Evaluation of Pipelined Dilated Banyan Switch Architectures for ATM Networks

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Abstract

In the Pipeline Banyan (PB) [1] the reservation cycle in the control plane is made several times faster than payload transmission in data plane. This enables pipelining multiple banyans. It is observed that the ratio of throughput to switching delay (service rate) is relatively low in the PB due to the banyan. For this, we present a scalable pipelined ATM switch architecture employing a family of Dilated Banyan (DB) networks together with their complexity analysis and performance. A DB can be engineered between two extremes: (1) a low-cost banyan with internal and external conflicts, or (2) a high-cost conflict-free fully-connected network with multiple outlets. Between the two extremes lies a family of DBs having different switching delays and throughputs. Increasing the dilation degree reduces path conflicts, which produces noticeable increase in service rate due to increase in throughput and decrease in path delay. Compared to PB, the Pipelined Dilated Banyan (PDB) requires smaller number of data planes for the same throughput, or provides higher throughput for a given number of data planes. Simulation of PDB is carried out under uniform traffic and simulated ATM traffic. We study the switch performance while varying the load, buffer size, and number of data planes. To analyse the robustness of the switch, we show that performance is not degradable under ATM traffic with temporal and spatial burstiness generated using the ON-OFF model. The PDB is scalable with respect to service rate and can be engineered with respect to: (1) cell loss rate, (2) hardware resources, (3) size of buffers, (4) switching delays, and (5) delay incurred to higher priority traffic. The PDB can deliver up to 3.5 times the service rate of the PB with only linear increase in hardware cost.

1 Introduction

Broadband integrated services digital networks (B-ISDN’s) [2, 3, 4] are high speed networks based on the use of asynchronous transfer mode (ATM) as the switching technology. To match the switching speed with the high transmission speed, ATM technology operates at extremely high speed with short fixed-length packets called cells. To support quality of service per traffic source, ATM uses the connection-oriented approach [5, 6]. Statistical multiplexing has been adopted support a wide variety of data rates and isochronous traffic

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sources. Providing acceptable quality-of-service (QoS) for isochronous traffic sources requires minimal cell delay and delay jitter. No link-by-link flow control was planned to ensure minimal delays in buffering and switching. Only preventive actions against cell loss are provided by ATM. At connection set-up, resource are allocated based on currently available resources and active connections. For the above reasons, the cell loss probability (CLP) must be kept extremely low.

We are concerned with the design of a low complexity switch architecture capable of switching bursty traffic at typical rates on the order of 1 Gega cells per second, with a cell loss probability as low as $10^{-8}$. The progress in the field of VLSI technology has brought new design concepts, high performance, high capacity, and low cost. Banyan-based multistage networks are suitable for VLSI implementation because of their modularity, self routing, and low hardware complexity. Several strategies have been suggested to overcome the low throughput of banyan networks [7, 8, 9].

In the Tandem Banyan Switching Fabric (TBSF) [10] the cells are issued to banyans arranged in series. One conflicting cell is misrouted in current banyan and re-issued to next banyan. Thus by properly adjusting the number of banyans the CLP can be made as low as needed. The cost is the relatively large number of banyans and implied propagation delays.

Multi-parallel banyans use vertical connections to shorten propagation delays such as in the Piled Banyan Switching Fabric (PBSF) [11] and Parallel-Tree Switching Fabric (PTBSF) [12] which have no input buffering. In the case of conflict some cells are vertically forwarded to a corresponding switching element in a parallel banyan to reduce the propagation delays due to different switching paths. However, achieving low CLP requires complex hardware and a large number of connections.

The Pipeline Banyan (PB) [1] uses one single control plane for path reservation and a number of data planes for payload transfer. Each time slot consists of a number of reservation slots. In a reservation slot, a successfully self-routed header makes reservation of a path on a free data plane to enable transfer of its payload. An input buffer may be notified to re-submit its cell header in the next slot if its cell (header) cannot continue its self-route because of conflicts. Since the cell header is much shorter than the payload, multiple reservation slots can be done during one payload transfer slot which enables pipelined operations. Thus, PB has low switching delay and relatively high throughput.

In this paper, we investigate pipelined switch architectures that employ a family of Dilated Banyans (DBs). Given the relatively low throughput and large switching delay of banyan our objective is to find an architecture that can provide higher throughput with less switching delays which enables increasing switch service rate. The primary DB was originally proposed [13] for the Burroughs FMP multiprocessor. It was later studied as an unbuffered shuffle-exchange network in [14] and as packet a switch in [15]. We show how the architecture of DBs can be made scalable with respect to throughput and propagation delays. We study pipelined switches employing few DBs for which we evaluate the: 1) switching delays, 2) number of data planes needed to guarantee some CLP, and 3) overall hardware complexity. This will allow us to find some pipelined schemes employing specifically designed DBs that can provide high throughput with reasonable switching delay.

The organization of this paper is as follows. Section 2 presents some background on
related switch architectures. Section 3 presents the topology and complexity analysis of
the proposed pipelined dilated banyan. Section 4 presents evaluation of the proposed
switch architecture under uniform traffic. In section 5 we present performance results of
proposed switch under some ATM traffic mixes. We conclude in Section 6.

2 Background

One fundamental problem to minimize CLP in ATM switches is to find an efficient method
for partitioning the set of head-of-line (HOL) cells into subsets so that the cells within each
subset are free of internal and external conflicts. Each subset of cells can then be switched
out without conflicts by using a separate banyan. A number of proposals have been made
to provide partial solutions. The idea is to issue all HOL cells to one banyan, perform
self-routing, retrieve cells which reach their destinations, and re-issue all unsuccessful cells
to the next banyan, and so on. We call this approach Iterative Conflict Resolution (ICR)
in which a cell loss occurs: 1) at the last banyan in the case of no input buffering, or 2)
at the input buffers in the case of full buffers.

The ICR strategy can be used to provide arbitrary low CLP by selecting an appropriate
number of iterations for a given switch size. Example of switch architectures employing
the ICR method is the TBSF [10] in which cells are applied to banyans arranged in series.
When a conflict occurs within some banyan, one of the cells is routed correctly while the
other is misrouted and marked as such. At the banyan output, those cells that arrive to
each output port without being misrouted are forwarded to corresponding output buffers.
The misrouted cells are applied to the next banyan in series. Cells reaching incorrect
outputs of the last banyan are lost. This architecture can achieve arbitrary low CLP at
the cost of relatively long switching delay.

Multi-parallel banyans use vertical connections among banyans to reduce sequential
propagation delays. The PBSF [11] and PTBSF [12] are two examples. If two cells conflict
within a switching element (SE) one cell is routed to the correct output and the other is
routed to the corresponding SE of a banyan located in the next lower level. Each SE has
two horizontal inputs and outputs as well as two vertical inputs and outputs. By avoiding
misrouting of cells, vertical links shorten the path delay. Cell loss can occur in: (1) SEs
of arbitrary banyan of PBSF, and (2) in SEs of lowest banyan of PTBSF. Hence, the
throughput of the piled banyan saturates at 98% under full load, while the PTBSF can
be scaled up to achieve arbitrary low CLP. Both switches have short propagation delay
but they use a relatively large amount of hardware and interconnections.

A recent switch that addresses the above issues is the PB [1]. It consists of one single
control plane and a number of data planes. In a reservation slot, headers of all HOL
cells are self-routed to their destinations within the control plane. In the case of conflict
between two headers one of them is dropped and a back-pressure mechanism is used to
notify the corresponding input buffer to re-submit its cell header in the next slot. Headers
that successfully reach their destinations make path reservations on a data plane to
transfer their payloads from input buffers to destination output buffers. Since cell header
is much shorter than payload, multiple reservations can be done during payload trans-
mition time. Self-routing of cell headers without payload contributes in shortening the
reservation time. The reservation time is several times faster than payload transmission
time which enables pipelined operations. The separation between control and data planes
significantly contributes in reducing switching delay. Pipelining helps improve overall switch throughput.

The PB is characterized by its throughput, control plane delay, and data plane delay. The number of reservation slots in each time slot is dictated by the need to achieve some guaranteed CLP. Estimating the delays in control and data planes enables finding the time slot of the switch. The ratio of switch throughput to the duration of time slot is the service rate of the pipelined switch, which is one structural feature of the banyan used. The PB can still guarantee some CLP if overall cell arrival rate is below the service rate of the switch. However, when cell arrival rate exceeds the service rate of PB, there is need for a scalable banyan architecture that can provide higher throughput without dramatically increasing switching delays. Our objective is to investigate a class of dilated banyan networks that can be engineered between two extremes: (1) a low-cost banyan with internal and external conflicts, or (2) a high-cost conflict-free fully-connected network with multiple outlets. The idea is that reducing the degree of conflicts in dilated banyans is accompanied with shorter switching delays due to simplified hardware. For this reason, the service rate of pipelined dilated banyan (PDB) might largely exceed that of the PB.

In the next section we present a class of DBs, investigate their basic building blocks, analyse their hardware complexity, and build a delay model for their modular components.

3 Pipelined dilated banyans

Our objective is to find a pipelined switch architecture having high service rate. For this we study in this section a class of DBs to be used as banyan planes in the pipelined scheme. We provide the basis for general evaluation of hardware complexity and propagation delays of arbitrary banyans made of switching elements having 2 input channels and 2 output channels where each input or output channel has power-of-2 number of ports.

In the next sub-section, we present a class of DBs for which the building blocks are $2 \times 2$ binary sorter and $1 \times 2$ demultiplexers (DM). Next we study the complexity of DBs through evaluation of the number of sorters and DMs, interconnection links, and propagation delays. Using a design approach for the sorters and DMs, we also find the complexity of needed hardware in terms of number of gates and express propagation delay as a function of gate delay.

3.1 A taxonomy of dilated banyans

The Dilated Banyan DB belongs to a class of dynamic, full access, single path, blocking multistage interconnection networks. The architecture of DBs is based on expanding the internal channel bandwidth (multiple ports) to reduce the CLP. A DB in which the degree of expansion of each link is $d$ ($1:2^d$ DB) is said to have $2^d$-dilation. Figure 1 shows an 8-input 1:4 DB that has 3 stages and 8 output channels allowing up to 4 cells to be simultaneously transmitted over distinct links of a given channel. Each output channel has 4 ports (4-dilation). The dashed lines show one path through the binary expansion phase. The first two stages expand the input ports in the form of a binary tree by using DMs. The third stage is a routing phase for which the basic switch is called (D-SW) switch which has 2 input channels and two output channels. In the example shown in Figure 1 each channel of the D-SW switch has 4 ports. There is no cell loss in the first two stages. However, cell loss can occur in any D-SW of the routing phase. Here the
The routing phase consists of 4-stage binary sorters that sort the incoming cells based on their priority bit. This allows allocating the four output ports to most prior cells.

The general architecture of an $n$-input $1:2^d$ DB is shown on Figure 2, where $d$ always satisfies $0 \leq d \leq n$. It has two phases: 1) **expansion phase** (non-blocking), and 2) **routing phase** (blocking). In the expansion phase the internal links are doubled at each of the $d$ stages until reaching a $2^d$ dilation at the $d$th stage. The $k$th stage of the expansion consists of $2^n \times 2^k$ DMs which are $1 \times 2$. Each DM routes an incoming cell to its upper or lower outputs depending on the cell destination bit.

The routing phase consists of $n - d$ stages, each having $2^{n-1}$ switches called $D$-SW switches as shown on Figure 2. Each $D$-SW switch has two input channels ($I_{C0}$ and $I_{C1}$) and two output channels ($O_{C0}$ and $O_{C1}$). Each of these channels has $2^d$ ports. Input and output channels are shown on Figure 1 for $n = 3$ and $d = 2$. At most $2^d$ cells with identical output can be simultaneously transmitted (one cell per port) without conflicts from input to output. A DB switch becomes non-blocking if the dilation degree is equal to the number of stages, i.e. there is no routing phase. In this case the hardware requirements are maximal due to its $n$-level $2^n$ binary trees. A non-blocking DB is one **Fully-Connected Network** with multiple outlets. On the other hand, a DB without an expansion phase is a simple banyan network (1:1 or 1-dilation) with internal and external conflicts.

The $D$-SW has 2 input channels and 2 output channels where each channel has $2^d$ ports. Internally, the $D$-SW switch consists of one stage of DMs and two binary concentrators called **Upper Concentrator** ($UC$) and **Lower Concentrator** ($LC$). Each of the $UC$ and $LC$ may at most have $2^d$ stages as will be shown latter.

One may consider $D$-SW switches in which $UC$ and $LC$ are $k$-stage networks which
we denote by $D-SW(d,k)$. Simple banyans use $D-SW(0,1)$, shown in Figure 3-(a), as a $2 \times 2$ switching element (SE). A 2-dilation DB uses $D-SW(1,2)$, shown in Figure 3-(b), as basic SE. Figures 3-(c) and (d) show two $D-SW(2,k)$ for which $k = 1$ and $k = 4$, respectively. In $D-SW(2,1)$ an internal conflict occurs at any of the one-stage sorters if there are two input cells at the sorter. In $D-SW(2,4)$ a loss may occur only at any sorter of the last stage (fourth) if there are more than four cells at the inputs of the corresponding concentrator. If the DB is used as control plane in the pipelined scheme [1] then a cell that is one of the four higher priority cells succeeds in making path reservation in $D-SW(2,4)$ and any of the other cells must be resubmitted again in the next cycle. Between $D-SW(2,1)$ and $D-SW(2,4)$ one can use other $D-SW$s ($k = 2$ and $k = 3$) with intermediate bandwidth. For example, in $D-SW(2,2)$ a loss may occur if there are three cells on four input ports of any given channel.

For maximum throughput a 4-dilation DB may use $D-SW(2,4)$ as the basic SE, but $D-SW(2,1)$ or $D-SW(2,2)$ or $D-SW(2,3)$ could also be used to reduce cost and bandwidth. However, any cell of the four highest priority cells is guaranteed to be allocated (succeeds) one output port of $D-SW(2,4)$. An $n$-stage, $1:2^d$ DB in which the routing phase is made of $D-SW(d,k)$ is denoted by $DB(n,d,k)$, where $k \leq 2^d$. For example, the DBs shown in Figure 1 are identical to $DB(3,2,4)$.

The $UC$ and $LC$ have identical architecture and each has $2^{d+1}$ input ports and $2^d$ output ports. Depending on cell destination bit ($x$), the DM stage routes an incoming cell (up to $2^{d+1}$ cells) to one of the $2^d$ input ports of $UC$ (for cells with $x = 0$) or to one of the $2^d$ input ports of $LC$ (for cells with $x = 1$). Each of $UC$ and $LC$ sorts incoming cells (at most $2^{d+1}$ cells) based on cell priority bit and allocate at most $2^d$ cells.

A $D-SW(n,d,2^d)$ outputs at most $2^d$ cells among the most prior cells. In this case, in each of $UC$ and $LC$ the lower priority cells in excess of $2^d$ cannot be correctly routed and must be resubmitted in the next reservation cycle. In other terms, each concentrator takes at most $2^{d+1}$ input cells, selects at most $2^d$ cells among the most prior regardless of their source input ports, and forwards the selected cells to their $2^d$ output ports. Therefore, cell loss can occur within $D-SW$ only if there are more than $2^d$ cells at the input of concentrator.
The UC and LC are made of stages of $2 \times 2$ binary sorters (up-sorters and down-sorters) interleaved with a perfect-shuffle permutation. If there are two input cells to an up-sorter (down-sorter), then the cell with highest priority exits at the upper (lower) output and the other cell exits at lower (upper) output. A single input cell is always sent to upper output regardless of its priority. The up-sorter and down-sorter have symmetric functions.

In the first sorter stage, the upper output of up-sorter transmits a cell if there is at least one cell at either inputs. The lower output transmits a cell if there are at least 2 input cells. In the $i$th sorter stage, the lower output of up-sorter transmits a cell if there are at least $i$ cells at input of concentrator. In Figure 3-(d), we labeled some links with the minimum number of cells arriving on concentrator inputs so that at least one cell is transmitted on that link. Since the lower output has least priority, the lower outputs of $i$th stage up-sorter transmits a cell if and only if there are at least $i + 1$ cells at inputs of concentrator. There are $2^{d+1}$ input ports and $2^{d}$ output ports for each of UC and LC. Therefore, there must be $2^{d}$ stages in each of the UC and LC to guarantee that no cell loss can occur in last stage of concentrator as long as the number of input cells is no more than $2^{d}$ at concentrator input. Moreover, a $D$-$SW(n, d, k)$ for which $k = 2^{d}$ guarantees that $2^{d}$ cells among the most prior cells are successfully routed to the $2^{d}$ concentrator outputs among a set of $2^{d+1}$ input cells. The throughput of $D$-$SW(n, d, k)$ is presented in the Appendix. It is shown that increasing the number of stages $k$ in the concentrator produces: (1) increase in the throughput at output of concentrator, and (2) decrease in the probability of delaying higher priority cells.
3.2 Pipeline switch architecture by using dilated banyans

The architecture of an $8 \times 8$ pipelined dilated banyan PDB with 2-dilation is shown in Figure 4. The PSB has: (1) one dilated banyan control plane, (2) $L$ dilated banyan data planes, (3) a set of input multiplexers, and (4) a set of output multiplexers. The vertical links of control plane provide the data planes with the state of all switching elements of control plane. Details about pipeline switch architecture can be found in [1].

3.3 Complexity analysis

In this section we evaluate hardware complexity and delays for a number of useful DBs which are simple banyan ($DB(n, 0, 1)$), partially-dilated banyan ($DB(n, d, 1)$), and fully-dilated banyan ($DB(n, d, 2^d)$). The concentrator of each $D$-SW consists of: (a) only one stage ($k = 1$) of sorters for simple banyan and partially-dilated banyan, and (b) $2^d$ stages of sorters for the fully-dilated banyan. The number of sorters, DMs, and links are shown in Table 1.

We evaluate delays in control and data planes. There are $n$ DMs along a path from input to output in a DB. A path through a $D$-SW($d, k$) encounters $k$ sorters. The total delay due to sorters in the routing phase is $k(n - d)\tau_S$. Denote by $\tau_{DM}^C$, $\tau_{DM}^D$, $\tau_S^C$, and $\tau_S^D$ the delays through a DM and a sorter in control (C) and data (D) planes, respectively. The overall delay in DB due to expansion and routing phases is then $DB_{C}^C(n, d, k) = n\tau_{DM}^C + k(n - d)\tau_S^C$ in control plane and $DB_{D}^D(n, d, k) = n\tau_{DM}^D + k(n - d)\tau_S^D$ in data plane. Note that paths in data plane are pure combinational logic.

A sorter requires three D-latches to store the activity bits of two cells and the state, state logic, and two parallel $2 \times 1$ multiplexers. Using dual input gates, 12 gates are needed for 3 D-latches. The delay of one D-latch is $2\tau_{gate}$, where $\tau_{gate}$ is one gate delay. The state is $pr_0a_0 + \pi_1$, where $pr_0$, $a_0$, and $a_1$ are the priority of upper input cell ($C_0$), activity of $C_0$, and activity of lower input cell, respectively. The state requires 3 gates and has a delay of $2\tau_{gate}$. The two multiplexers require 6 gates and each has a delay of $2\tau_{gate}$. Hence a sorter in control plane requires $N_{S}^C = 21$ gates and has a delay of $\tau_S^C = 8\tau_{gate}$. In data plane the sorter is reduced to two $2 \times 1$ multiplexers that are set by using one D-latch. Thus a sorter in data plane requires $N_{S}^D = 10$ gates and has a delay of $\tau_S^D = 2\tau_{gate}$. 

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![Figure 4: An 8 x 8 pipelined dilated banyan using DB(3,2,1) (diagram)](image_url)
A DM requires two D-latches to store the activity bit of input cell and the state, state logic (up or down), and one 1\times 2 demultiplexer. The state is \(x_0a_0\), where \(x_0\) is destination bit. Hence a DM in control plane requires \(N_{DM}^C = 11\) gates and has a delay of \(\tau_{DM}^C = 6\tau_{gate}\). In data plane the DM is reduced to a 1\times 2 demultiplexer that is set by using one D-latch. Thus a DM in data plane requires \(N_{DM}^D = 6\) gates and has a delay of \(\tau_{DM}^D = \tau_{gate}\).

Table 1 lists the delays \(\tau_{control}\) and \(\tau_{data}\) of dilated banyans as a function of gate delay. The total complexity of \(DB\) is evaluated (Table 1) by using the expression of \(DB_{Sorters}\) and \(DB_{Dmux}\) and the values of \(N_{S}^C\), \(N_{DM}^C\), \(N_{S}^D\), and \(N_{DM}^D\).

A pipelined dilated banyan (Figure 4) with \(L\) data planes requires a set of \(2^n\) input demultiplexers (1-to-L). This allows interfacing the \(i\)th \((0 \leq i \leq 2^n - 1)\) input buffer to the \(i\)th input of any data plane. Similarly, we need another set of \(2^n\) output multiplexers (L-to-1). The overall complexity of all the above is \(2^{n+1}M(M-1)\) dual-input gates, where \(M\) is the least power-of-2 integer satisfying \(L \leq M\). For example a \(256 \times 256\) pipelined switch with 6 data planes requires about 28.7 K-gates due to the above multiplexers and demultiplexers.

In the next Section we carry out simulation of pipelined DBs under uniform traffic and find the number of needed reservation cycles to achieve some CLP. This will enable evaluation of service rate.

### 4 Evaluation under uniform traffic

In this section we study the CLP and cell delay of pipelined switch architectures under uniform traffic pattern. We assume time slotted synchronized operations for which the slot time is greater than or equal to the switch processing time. Cells must be synchronized and aligned with the local slot boundaries before being routed by the switch. All switch inputs are identical and independent. The cell destinations are uniformly distributed over all the switch outputs.

In pipelined switches the cells are generated in the beginning of each time slot which consists of a number of reservation slots. HOL cells are submitted to the control plane.
in each reservation slot. In unbuffered switches only cells generated during the current
time slot are submitted to the control plane for each reservation slot until they succeed
in reserving a path, otherwise they are lost. In buffered switches the same process occurs
with the difference that an HOL cell runs for each reservation slot and remains as HOL
if it fails in making a reservation. Therefore, cell loss can occur in a buffered switch only
if a cell is generated for a given input buffer and that buffer is full.

In the next sub-sections we study performance of pipelined switches employing: 1)
fully-dilated banyans, 2) partially-dilated banyans, and 3) simple banyans.

4.1 Performance of pipelined fully-dilated banyans

The partially-dilated banyan $DB(n, d, 1)$ has the least throughput and least hardware
requirements among all DBs with $n$-stages and $2^d$-dilation. At the other extreme we
have the fully-dilated banyan $DB(n, d, 2^d)$ for which there are $2^d$-sorter stages in each
$D$-SW switch of the routing phase. The throughput of $DB(n, d, 2^d)$ is the highest among
the family of DBs. In this section we evaluate performance of Pipelined Fully-Dilated
Banyan (PFDB) under uniform traffic. This means we use one $DB(n, d, 2^d)$ in the control
plane and in each data plane of the pipelined switch. Figure 5 shows the CLP of single
$DB(n, d, 2^d)$ plane versus the dilation degree $d$ as obtained from analysis and simulation.
See the Appendix for a derivation of the analytical model of $DB(n, d, k)$, where $1 \leq k \leq 2^d$.

Here a dilation degree of 3 (8-dilated) is sufficient to achieve a CLP around $10^{-6}$ at full
load but at a high hardware cost compared to that of simple banyan. This is discussed
later in the hardware analysis of Sub-Section 4.5. Note that fully-dilated banyans exhibits
nearly the same CLP regardless of the switch size.

The analytical results were more optimistic than simulation results because the ana-
typical model assumes uniform traffic at all the stages. In reality the uniform traffic gets
corrupted by the deterministic stage routing which increases from one stage to the next.
In other terms, the traffic uniformity decreases with increasing stage number. We validated the above interpretations by uniformly re-generating the cell destination after each stage of the simulation which gave CLPs that were very close to those obtained from the analysis. Therefore, the simulation results are more representative of real switch performance than the analytical results. We also evaluated the performance of pipelined DBs by using our analytical model of $DB(n, d, 2^d)$ and the discrete-time probability state transition model introduced in [1] to model the number of backlogged cells in input queues. The CLP obtained from analysis was always more optimistic than that obtained from simulation due to the effects described above.

Buffering of single plane DB did not produce significant drop in CLP except when the dilation degree was 3 or above, i.e. when CLP is very low. For the pipelined scheme we used fully-dilated banyan with dilation degree 1 (2-dilation) and observed good results. As shown in Figure 6, an unbuffered PFDB requires only 5 or 6 data planes as compared to 12 data planes for an unbuffered PB (see Figure 10). The use of input buffering significantly reduces the number of data planes (reservation cycles) needed to achieve some CLP. From Figure 6, a 256-input unbuffered PFDB requires 5 data planes to achieve a CLP below $10^{-6}$, while the same level of performance can be achieved by using only 4 data planes if 2 buffers were used. When input buffering is used, the number of data planes needed to achieve a CLP of nearly $10^{-8}$ gradually decreases from 4 to 2 when the buffer size is increased from 2 to 10 as shown on Figure 7. This Figure shows the gain of increasing input buffer size on a 256 × 256 PFDB.

4.2 Performance of pipelined partially-dilated banyans

The PB uses simple banyan in control and data planes. Since simple banyan has relatively low throughput the PB requires relatively large number of reservation slots to achieve some CLP. To reduce the number of data planes used in the PB, one may use higher throughput banyan switches such as partially-dilated banyan $DB(n, d, 1)$.

In this section we present the simulation results of Pipelined Partially-Dilated Banyan (PPDB) which uses one $DB(n, d, 1)$ in each data plane. We have simulated the PPDB for $d = 1$ (2-dilation) and $d = 2$ (4-dilation). Figures 8 and 9 show the CLP of PPDB in the case of 4-dilation. With no input buffering the 4-dilated PPDB requires 6 or 7 data planes (8 or 9 data planes for 2-dilation) against 12 for the PB (See Figure 10).

The CLP reduces further when input buffers are used. We observe a similar trend as in unbuffered pipelined switches. When a CLP below $10^{-6}$ is needed, the profitability of input buffering is significant as it reduces the number of data planes from 3 or 4 for 2-dilation to 2 for 4-dilation (Figure 9). Notice that switching delay of partially-dilated banyan is shorter than that of simple banyan because of the expansion phase. Therefore, the use of partially-dilated banyans allows reducing the number of data planes in the pipelined switch while decreasing the reservation time compared to that of simple banyan. This reduction in the number of data planes and delay in pipelined partially-dilated banyan is gained at the cost of increased hardware and number of interconnection links in both control and data planes.

4.3 Performance of pipelined banyans

In this section we present simulation results of the pipelined banyan (PB) switch presented in [1]. Notice that the simple banyan network is a particular DB ($DB(n, 0, 1)$) for which
Figure 6: CLP of unbuffered pipelined fully-dilated banyans

Figure 7: CLP of buffered pipelined fully-dilated banyans
Figure 8: CLP of unbuffered pipelined partially-dilated (4) banyans

Figure 9: CLP of buffered pipelined partially-dilated (4) banyans
the dilation degree \( d = 0 \) and the number of sorter stages in each \( D-SW \) is 1.

In unbuffered \( PB \), the loss can occur in the last reservation slot, while in buffered \( PB \) the loss occurs at input of full buffers. Figure 10 shows the CLP as function of the number of data planes for unbuffered \( PB \). To achieve a CLP below \( 10^{-6} \) the number of data planes required is 12 for large switches such as 256 \( \times \) 256 or 1024 \( \times \) 1024. The number of data planes required can be reduced when input buffers are available. In buffered \( PB \), a cell that fails in a reservation slot remains in input buffer until it succeeds in some subsequent reservation slot. Figure 11 shows the profitability of input buffering for pipelined simple banyan. This Figure shows the effect of varying input buffer size for a 256 \( \times \) 256 switch. When the input buffer size is increased from 2 to 10 we observe a decrease in the number of data planes from 9 to 5 while achieving a CLP around \( 10^{-8} \).

Though the simple banyan has relatively poor throughput, input buffering contributes significantly in boosting the performance of the pipeline banyan. Our simulation indicates that six or seven data planes are required to achieve a CLP below \( 10^{-6} \) for large switches with 10-input buffering.

### 4.4 Queuing delays

In this section we study total switch delay which is defined as the sum of HOL delay and queuing delay. It is expressed in reservation cycles. The HOL delay is counted from the time a cell becomes HOL to the time the cell succeeds in making a reservation. The queuing delay is counted from the time a cell enters some input buffers until the time at which the cell becomes HOL.

Figure 12 shows the average total delay for \( PB \), \( PFDB \) with 2-dilation, \( PPDB \) with 2-dilation, and \( PPDB \) with 4-dilation when 2 input buffers are available for each input port. Lighter input loads certainly contributes in shortening the HOL delay and total delay for all switches. Mainly, delays are important when input traffic is close to full load. Pipelined switches employing 2-dilation \( PFDB \) or 4-dilation \( PPDB \) have high
Figure 11: CLP of buffered pipelined banyans

Figure 12: Comparison of average total delay
throughput which indicates that a cell remains, on the average, HOL for small fraction (about $10^{-2}$) of a reservation cycle at full load. At full load, the throughput of $PB$ and $PPDB$ (2-dilation) is much less compared to that of the other switches which causes an HOL delay of $10^{-1}$. The total delay (Figure 12) for $PB$ and $PPDB(2)$ with the same conditions becomes closer to 1 reservation cycle. In the case of $PB$ and $PPDB(2)$ at full load, a cell may: 1) wait in input buffer for 1 clock before becoming HOL, or 2) find the buffer empty and succeeds in the reservation attempt at second cycle. It is observed that HOL delay dominates the total delay in the case of 2-input buffers. Therefore, input buffering is profitable and buffer size significantly affects CLP.

In the next section we compare the CLP for the above pipelined switches together with their hardware requirements, costs, propagation delays, and service rates.

### 4.5 Comparisons

The performance of pipelined switches cannot be characterized only by the achieved throughput or the corresponding CLP. The time it takes to achieve a level of performance is one important factor in the evaluation. For example, the pipelined banyan can achieve a CLP below $10^{-6}$ when there are six data planes. Whether this performance is adequate for ATM rates or not depends on: (1) the number of needed reservation slots, (2) switching delay in control plane, and (3) payload transmission time in data plane. In this section we present analysis of the above factors which will allow evaluation of the switch service rate.

Using the results from Section 3.3 for a 256-input dilated banyan switch we list the hardware requirements in Table 2. To compare performance of the pipelined switches that employ DBs we need to evaluate the service rate as function of gate delays. The service rate of the switch is the number of cells that can be switched per unit of time. Though wires occupy significant space and cause complexity in VLSI and PCB we do not account for wiring and wire delays for simplicity. This assumes that delays through a logical gate in VLSI dominate delays on wire.

A payload of $N_{cell}$ bits can be transmitted over a free data plane in $T_{data} = N_{cell} \tau_{bit} + \tau_{data}$ gate delays, where $\tau_{data}$ is the delay in one data plane and $\tau_{bit}$ (1/$\tau_{bit}$ is the rate) is the time in gate delays to remove one bit from input buffer. In the control plane, the switching time (also reservation slot) of the header is $T_{control} = \tau_{control}$ gate delays which accounts for self-routing of the header along a path from input to output. Note that

<table>
<thead>
<tr>
<th></th>
<th>$PB$</th>
<th>$PDB_2$</th>
<th>$PDB_3$</th>
<th>$PDB_8$</th>
<th>$FDB_2$</th>
<th>$FDB_4$</th>
<th>$FDB_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DB_{Sorters}$</td>
<td>2048</td>
<td>3584</td>
<td>6144</td>
<td>10240</td>
<td>7168</td>
<td>24576</td>
<td>81920</td>
</tr>
<tr>
<td>$DB_{Dmux}$</td>
<td>2048</td>
<td>3840</td>
<td>6912</td>
<td>12032</td>
<td>3840</td>
<td>6912</td>
<td>12032</td>
</tr>
<tr>
<td>$DB_{Link}$</td>
<td>6400</td>
<td>11520</td>
<td>20224</td>
<td>34560</td>
<td>18688</td>
<td>57088</td>
<td>177920</td>
</tr>
<tr>
<td>$\tau_{control}$ (in Gate delay)</td>
<td>112</td>
<td>104</td>
<td>96</td>
<td>88</td>
<td>160</td>
<td>240</td>
<td>368</td>
</tr>
<tr>
<td>$\tau_{data}$ (in Gate delay)</td>
<td>24</td>
<td>22</td>
<td>20</td>
<td>18</td>
<td>36</td>
<td>56</td>
<td>88</td>
</tr>
<tr>
<td>Control Plane Complexity (Gates)</td>
<td>65536</td>
<td>117504</td>
<td>205056</td>
<td>347392</td>
<td>192768</td>
<td>592128</td>
<td>1852672</td>
</tr>
<tr>
<td>Data Plane Complexity (Gates)</td>
<td>32768</td>
<td>58880</td>
<td>102912</td>
<td>174592</td>
<td>94720</td>
<td>287232</td>
<td>891392</td>
</tr>
</tbody>
</table>

Table 2: Hardware requirements and delays for $256 \times 256$ dilated banyans
Buffered pipelined dilated banyan switches (input buffer size is 10)

<table>
<thead>
<tr>
<th></th>
<th>PB</th>
<th>PPDB₂</th>
<th>PPDB₄</th>
<th>PFDB₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of reservation slots $L$</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Achieved CLP</td>
<td>$4 \times 10^{-7}$</td>
<td>$1.2 \times 10^{-8}$</td>
<td>$2 \times 10^{-8}$</td>
<td>$1.2 \times 10^{-8}$</td>
</tr>
<tr>
<td>$T_{data}$ (in Gate delay)</td>
<td>448</td>
<td>446</td>
<td>444</td>
<td>460</td>
</tr>
<tr>
<td>$T_{control}$ (in Gate delay)</td>
<td>112</td>
<td>104</td>
<td>96</td>
<td>160</td>
</tr>
<tr>
<td>$N_{dp} = \lceil T_{data}/T_{control} \rceil$</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>$N_{phys}^{dp} = \text{Min}{N_{dp}, L}$</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Gates in control plane</td>
<td>65536</td>
<td>117504</td>
<td>205056</td>
<td>192768</td>
</tr>
<tr>
<td>Gates in $N_{phys}^{dp}$ data planes</td>
<td>131072</td>
<td>176640</td>
<td>205824</td>
<td>189440</td>
</tr>
<tr>
<td>Links in control and $N_{phys}^{dp}$ data planes</td>
<td>32000</td>
<td>46080</td>
<td>60672</td>
<td>56064</td>
</tr>
<tr>
<td>Gates in pipelined switch</td>
<td>196608</td>
<td>294144</td>
<td>410880</td>
<td>382208</td>
</tr>
<tr>
<td>Service Rate (Gega Cells/s)</td>
<td>0.381</td>
<td>0.821</td>
<td>1.333</td>
<td>0.800</td>
</tr>
</tbody>
</table>

Table 3: Service rates of $256 \times 256$ pipelined dilated banyans

$N_{dp} = \lceil T_{data}/T_{control} \rceil$ is the least number of needed data planes in the pipelined switch to guarantee there is a free data plane for payload transmission at the end of each reservation slot.

Denote by $p$ the cell input load and let $clp$ be the switch CLP when each time slot is formed by $L$ reservation slots. In steady state, the average number of cells that can be switched in one time slot is $p(1 - clp)2^n$ for $2^n$-input switch. For pipelined switches each time slot consists of $L$ reservation slots but this requires $N_{phys}^{dp} = \text{Min}\{N_{dp}, L\}$ physical data planes. The duration of one time slot is then $T = L \tau_{control}$ gate delays. The time to switch $p(1 - clp)2^n$ cells is $T$ gate delays. If one assumes $N_{phys}^{dp}$ data planes, then the service rate ($S$) of the pipelined switch will be:

$$S = \frac{p(1 - clp)2^n}{L \tau_{control}}$$

We assume header and payload are stored into the input buffers and retrieved at a rate of one bit per gate delay ($\tau_{bit} = 1$). One gate delay is assumed to be 1 ns. Now we consider a number of buffered pipelined DBs which are $PB$, $PPDB₂$, $PPDB₄$, and $PFDB₂$ and list some of their parameters in Table 3. $L$ is being the number of reservation slots required to achieve a CLP below $10^{-8}$.

For $PB$ the number of needed reservation slots is $L = 6$. Since $N_{dp} = 4$ we can make 6 reservation cycles by using 4 data planes. The same considerations apply to the other switches. Due to low the throughput of simple banyan, $PB$ requires a higher number of data planes than a pipelined DB which requires between 2 and 3 more complex data planes.

The service rate is evaluated for $PB$, $PPDB₂$, $PPDB₄$, and $PFDB₂$ with 256 inputs and outputs. Since the number of reservation slots $L$ is dictated by the need for an acceptable CLP, Pipelining allows minimizing the cost of $L$ reservation slots through the use of only $N_{phys}^{dp}$ data planes. Therefore, the service rate $S$ is one structural feature of the used banyan which cannot be increased beyond some limit for a given design technology.
The service rate of PB is the least for the family of pipelined DBs. According to our design model and assumptions, a 256-input PB can switch cells with a CLP below $10^{-8}$ only when overall cell arrival rate is below $381 \times 10^6$ cells/s. This represents a structural limit of pipelined banyans.

One way to achieve higher service rates than that of PB is to use DBs. The highest service rate is achieved for PPDB, which indicates that a 4-dilation with one-stage of sorters in the D-SW switch was critical in providing high throughput without dramatically increasing the reservation time. It is shown in Table 3 that a PPDB has nearly 3.5 times the service rate of PB at hardware cost of nearly three times that of PB. There are two reasons for the relatively high service rates of partially-dilated banyans. First, the non-blocking binary expansion phase is responsible for significantly increasing the rate of successful reservations compared to simple banyans. Second, the use of one sorter stage in each D-SW switch of partially dilated banyans was the key factor to maintain a low propagation delay compared to fully-dilated banyans. One can think of partially-dilated banyans as a banyan whose service rate can be scaled up by horizontally expanding the banyan architecture (increasing dilation) without increasing the path delay (stages of D-SW).

It is shown in the Appendix that the number of stages in the D-SW controls the probability of delaying higher priority traffic. Increasing the number of stages in D-SW causes a decrease in the probability of delaying higher priority traffic. Although PPDB generally provides higher service rate, the PFDB incurs the least delays to higher priority traffic.

In the next section we study the performance of the above pipelined switches under ATM traffic which will allow us to comments on the robustness of each switch.

5 Performance under ATM traffic conditions

A realistic ATM workload is a mixture of bursty and non-bursty sources with the load originated from a variety of traffic sources which exhibit correlation in space as well as in time. Traffic source characterization has been an extensive area of research[16]. A simple and widely adopted traffic source model is the ON-OFF model. According to this model, during the lifetime of a virtual connection, the traffic source will be in one of two states, active or idle. During the active state the source is transmitting cells at some given rate. Each active state may be followed by an idle period during which the source is silent. The cells generated during the same ON-period form a burst. Furthermore, it is always assumed that successive active and idle periods are statistically independent and exponentially distributed.

For simulation purposes, several parameters have been identified, which together, completely characterize an ON-OFF traffic source. These are, the peak cell rate ($pcr$), the sustainable cell rate ($scr$), and the average duration of the ON-state ($t_{on}$). Other parameters of interest such as the source burstiness ($\beta$) or the average duration of the OFF-state ($t_{off}$) are easily derived from these three parameters. For example, $\beta = pcr/scr$ and $t_{off} = (\beta - 1)t_{on}$. Typical values for the traffic parameters for some traffic sources are summarized in [16].

In our simulation study, we assumed that $pcr$, $t_{on}$, and $\beta$ are known for each source. Furthermore, we assumed that the active and idle periods are exponentially distributed [17].
<table>
<thead>
<tr>
<th>Traffic</th>
<th>Source type</th>
<th>Peak cell rate (Mbps)</th>
<th>Sustainable cell rate (Mbps)</th>
<th>Mean burst length (cells)</th>
<th>Burstiness</th>
<th>Percentage channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traffic 1</td>
<td>CBR</td>
<td>0.064</td>
<td></td>
<td></td>
<td></td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>CBR</td>
<td>1.4</td>
<td></td>
<td></td>
<td></td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>VBR</td>
<td>0.7</td>
<td>200</td>
<td>5</td>
<td>20%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>VBR</td>
<td>25</td>
<td>20</td>
<td>5</td>
<td>20%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>VBR</td>
<td>21</td>
<td>30</td>
<td>4</td>
<td>40%</td>
<td></td>
</tr>
<tr>
<td>Traffic 2</td>
<td>CBR</td>
<td>0.064</td>
<td></td>
<td></td>
<td></td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>CBR</td>
<td>1.4</td>
<td></td>
<td></td>
<td></td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>VBR</td>
<td>0.7</td>
<td>200</td>
<td>5</td>
<td>12%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>VBR</td>
<td>20</td>
<td>25</td>
<td>5</td>
<td>13%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>VBR</td>
<td>2</td>
<td>25</td>
<td>10</td>
<td>6%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>VBR</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>6%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>VBR</td>
<td>30</td>
<td>21</td>
<td>4</td>
<td>6%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>VBR</td>
<td>3</td>
<td>6</td>
<td>5</td>
<td>7%</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Features of traffic 1 and Traffic 2

with parameters $a = 1/t_{on}$ and $b = 1/t_{off}$, respectively. We experimented with the ON-OFF traffic mixes defined in Table 4 (see Traffic Mix 1 and Traffic Mix 2 in [12] for more details).

We subjected the pipelined banyan $PB$, pipelined partially-dilated banyan $PPDB$, and pipelined fully-dilated banyan $PFDB$ switches to Traffic 1 and Traffic 2. For Traffic-1, it was observed that less number of data planes were required to achieve some CLP than that needed in the case of uniform traffic for equal number of input buffers. The above observation was true for all three switches and for all switch sizes.

The reason is that Traffic-1 and Traffic-2 causes less conflicts compared to full load uniform traffic. Figure 13 shows the CLP for $PFDB$ switch with 2-dilation and 2 input buffers under Traffic-1. Two data planes were required to achieve a CLP close to $10^{-6}$ for $PFDB$ and between 3 and 4 for $PPDB$ as shown on Figure 14. Using the same number of data planes, the CLP of $PFDB$ dropped below $10^{-8}$ for all switch sizes when the buffer size was increased to 5.

We repeated the above experiments by using Traffic-2 with 2 input buffers and noticed that the CLP is much less than that obtained under Traffic-1 for the same number of data planes. Specifically, the CLP of $PB$, $PPDB$, and $PFDB$ was below $10^{-8}$ when 2 data planes and two or more input buffers were used with all switch sizes. The reason is that in Traffic-2 only 50% of the sources are VBR sources compared to 80% in the case of Traffic-1.

Figure 15 shows the CLP for $PPDB$ with 4-dilation when 2 input buffers are used under Traffic-1 which can be compared to the case of 2-dilation shown on Figure 14. Increasing the dilation degree of $PPDB$ under Traffic-1 leads to lower CLP or lesser number of needed data planes which indicates that CLP is significantly affected by the degree of dilation under both uniform traffic and ATM traffic. Figure 16 shows the effect of varying the input buffer size for $PPDB$ with 2-dilation under Traffic-1.

Figures 17 and 18 show the CLP for $PB$ when using 2 input buffers under Traffic-1 and Traffic-2, respectively. In the case of Traffic-2, much less number of data planes are needed to achieve equal CLP which indicates that $PB$ is also very sensitive to percentage of VBR sources. Using other experiments, we find that increasing the buffer size results
Simulation of PFDB (2-dilation), full load, Queue size = 2, Traffic 1 mixture:
10% CBR(0.064), 10% CBR(1.4), 20% VBR(0.7,200,5), 20% VBR(25,20,5), 40% VBR(21,30,4)

Cell loss is < 10e-8 beyond each terminal point.

Figure 13: PFDB with 2-dilation under ATM Traffic-1

Simulation of PPDB (2-dilation), Queue size = 2, Traffic 1 mixture:
10% CBR(0.064), 10% CBR(1.4), 20% VBR(0.7,200,5), 20% VBR(25,20,5), 40% VBR(21,30,4)

Cell loss is < 10e-8 beyond each terminal point.

Figure 14: PPDB with 2-dilation under ATM Traffic-1
Figure 15: PPDB with 4-dilation under ATM Traffic-1

Figure 16: Effect of increasing buffer size on 1024-input PPDB(2) under Traffic-1
in a significant drop in the number of data planes needed to achieve a CLP of $10^{-6}$ or less. For example, the number of data planes dropped from 6 to 4 when increasing input buffering from 2 to 10 in the case of a 1024-input PB. An input buffering of 38 requires 3 data planes to achieve a CLP of $10^{-8}$, but in this case the total switch delay is about 22 times that of 2 input buffers. Similar effects were observed for PDBs. Unfortunately, this hardware saving is accompanied with significant increase in switching delays. Excessive switching and buffering delays should be imperatively avoided especially for delay-sensitive real-time traffic. We favor implementations with relatively small buffer sizes in order not to offset the benefit of the high speed cell transmission and switching.

6 Conclusion

In this paper we evaluated pipelined ATM architectures employing a family of Dilated Banyans for increasing throughput and reducing switching delay in banyan-based ATM switches. This proposed scheme is a generalization of the pipeline banyan [1] which was proposed earlier. The number of reservation cycles required to achieve some CLP depends on the throughput of the dilated banyan used in the pipelined scheme. This determines overall switching delay and allows finding the corresponding service rate for a given design methodology under some guaranteed CLP. To allow scaling up the service rate, we proposed a family of dilated banyan networks for which the throughput and switching delays can be scaled up with increasing dilation degree and size of input buffers. A DB can be engineered between two extremes: (1) a low-cost banyan with internal and external conflicts, or (2) a high-cost conflict-free fully-connected network with multiple outlets. Between the two extremes lies a family of DBs having different switching delays and throughput. Increasing the dilation degree reduces path conflicts which produces noticeable increase in service rate due to increase in throughput and decrease in path delay. Dilated banyans were studied with respect to their hardware requirements, number
of interconnections, input buffering, switching delay, and CLP. We carried out complexity analysis and simulation of pipelined dilated banyans which we subject to uniform traffic and ATM traffic. We also studied the robustness of the proposed pipelined switches under a variety of ATM traffic. Pipelining partially-dilated banyan can provide up to 3.5 times the service rate of pipeline banyan with linear increase in hardware cost. This provides insight on banyan architectures that are most suitable to maximize the service rate in pipelined switch architectures.

7 Appendix

The analytical model of pipelined DBs uses a discrete-time probability state transition to model the number of backlogged cells in each input buffer which was proposed in [1]. The model requires knowledge of throughput of $DB(n, d, k)$ which we derive here. All inputs are assumed to be identical and independent in time as well as in space. The cell destinations are uniformly distributed over all the outputs. In every time slot, the load on each input of $DB(n, d, k)$ is $q$. A $DB(n, d, k)$ has $k$ expansion stages and $n - d$ routing stages. The throughput of the expansion phase is $T_{\text{expansion}} = q2^{-d}$. The throughput of the routing phase ($T_{\text{routing}}$) can be recursively by computing the throughput ($T_{D-SW}$) of each stage as function of input load and iterating from first to the $(n - d)$th $D-SW$ stage. Since each $D-SW$ has $k$-stages, we iteratively compute $T_{D-SW}$ by evaluating the throughput of each sorter stage as function of input load and iterating from first to $k$th sorter stage, where $1 \leq k \leq 2^d$.

We assume uniform cells rate at inputs of $D-SW$ with load $q$. The $D-SW$ uses cell destinations to partition incoming cells into: (1) set $G_0$ formed by inputs of $UC$, and (2) set $G_1$ formed by inputs of $UC$. Each set has at most $2^{d+1}$ cells. The load on any input of $UC$ or $LC$ is $q/2$. The $k$-stages of sorters ($UC$ and $LC$) sort the cells based on cell priority bit ($pr = 0/1$). Let’s denote by $q_i^0$ and $q_i^1$ the load on inputs 0 (upper) and 1 (lower) of a

Figure 18: CLP of PB with 2 input buffers under Traffic-2
Throughput of \( i \)th sorter as function of load on upper \((q^0_i)\) and lower \((q^1_i)\) inputs

<table>
<thead>
<tr>
<th>INPUTS</th>
<th>OUTPUTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>load ( pr=0/1 )</td>
<td>Throughput ( pr=0/1 )</td>
</tr>
<tr>
<td>Upper (0 as superscript)</td>
<td>( q^0_i )</td>
</tr>
<tr>
<td>Lower (1)</td>
<td>( q^1_i )</td>
</tr>
</tbody>
</table>

Table 5: Throughput of \( i \)th stage sorter in a D-SW switch

sorter in the \( i \)th stage of concentrator, respectively. We also denote by \( p^0_i \) and \( p^1_i \) the loads due to higher priority cells \((pr = 1)\) on upper input \((0)\) and 1 of the sorter, respectively. Table 5 gives the up-sorter throughput at upper output \((q^0_{i+1} = TU(q^0_i, q^1_i))\) and lower output \((q^1_{i+1} = TL(q^0_i, q^1_i))\). The throughput of down-sorter is \( q^1_{i+1} \) for upper output and \( q^0_{i+1} \) for lower output, i.e. swapped. All sorter stages except last are non-blocking in the concentrator. In last stage each sorter has only one output. Thus the probability of internal conflicts within the concentrator is the throughput \( q^1_k \) of lower output of up-sorter in the \( k \)th stage, i.e. last stage. The probability \((p^1_k(1))\) of a cell with priority \( pr = 1 \) to exit the lower output of last sorter stage is the least when \( k = 2^d \) for the family of dilated banyans \((1 \leq k \leq 2^d)\).

The number of stages \((k)\) in the concentrator affects two parameters: (1) the throughput of the concentrator, and (2) the probability of allocation of concentrator outputs to higher priority cells. A \( k \)-stage concentrator is non-blocking in the first \( k - 1 \)-stages and blocking only for the last stage. Therefore, the throughput of all output ports of any stage form 1 to \( k - 1 \) is identical to the sum of loads on concentrator inputs. At each stage the concentrator performs sorting of the cells according to cell priority. Thus the probability of allocating the concentrator output ports to higher priority cells increases with increasing \( k \). In the pipelined scheme, a cell header that is not allocated an output port in control plane will be resubmitted in the next time slot. This affects cell service delay. Figure 19 shows the throughput of higher priority cells at upper and lower output of up-sorters versus the number of stages \((1, \ldots, 16)\) and load \((0.2, \ldots, 1)\) on concentrator inputs (marked as stage 0 in Figure 19). It is assumed that load on each concentrator input due to higher priority cells is 50% of total load. The probability that a cell with higher priority to exit the concentrator without conflicts increases with increase in \( k \). Thus the number of stages in the concentrator affects the probability of delaying higher priority traffic. Therefore, increasing the number of stages \( k \) in the concentrator produces: (1) increase in the throughput at output of concentrator, and (2) decrease in the probability of delaying higher priority cells.
Throughput of higher priority cells at outputs of sorter stages 1, ..., 16 for loads 0.2, ..., 1 at concentrator input (stage 0) assuming 50% of load is for higher priority traffic.

References


