Lecture 6

16/9/2020

* 3.2 Probabilistic System

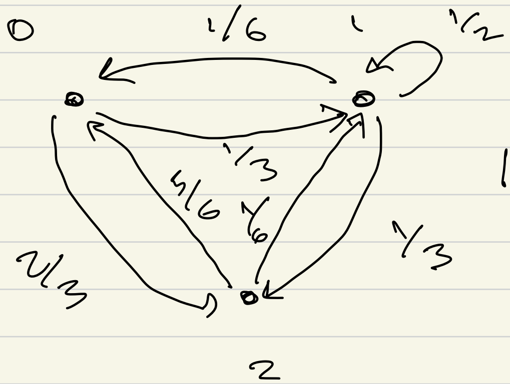
* Let's focus on a single marble system
* We will use a graph with 3-vertices

let the state $\begin{array}{ccc} [1/5, & 3/10, & 1/2]^T \\ 0 & 1 & 2 \end{array}$

$$\left(1/5 + 3/10 + 1/2 = \underline{1}\right)$$

* To model the system, we will use weighted graphs where edges are labled: represent the probability that the marble moves from one node to another

Ex



$$M = \begin{bmatrix} 0 & 1/6 & 5/6 \\ 1/3 & 1/2 & 1/6 \\ 2/3 & 1/3 & 0 \end{bmatrix} \begin{matrix} =1 \\ =1 \\ =1 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 1 \end{matrix}$$

Doubly stoch. matrix

Let $X = [1/6 \;\; 1/6 \;\; 2/3]^T$

$$MX = Y_1 = \begin{bmatrix} 0 & 1/6 & 5/6 \\ 1/3 & 1/2 & 1/6 \\ 2/3 & 1/3 & 0 \end{bmatrix} \begin{bmatrix} 1/6 \\ 1/6 \\ 2/3 \end{bmatrix}$$

X: Initial State, Prob. Position of marble at t_0

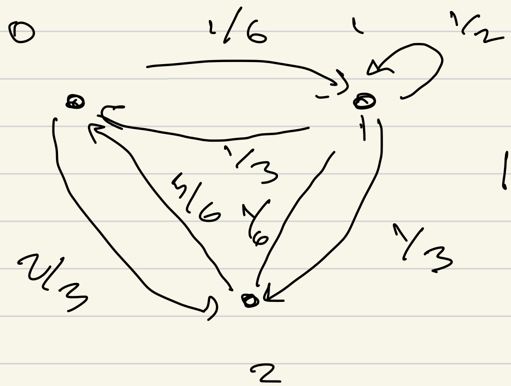
Y_1: t_1

$$= \begin{bmatrix} 21/36 \\ 9/36 \\ 6/36 \end{bmatrix}$$

$$M^2 X = Y_2 \longrightarrow$$

$$M^k X = Y_k$$

$$(MX)^T = X^T M^T$$



$$M = \begin{bmatrix} 0 & 1/6 & 5/6 \\ 1/3 & 1/2 & 1/6 \\ 2/3 & 1/3 & 0 \end{bmatrix} \begin{matrix} =1 \\ =1 \\ =1 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 1 \end{matrix}$$

Sl st ch

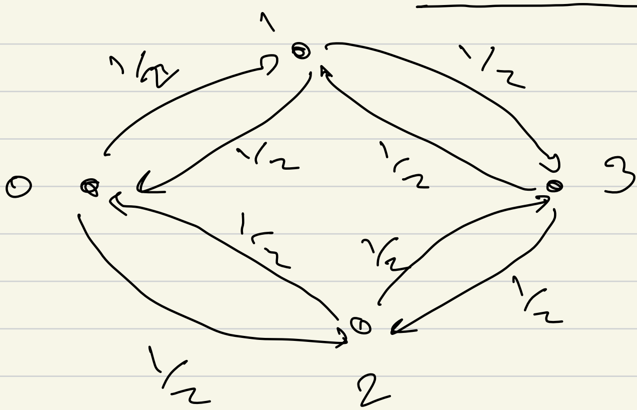
$$M^T = \begin{bmatrix} 0 & 1/3 & 2/3 \\ 1/6 & 1/2 & 1/3 \\ 5/6 & 1/6 & 0 \end{bmatrix}$$

$X^T M^T = Y^T$ t to $t-1$

* Let M and N be two doubly stoch. matrices that repre G_M & G_N, resp.

$$(M * N)_{[ij]} = \sum_{k=0}^{n-1} M_{[i,k]} N_{[k,j]}$$

Ex Stochastic Billiard Ball (Complex)



$$A = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 0 & 1/2 & 1/2 & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$$

$$AX = \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \\ 0 \end{bmatrix}, \quad A^2X = \begin{bmatrix} 1/2 \\ 0 \\ 0 \\ 1/2 \end{bmatrix}$$

$$A^3X = \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \\ 0 \end{bmatrix}$$

Ex

Prob. double-slit Ex

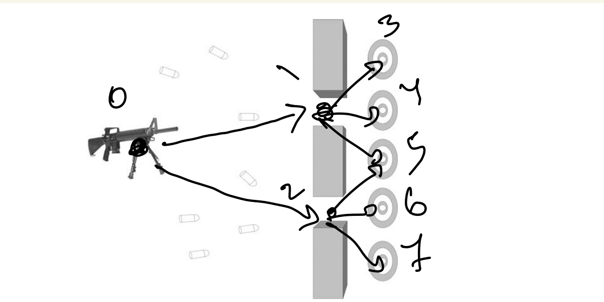
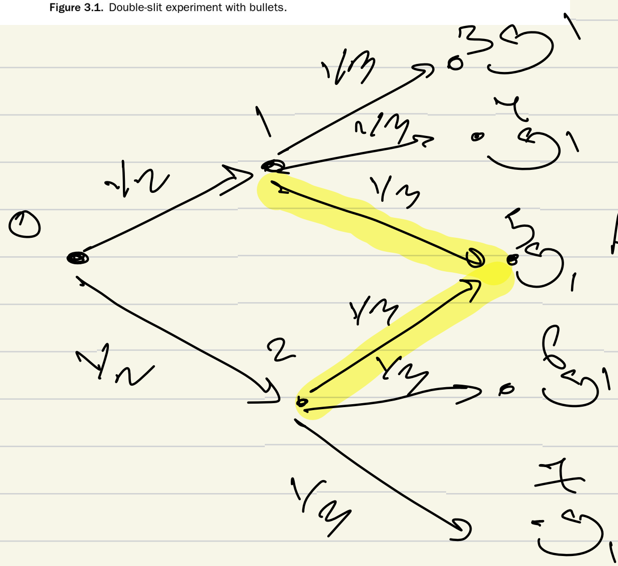


Figure 3.1. Double-slit experiment with bullets.



3-Col

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \\ 1/2 & 0 & 0 & 0 & 0 & \\ 1/2 & 0 & 0 & 0 & 0 & \\ 0 & 1/3 & 0 & 1 & 0 & \cdots \\ 0 & 1/3 & 0 & 0 & 1 & \\ 0 & 1/3 & 1/3 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \end{bmatrix}$$

Not good?

$$B^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/6 & 1/3 & 0 \\ 1/6 & 1/3 & 0 \\ 1/3 & 1/3 & 1/3 \\ 1/6 & 0 & 1/3 \\ 1/6 & 0 & 1/3 \end{bmatrix}$$

$$X = [1\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$$

$$B^2 X = [0, 0, 0, 1/6, 1/6, 1/3, 1/6, 1/6]^T$$

* 3.3 Quantum System

* Quantum system uses complex number
* So, what?

  - Real number prob. can only increase in addition

    if $0 \leq P_1, P_2 \leq 1$, than

    $(P_1 + P_2) \geq P_1$ & $(P_1 + P_2) \geq P_2$

$$|c_1| = \sqrt{a^2+b^2}$$

$$|c_1|^2 = a^2+b^2$$

- In Complex numbers, this is not nec.

correct $c_1, c_2 \in \mathbb{C}$

$$|c_1+c_2|^2 \neq |c_1|^2$$

Ex

$c_1 = 5+3i$, $c_2 = -3-2i$

a)

$$c_1+c_2 = 2+i$$

$$|c_1|^2 = 34 \quad , \quad |c_2|^2 = 13$$

$$|c_1+c_2|^2 = 5$$

$$5 \neq 13$$

b) $c_1 = 5+3i$, $c_2 = -5-3i$

→ This is called interference