

Lecture 13

14/10/20



+ Hadamard gate

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \rightarrow \text{Hadamard matrix}$$

- Hadamard gate creates in a superposition state

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{|0\rangle} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$$

$$= \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$$

* Pauli matrices

$$\sigma_a = \begin{pmatrix} 0 & \gamma_{ab} \\ \gamma_{ab} & 0 \end{pmatrix}$$

$$\gamma_{ab} = \begin{cases} +1 & \text{if } a=b \\ 0 & \text{if } a \neq b \end{cases}$$

$$X = X^T \quad X^T X = I = X^2 \quad a, b = 1, 2, 3$$

$$\sigma_1 = \sigma_X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = X \xrightarrow{\text{NOT}} \begin{matrix} +1 \\ -1 \end{matrix}$$

$$\sigma_2 = \sigma_Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = Y \quad \begin{matrix} +1 \\ -1 \end{matrix}$$

$$\sigma_3 = \sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad + \mid -$$

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \quad T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

Ex 5.4.3 Page 159 $\text{H}\omega$

$\pm \sqrt{N\Omega}$ State

$$\sqrt{N\Omega} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & + \\ 1 & - \end{bmatrix}$$

$$\sqrt{N\Omega} \neq \sqrt{N\Omega}^+$$

- why is it called $\sqrt{\text{NOT}}$?

$$\sqrt{\text{NOT}} + \sqrt{\text{NOT}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} + \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} - \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

NOT

(1)

$$\sqrt{N\sigma} \cdot |1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\sqrt{N\sigma} \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

= $-1|0\rangle$

* Measurement gate



* Qubit Representation

$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle$$

$$c_0 = r_0 e^{i\theta_0}$$

$$c_1 = r_1 e^{i\theta_1}$$

$$|\psi\rangle = r_0 e^{i\theta_0}|0\rangle + r_1 e^{i\theta_1}|1\rangle$$

$$e^{i\phi} |\psi\rangle = r_0 |0\rangle + r_1 e^{i(\phi - \phi_0)} \cancel{|1\rangle}$$

$$|r_0|^2 + |r_1|^2 = 1 = |r_0 e^{i\phi_0}|^2 + |r_1 e^{i\phi_1}|^2$$

$$r_0^2 + r_1^2 = 1$$

$$r_0 = \cos(\theta), \quad r_1 = \sin(\theta)$$

$$|\psi\rangle = \cos\theta |0\rangle + e^{i\phi} \sin\theta$$

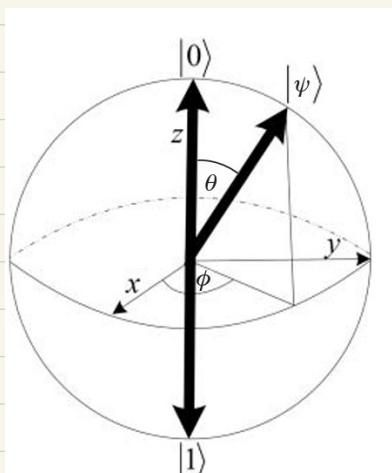


Figure 5.6. Bloch sphere.

Two parameters

θ latitude

ϕ longitude

$$x = \cos\theta \sin\phi$$

$$y = \sin\theta \sin\phi$$

$$z = \cos\theta$$

$$0 \leq \theta \leq 2\pi \quad 0 \leq \phi \leq \pi$$

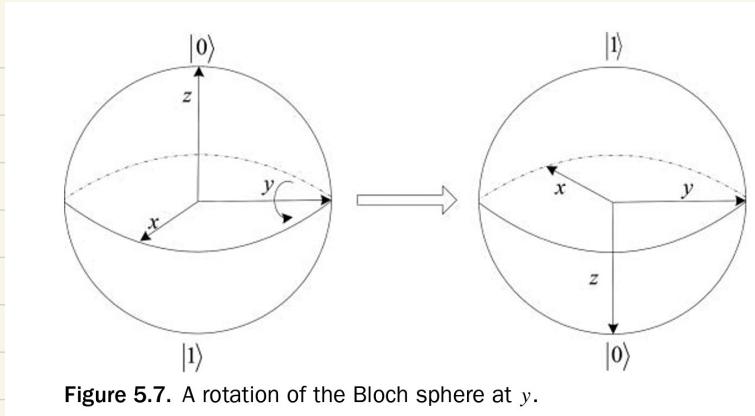
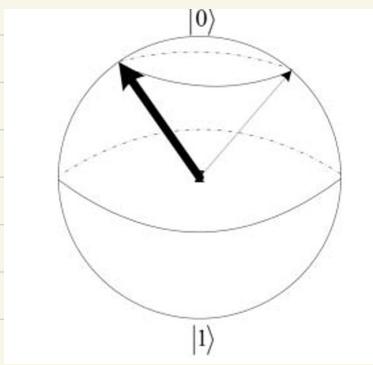


Figure 5.7. A rotation of the Bloch sphere at y .

* Phase shift gate

$$R(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$$



$$(\cos(\theta) | 0 \rangle + e^{i\phi} \sin(\theta) | 1 \rangle)$$

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{\theta} \end{bmatrix} \begin{bmatrix} \cos(\theta) \\ e^{i\phi} \sin(\theta) \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ e^{i\phi+\theta} \sin(\theta) \end{bmatrix}$$

Change longitude
only

$$R_x(\theta) = \cos \frac{\theta}{2} I - i \sin \frac{\theta}{2} X$$

$$R_y(\theta) = \cos \frac{\theta}{2} I - i \sin \frac{\theta}{2} Y$$

$$R_z(\theta) = \cos \frac{\theta}{2} I - i \sin \frac{\theta}{2} Z$$

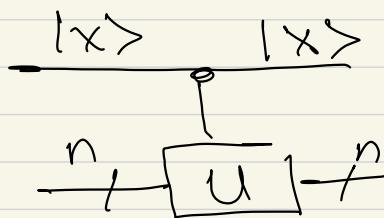
$$R_D(\theta) = \cos \frac{\theta}{2} I - i \sin \frac{\theta}{2} (D_x X + D_y Y + D_z Z)$$

$$D_x = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, D_y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$D_z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

, Controlled-U (C_U)

- Equivalent IF-THEN



- Performs U operation if $Ix>=1$
 - if $Ix>$ is 0, U becomes I

A diagram illustrating a controlled-U operation with a condition. On the left is a circle containing the text 'U ='. To its right is a matrix with four entries: 'a' and 'b' in the top row, and 'c' and 'd' in the bottom row.

$$\begin{matrix} a & b \\ c & d \end{matrix}$$

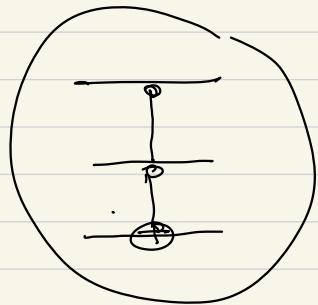
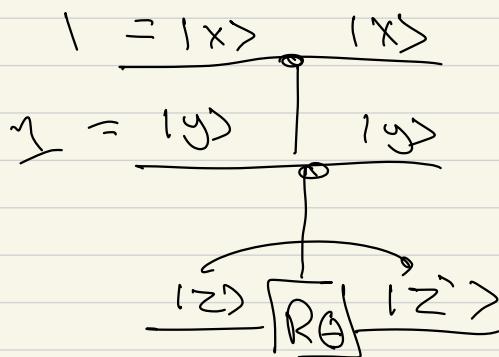
A diagram illustrating a controlled-U operation with a condition and a matrix. On the left is a circle containing the text 'C_U ='. To its right is a matrix with four rows and four columns. The first three rows are identical to the one in the previous diagram: [1 0 0 0], [0 1 0 0], and [0 0 a b]. The fourth row is [0 0 c d].

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \end{bmatrix}$$

* Set of universal quantum gates

$$\{ H, \text{NOT}, R\left(\cos^{-1}\left(\frac{3}{5}\right)\right) \}$$

* Deutsch gate



- Toffoli gate with $R(\theta)$ to change $|z\rangle$

No-Cloning Theorem

No-Hidr