frac{1}{\sqrt{2}}\left(|0\rangle|-\rangle - |1\rangle \quad |-\rangle\right)$

$= |-\rangle \qquad |-\rangle$

$|\varphi_4\rangle = |1\rangle|-\rangle$

How f^4

$|\varphi_4\rangle = |1\rangle|-\rangle$