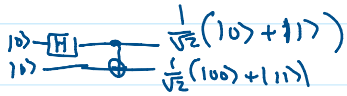
Lecture 10

Tuesday, October 1, 2024 8:57 AM

Universal gates

Classically, NAND is a universal gate

- Required components
  1. Superposition: H
  2. Entanglement: CNOT



  3. Complex amplitudes: Y, S, T
  4. Contains more than Clifford group {CNOT, H, S} (see Gottesman-Knill Theorem*)

Examples of universal Gate sets:

1. {CNOT, all single qubit gates} → $U_{\phi,\theta,\gamma}$
2. {CNOT, H, T}
3. {CNOT, $R_{\frac{\pi}{8}}$, S}
4. {Toffoli, H, S}
5. {H, "almost" any two-qubit gate}

Solovay-Kitaev Theorem*

With any universal gate set you can approximate an n-qubit quantum gate with $\Theta(2^n \log^c(1/\epsilon))$ gates

Chapter 6 Entanglement & Quantum Protocols

$$|\psi\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$$ (superposition state form)

$$= \underbrace{\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right)}_{=|+\rangle} \otimes \underbrace{\left(\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right)}_{|-\rangle}$$

$= |+\rangle \otimes |-\rangle$ not entangled (product state)

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$ $C_0 = \alpha^2$ $C_3 = \beta^2$ $C_1 = \alpha\beta_0$ $C_2 = \alpha\beta = 0$

Entangled state

Maximally Entangled states

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$
$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Partially Entangled States

$$|\varphi\rangle = \frac{\sqrt{3}}{2\sqrt{2}}|00\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|01\rangle + \frac{\sqrt{3}}{4}|10\rangle + \frac{1}{4}|11\rangle$$

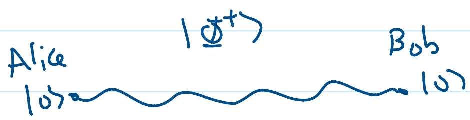
Measure left qubit

- $|0\rangle$ with $\left|\frac{\sqrt{3}}{2\sqrt{2}}\right|^2 \cdot 2 = 2\left|\frac{3}{8}\right| = \frac{3}{4}$ and qubit collapses $|0\rangle\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
- $|1\rangle$ with $\left|\frac{\sqrt{3}}{4}\right|^2 + \left|\frac{1}{4}\right|^2 = \frac{3}{16} + \frac{1}{16} = \frac{1}{4}$ " " " $|1\rangle\left(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle\right)$

If we firs measure left qubit

- Get $|0\rangle$, measuring right qubit: $|0\rangle \to \frac{1}{2}$, $|1\rangle \to \frac{1}{2}$
- Get $|1\rangle$, " " " " $\to \frac{3}{4}$, $\to \frac{1}{4}$

EPR Paradox and Local Hidden Variable



CHSH