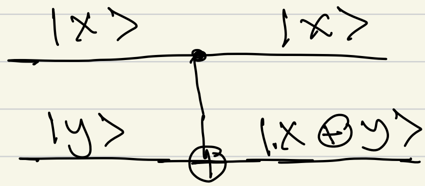
Lecture 12

12/10/20

- CNOT (Controlled-NOT, ^{C}NOT)
  - It takes two inputs, produces two output;



XOR $\oplus$

x	y	
0	0	0
0	1	1
1	0	1
1	1	0

- $|x\rangle$ is a control bit that controls the ou

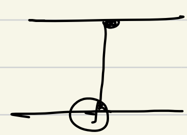
- if $|x\rangle = 0$, $|0 \oplus y\rangle = |y\rangle$
- if $|x\rangle = 1$, $|1 \oplus y\rangle = |\neg y\rangle$

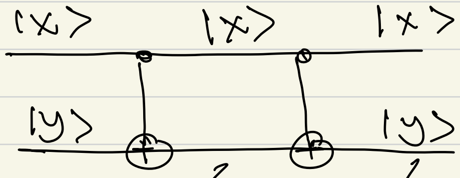
$$|x, y\rangle \longmapsto |x, x \oplus y\rangle$$

$$\begin{array}{c} \\ 00 \\ 01 \\ 10 \\ 11 \end{array}
\begin{array}{c} \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

CNOT is reversibl

CNOT



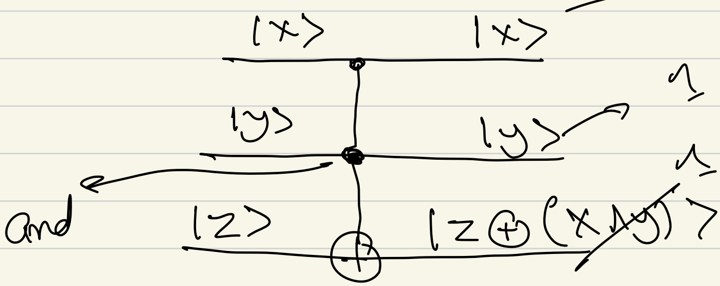


$x \oplus x = 0$

$|x \oplus y\rangle$ $\quad$ $|x \oplus x \oplus y\rangle$

$|0 \oplus y\rangle = |y\rangle$

* Toffoli gate



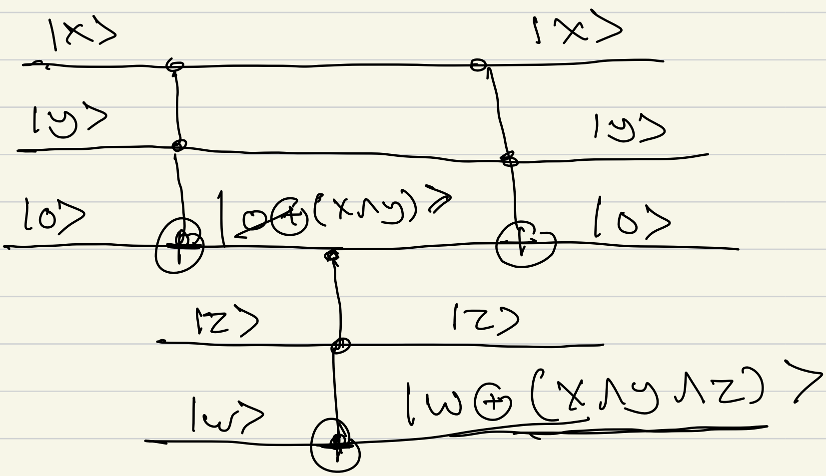
$|1-z\rangle$

- Similar to CNOT, but it has two controlling bits
- $|z\rangle$ is flipped only when both $|x\rangle$ & $|y\rangle$ are $|1\rangle$

- Matrix of Toffoli gate

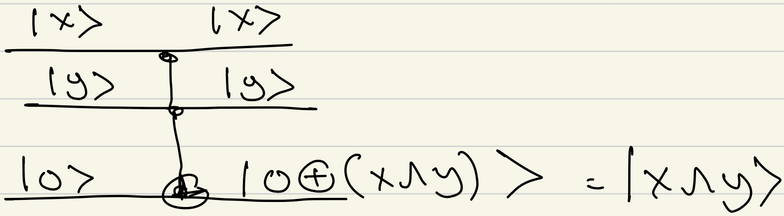
$$
\begin{array}{cc}
 & \begin{matrix} \mathbf{000} & \mathbf{001} & \mathbf{010} & \mathbf{011} & \mathbf{100} & \mathbf{101} & \mathbf{110} & \mathbf{111} \end{matrix} \\
\begin{matrix} \mathbf{000} \\ \mathbf{001} \\ \mathbf{010} \\ \mathbf{011} \\ \mathbf{100} \\ \mathbf{101} \\ \mathbf{110} \\ \mathbf{111} \end{matrix} &
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{array}.
$$

- Can we have a gate with 3 Controling bits?

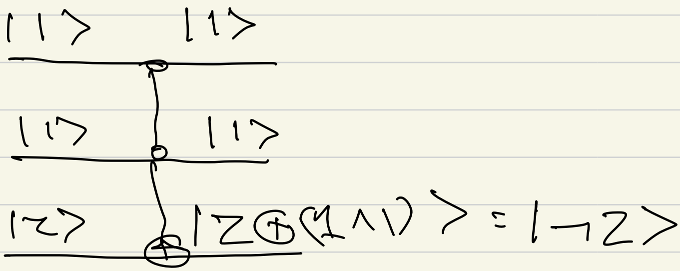


- Toffoli gate is a universal gate

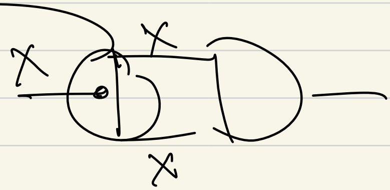
- AND gate

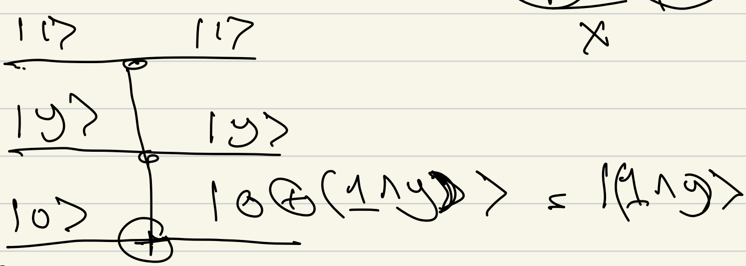


- NOT



- Fan-out

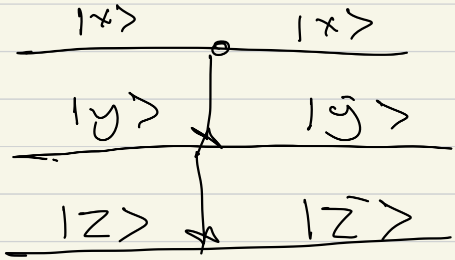




$100 \rightarrow 100$

$110 \rightarrow 111$

* Fredkin gate

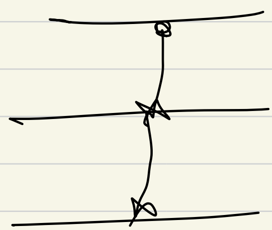


- $|x\rangle$ is a controling bit
- if $|x\rangle = 0$, $|y'\rangle = |y\rangle$ & $|z'\rangle = |z\rangle$
- if $|x\rangle = 1$, $|z'\rangle = |y\rangle$ & $|y'\rangle = |z\rangle$

$$|0, y, z\rangle \longmapsto |0, y, z\rangle$$

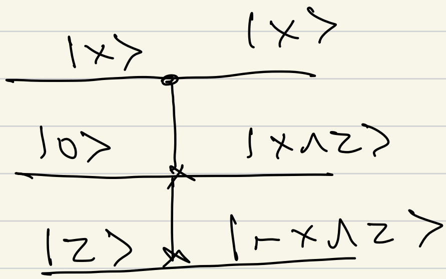
$$|1, y, z\rangle \longmapsto |1, z, y\rangle$$

$$\begin{array}{c c}
 & \begin{array}{cccccccc} \mathbf{000} & \mathbf{001} & \mathbf{010} & \mathbf{011} & \mathbf{100} & \mathbf{101} & \mathbf{110} & \mathbf{111} \end{array} \\
\begin{array}{c} \mathbf{000} \\ \mathbf{001} \\ \mathbf{010} \\ \mathbf{011} \\ \mathbf{100} \\ \mathbf{101} \\ \mathbf{110} \\ \mathbf{111} \end{array} &
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\end{array}.$$



- Fredkin gate is a universal gate

- AND gate



$x\wedge z$

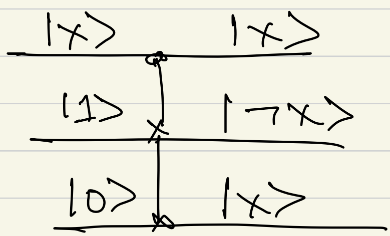
$000 \to 000$

$001 \to 001$

$100 \to 100$

$101 \to 110$

- NOT gate



$010 \to 010$

$110 \to 101$

$x=0$

x

$010 \to 010$

$110 \to 101$

$x=1$

Sec 5.4 Quantum gate

Def 5.4.1 : A quantum gate is simply an operator that acts on qubits. Such an operator will be represented by unitary matrixes

* Example of quantum gates

1 - Identity

2 - NOT gate

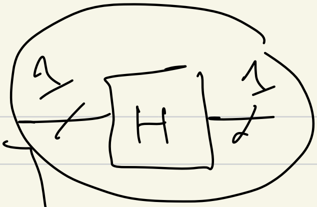
3 - CNOT

4 - Toffoli

5 - Fredkin

6 - Hadamard

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} = \begin{bmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & -1/\sqrt{2}\end{bmatrix}$$



$|1\rangle$ or $|0\rangle$ $\rightarrow \dfrac{|0\rangle + |1\rangle}{\sqrt{2}}$ $\quad \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$