Lecture 9

30/9/2020

Ex 4.1)

Assume a particle can be four. ($\mathbb{C}^4$)

$$|\psi\rangle = \begin{bmatrix} -3-i \\ -2i \\ i \\ 2 \end{bmatrix} \qquad P(x_2) = \frac{|i|^2}{(4.3589)^2}$$

$$|\psi| = 4.3589 \qquad P(x_2) = 0.052624$$

* Ket prop.

① Addition

$$|\psi\rangle = c_0|x_0\rangle + c_1|x_1\rangle + \cdots + c_{n-1}|x_{n-1}\rangle$$

$$|\psi'\rangle = c_0'|x_0\rangle + c_1'|x_1\rangle + \cdots + c_{n-1}'|x_{n-1}\rangle$$

$$|\psi\rangle + |\psi'\rangle = (c_0 + c_0')|x_0\rangle + \cdots + (c_{n-1} + c_n')|x_n\rangle$$

b) Scalar mult.

$$c|\psi\rangle = [cc_0, cc_1, \ldots, cc_{n-1}]^T$$

Ex

what $|\psi\rangle + |\psi\rangle$?

$$|\psi\rangle + |\psi\rangle = [2c_0, 2c_1, \ldots, 2c_{n-1}]^T$$

$|\phi\rangle = 2|\psi\rangle$

$P(x_i) = P(x_i)$ ¿ both cases?

* We can mult. a ket by any (complex) number and get the physical state

* We will work normalized $|\psi|$

$$\frac{|\psi\rangle}{||\psi\rangle|}$$ which has a length of 1

* if $|\psi\rangle$ is normalized

$$P(x_i) = |c_i|^2$$

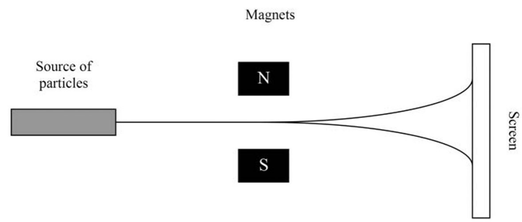

Figure 4.3. The Stern–Gerlach experiment.

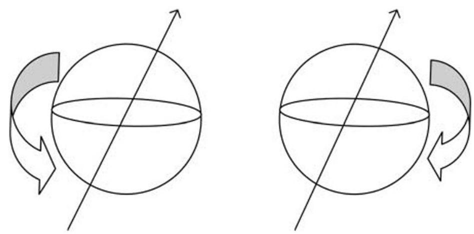

Figure 4.4. Particles with spin.

* There are two basic spin states for vertical axes: up spin $|\uparrow\rangle$, down spin $|\downarrow\rangle$

$$|\psi\rangle = c_0 |\uparrow\rangle + c_1 |\downarrow\rangle$$

amp of finding a particle with up state

Ex 4.1.4

$$|\psi\rangle = (3-4i)|\uparrow\rangle + (7+2i)|\downarrow\rangle$$

length $\sqrt{|3-4i|^2 + |7+2i|^2} = 8.8318$

$$P(\uparrow) = \frac{|3-4i|^2}{(8.8318)^2} = \frac{25}{78}, \quad P(\downarrow) = \frac{53}{78}$$

* Inner product of stase space gives us transition amplitude : determines how likely the start state of system (before a measure) will change to another (end) state (after measurement)

$$|\psi\rangle = \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_{n-1} \end{bmatrix} \text{ and } |\psi'\rangle = \begin{bmatrix} c_0' \\ \vdots \\ c_{n-1}' \end{bmatrix}$$

normalized

bra-ket

* The end state will be a row vector whose cordinates are complex conjugate of $|\psi'\rangle$ coord—

* This is call bra $\langle \psi' |$

$$\langle \psi' | = |\psi'\rangle^{\dagger} = [\bar{c}_0, \bar{c}_1, \ldots, \bar{c}_{n-1}]$$

* Transition amplitude

$$\langle \psi' | |\psi\rangle = \langle \psi' | \psi \rangle$$

$$= [\bar{c}_0, \bar{c}_1, \ldots, \bar{c}_{n-1}] \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_{n-1} \end{bmatrix}$$

$$= \bar{c}_0 c_0 + \bar{c}_1 c_1 + \cdots + \bar{c}_{n-1} c_{n-1}$$

$$\underset{\text{start}}{|\psi\rangle} \xrightarrow{\langle \psi' | \psi \rangle} |\psi'\rangle$$

* What does it mean to have $\langle \varphi | \psi \rangle = 0$?

once an electron in in the up state, it will never chage to down state.

* Orthogonal state are mutually exclusive alternatives

$$|\psi\rangle \begin{array}{l} \xrightarrow{b_0} |b_0\rangle \\ \xrightarrow{b_1} |b_1\rangle \\ \quad \vdots \\ \xrightarrow{b_{n-1}} |b_{n-1}\rangle \end{array}$$

$b_0 = \langle b_0 | \psi \rangle$

$b_i = \langle b_i | \psi \rangle$

$\sum |b_i|^2 = 1$

$\{ b_0, b_1, \dots, b_{n-1} \}$ orthonormal basis

* In other words, the result of measurement will be one of either state, but not in a superposition of them

$$A = \bar{A}^T = A^\dagger$$

Ex

$$|\psi\rangle = [3, 1-2i]^T$$

$$\langle\psi| = [3, 1+2i]$$

Ex 4.1.6

$$|\psi\rangle = \frac{\sqrt{2}}{2}[1, i]^T, \quad |\phi\rangle = \frac{\sqrt{2}}{2}[i, -1]$$

$$\langle\phi|\psi\rangle = \frac{\sqrt{2}}{2}[-i, -1]\frac{\sqrt{2}}{2}\begin{bmatrix}1\\ i\end{bmatrix}$$

$$= -i$$

Sec 4.2

Observables are physical quantities that can be observed at each state

Post. 4.2.1

To each physical observable there corrosponds a hermitian operator

* Think of an observable as a linear operater Ω that can be applied to $|\psi\rangle$ as. $\Omega|\psi\rangle$

Ex $|\psi\rangle = [-1, -1-i]^T$

$$\Omega = \begin{bmatrix} -1 & -i \\ i & 1 \end{bmatrix}$$

$$\Omega|\psi\rangle = [i, -1-2i]^T$$

Post. 4.2.2

For each observable, the ansers are the eigenvalues of the observable Ω

$$P = \begin{bmatrix} x_0 & 0 & 0 & \cdots & 0 \\ 0 & x_1 & 0 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & & \cdots & x_{n-1} \end{bmatrix}$$

Sec 4.3 Measurin

Pos. 4.3.1

Let Ω be an observable of $|\psi\rangle$

If the result of measuring Ω is the eigenvalue λ, the state after measurement will always be an eigenvector corrospondiy to λ

Ex 4.3.1

Material for Quiz 2

Chapter 5 Quantum Architecture

Def 5.1.1 A bit is a unit of infor. describing a two-dimensional classical system

* Boolean logic : T , F
* switch: on, off

* $|0\rangle = \begin{array}{c} \\ \end{array}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{array}{c} 0 \\ 1 \end{array}$, $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{array}{c} 0 \\ 1 \end{array}$

Def 5.1.2
A quantum bit (qubit) is a unit of information describing two-dim. quantum system

$\begin{array}{c} 0 \\ 1 \end{array}\begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$ $|c_0|^2 + |c_1|^2 = 1$

* Once we measure a qubit, it becomes a bit

$$[c_0 \ \ c_1]^T \begin{cases} \xrightarrow{|c_0|^2} \begin{bmatrix}1\\0\end{bmatrix} \\[2ex] \xrightarrow{|c_1|^2} \begin{bmatrix}0\\1\end{bmatrix}\end{cases}$$

$$\begin{bmatrix}c_0\\c_1\end{bmatrix} = c_0\begin{bmatrix}1\\0\end{bmatrix} + c_1\begin{bmatrix}0\\1\end{bmatrix}$$

$$= c_0|0\rangle + c_1|1\rangle$$