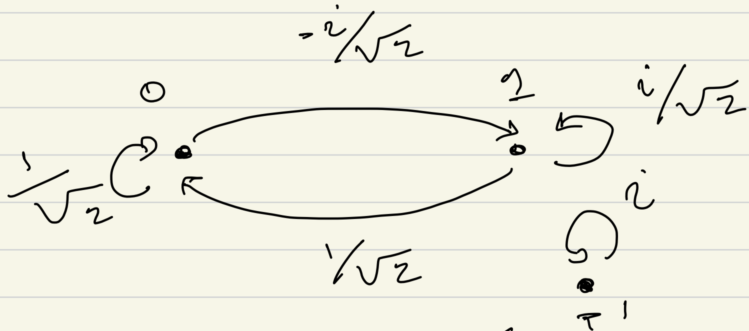
Lecture 7

21/9/2020

Ex



$$X = [1/\sqrt{3},\ 2i/\sqrt{15},\ \sqrt{2/3}]^T \neq 1$$

$$X^2 = [1/3,\ 4/15,\ 2/5]^T = 1$$

* Rather than use doubly stoc. matrices, we use unitary matrix

$$U = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -i/\sqrt{2} & i/\sqrt{2} & 0 \\ 0 & 0 & i \end{bmatrix} \quad |U^2[i,j]| =$$

$$|U[i,j]|^2 = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{array}{l} \text{doubly} \\ \text{stoch} \end{array}\right)$$

$$X = \left[\frac{1}{\sqrt{3}},\ \frac{2i}{\sqrt{15}},\ \sqrt{\frac{2}{3}}\right]$$

$$UX = \begin{bmatrix} \dfrac{5+2i}{\sqrt{30}} \\ \dfrac{-2-\sqrt{5}\,i}{\sqrt{30}} \\ \sqrt{\frac{2}{5}}\ i \end{bmatrix}$$

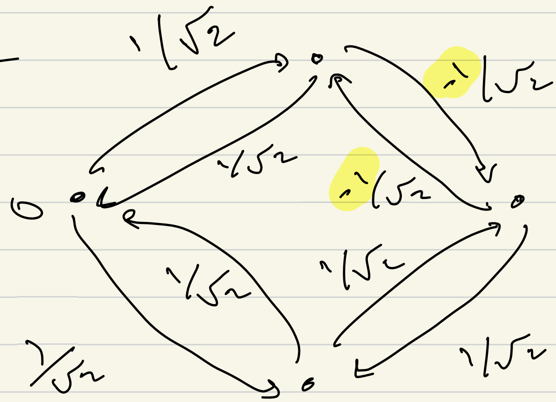
$$U \cdot U^{\dagger} = I$$

$$U^{\dagger} = \begin{bmatrix} 1/\sqrt{2} & i/\sqrt{2} & 0 \\ 1/\sqrt{2} & -i/\sqrt{2} & 0 \\ 0 & 0 & -i \end{bmatrix}$$

- U takes a state from $t \to t+1$
- $U^{\dagger}$ " " " $t \to t-1$

- $V \to UV \to U^{\dagger}UV \to I_3 V = V$

Ex 3.3.2



$$A = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

$$x_1 = [1 \quad 0 \quad 0 \quad 0]^T$$

$$Ax_1 = [0 \quad 1/\sqrt{2} \quad 1/\sqrt{2} \quad 0]^T = x_2 \qquad = 1$$

$$Ax_3 = [1 \quad 0 \quad 0 \quad 0]$$

Ex

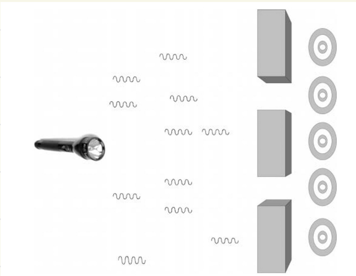
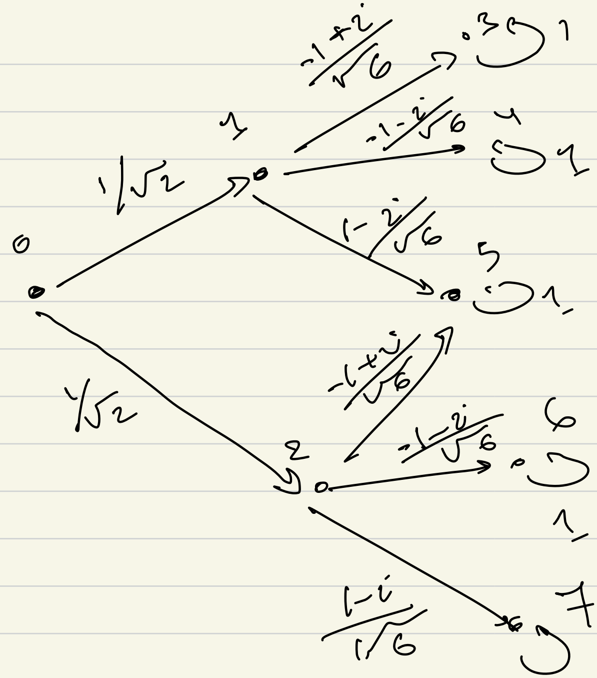


Figure 3.2. Double-slit experiment with photons.



$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1+i}{\sqrt{6}} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1-i}{\sqrt{6}} & \frac{-1+i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1-i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

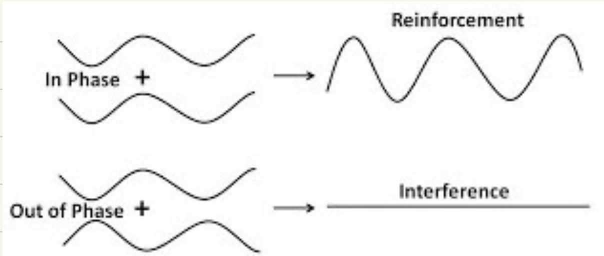
$$|P[i,j]|^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1+i}{\sqrt{12}} & \frac{-1+i}{\sqrt{6}} & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-1-i}{\sqrt{12}} & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{-1+i}{\sqrt{6}} & \frac{1-i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 \\ \frac{-1-i}{\sqrt{12}} & 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 0 & 1 & 0 \\ \frac{-1+i}{\sqrt{12}} & 0 & \frac{1-i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|P^2_{[i,j]}|^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/6 & 1/3 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1/6 & 1/3 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 0 & 1 & 0 & 0 \\ 1/6 & 0 & 1/3 & 0 & 0 & 0 & 1 & 0 \\ 1/6 & 0 & 1/3 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$1/\sqrt{2}\left(\frac{1-i}{\sqrt{6}}\right) + 1/\sqrt{2}\left(\frac{-1+i}{\sqrt{6}}\right) =$$

$$\frac{1-i}{\sqrt{12}} + \frac{(-1+i)}{\sqrt{12}} = 0$$



* Naive prob. intrepretation is not adequate
* $X = [c_0 \ c_1, \ldots, c_{n-1}]^T \in \mathbb{C}^n$, says that the prob. a photon being in position k is $|c_k|^2$.
* Matter, we need to think that a photon is in the same position at the same time
* A photon is not a single position, rather it is in many position, a <u>Superposition</u>
* Prob. of measuring an object = position $|c_i|^2$