Number Base Conversion

Objectives

Given the representation of some number (X_B) in a number system of radix B, this lesson will show how to obtain the representation of the same number (X) in another number system of radix A, i.e. (X_A).

Converting Whole (Integer) Numbers

Assuming X to be an Integer,

1. Assume that X_B has *n* digits $(b_{n-1}..........b_2\, b_1\, b_0)_B$,

   where $\mathbf{b}_i$ *is a digit in radix B system,*

   i.e. $\mathbf{b}_i \in$ **{0, 1, ….., "B-1"}**

2. Assume that X_A has *m* digits $(a_{m-1}..........a_2\, a_1\, a_0)_A$

   where $\mathbf{a}_i$ *is a digit in radix A system,*

   i.e. $\mathbf{a}_i \in$ **{0, 1, ….., "A-1"}**

Knowns

$$X_B = (b_{n-1}..........b_2\, b_1\, b_0)_B \Rightarrow (a_{m-1}..........a_2\, a_1\, a_0)_A$$

Unknowns

$$X_B = a_{m-1}*A^{m-1}+......+ a_2*A^2 + a_1*A^1 + a_0*A^0$$

Divisible by A **Not Divisible by A**

Where $a_i \in \{0\text{-}(A\text{-}1)\}$

Accordingly, dividing X_B by A, the remainder will be a_0.

In other words, we can write

$$\mathbf{X_B = Q_0 . A + a_0}$$

Where, $Q_0 = \underbrace{a_{m-1} * A^{m-2} + \ldots + a_2 * A^1}_{\text{Divisible by A}} + \underbrace{a_1 * A^0}_{\text{Not Divisible by A}}$

Divisible by A **Not Divisible by A**

$$\mathbf{Q_0 = Q_1 A + a_1}$$

$$\mathbf{Q_1 = Q_2 A + a_2}$$

…………..………………..

$$\mathbf{Q_{m-3} = Q_{m-2} A + a_{m-2}}$$

$$\mathbf{Q_{m-2} = a_{m-1} < A} \ \ \textbf{(not divisible by A)}$$

$$\mathbf{= Q_{m-1} A + a_{m-1}}$$

Where $Q_{m-1} = 0$

- This division procedure can be used to convert an integer value from some radix number system to any other radix number system
- An important point to remember is the first digit we get using the division process is a_0, then a_1, then a_2, till a_{m-1}

- In other words, we get the digits of the integer number starting from the radix point and moving lefts

Example :

Convert (53)$_{10}$ ⟶ (?)$_2$

Division Step	Quotient	Remainder	
53 ÷ 2	Q_0 =26	1 = a_0	LSB
26 ÷ 2	Q_1 =13	0 = a_1	
13 ÷ 2	Q_2 =6	1 = a_2	
6 ÷ 2	Q_3 =3	0 = a_3	
3 ÷ 2	Q_4 =1	1 = a_4	
1 ÷ 2	0	1 = a_5	MSB

↑ Stopping Point

Thus (53)$_{10}$=(110101.)$_2$

Since we always divide by the radix, and the quotient is re-divided again by the radix, the solution table may be compacted into 2 columns only as shown:

Binary Point

51	●		53	●	
25	1	a_0	26	1	← LSB
12	1		13	0	
6	0		6	1	
3	0		3	0	
1	1		1	1	
0	1	a_m	0	1	← MSB

$(51)_{10}=(110011.)_2$

$(53)_{10}=(110101.)_2$

Example :

Convert $(755)_{10}$ ⟹ $(?)_8$

Division Step	Quotient	Remainder	
755 ÷ 8	$Q_0=94$	3 = a_0	LSB
94 ÷ 8	$Q_1=11$	6 = a_1	
11 ÷ 8	$Q_2=1$	3 = a_2	
1 ÷ 8	0	1 = a_3	MSB

Thus, $(755)_{10}$ ⟹ $(1363.)_8$

The above method can be more compactly coded as follows:

755	●
94	3
11	6
1	3
0	1

Example :

Convert $(1606)_{10}$ ⟹ $(?)_{12}$

For radix twelve, the allowed digit set is:

{0-9, A, B}

1606 ÷12	●		
133 ÷12	10 = A	LSB	
11 ÷12	1		
0	11 = B	MSB	

$(1606)_{10} \Rightarrow (\text{B1A.})_{12}$

Converting Fractions

Assuming X to be a fraction (< 1),

1. Assume that X_B has *n* digits

$$X_B = (0.b_{-1}\ b_{-2}\ b_{-3}\ldots\ldots b_{-n})_B$$

2. Assume that X_A has *m* digits

$$X_A = (0.a_{-1}\ a_{-2}\ a_{-3}\ldots\ \ldots a_{-m})_A$$

Knowns

Thus, $X_B =\underbrace{(0.b_{-1}\ b_{-2}\ b_{-3}\ldots\ldots b_{-n})_B}_{\text{Knowns}} \Rightarrow \underbrace{(0.a_{-1}\ a_{-2}\ a_{-3}\ldots\ \ldots a_{-m})_A}_{\text{Unknowns}}$

Unknowns

$$X_B = a_{-1}*A^{-1}+a_{-2}*A^{-2}+\ldots\ldots\ldots a_{-m}*A^{-m}$$

Integer   Fraction

$$X_B*A = a_{-1} + X_{B1}$$

Repeating:

$$X_{B1}*A = a_{-2} + X_{B2}$$

……………………

$$X_{Bm-2}*A = a_{-m-1} + X_{Bm-1}$$

$$X_{Bm-1}*A = a_{-m}$$

Example :

Convert $(0.731)_{10}$ → $(?)_2$

● ← **Binary Point**

0.731*2=**1**.462

0.462*2=**0**.924

0.924*2=**1**.848

0.848*2=**1**.696

0.696*2=**1**.392

0.392*2=**0**.784

0.784*2=**1**.568

$$(0.731)_{10} = (.1011101)_2$$

Example :

Convert (0.731) $_{10}$ → (?)$_8$

● ← Octal Point

8*0.731 =5.848

8*0.848 = 6.784

8*0.784 = 6.272

8*0.272 = 2.176

(0.731) $_{10}$ =(0.5662) $_8$

Example :

Convert (0.357) $_{10}$ ⇒ (?)$_{12}$

- **For radix twelve, the allowed digit set is:**
  - **{0-9, A, B}**

● ← System Point

12*0.357 =4.284

12*0.284 = 3.408

12*0. 408= 4.896

12*0. 896= 10.752 ⇒ A=10

=A

(0.357) $_{10}$ ⇒ (0.434A)$_{12}$

IMPORTANT NOTE

For a number that has both integral and fractional parts, conversion is done separately for both parts, and then the result is put together with a system point in between both parts.

Conversion From Bases Other Than 10

Example

$(\quad)_7 \Rightarrow (\quad)_5$

$(\quad)_9 \Rightarrow (\quad)_{12}$

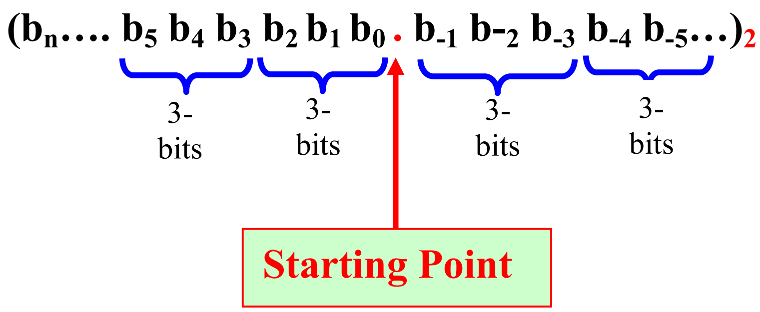
2 Approaches

Perform arith. in original base system (in the above example bases 7 & 9)

1. **Convert to Decimal**
2. **Convert from Decimal to new base (in the above example bases 5&12)**

Binary To Octal Conversion

$(b_n \ldots\ldots b_5\, b_4\, b_3\, b_2\, b_1\, b_0\, .\, b_{-1}\, b_{-2}\, b_{-3}\, b_{-4}\, b_{-5} \ldots\ldots)_2 \Rightarrow (\ ?\ )_8$



Group of 3 Binary Bits $b_{i+2}\ b_{i+1}\ b_i$	Octal Equivalent
0 0 0	0
0 0 1	1
0 1 0	2
0 1 1	3
1 0 0	4
1 0 1	5
1 1 0	6
1 1 1	7

Example :

Convert $(1110010101.1011011)_2$ ino Octal.

We first partition the Binary number into groups of 3 bits

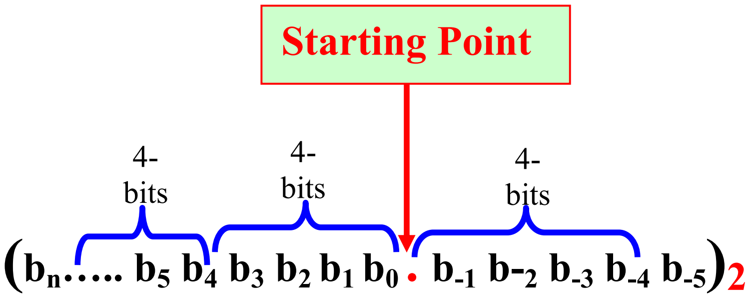
001__110__010__101_._101__101__100

1 6 2 5 5 5 4

$001\_\_110\_\_010\_\_101\_.\_101\_\_101\_\_100 = (1625.554)_8$

Binary To Hexadecimal Conversion

$(b_n \ldots\ldots b_5\, b_4\, b_3\, b_2\, b_1\, b_0\, .\, b_{-1}\, b_{-2}\, b_{-3}\, b_{-4}\, b_{-5} \ldots\ldots)_2 \Rightarrow (\,?\,)_{16}$



Group of 4 Binary Bits $b_{i+3}\, b_{i+2}\, b_{i+1}\, b_i$	Hexadecimal Equivalent
0 0 0 0	0
0 0 0 1	1
0 0 1 0	2
0 0 1 1	3
0 1 0 0	4
0 1 0 1	5
0 1 1 0	6
0 1 1 1	7
1 0 0 0	8
1 0 0 1	9
1 0 1 0	A
1 0 1 1	B
1 1 0 0	C
1 1 0 1	D
1 1 1 0	E
1 1 1 1	F

Example :

Convert $(1110010101.1011011)_2$ into Hexadecimal.

0011__1001__0101_._1011__0110

0011	1001	0101	.	1011	0110
3	9	5		B	6

= $(395.B6)_{16}$

To Convert Between Octal && Hexadecimal Convert to Binary as an Intermediate Step