KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
COLLEGE OF COMPUTER SCIENCES & ENGINEERING

COMPUTER ENGINEERING DEPARTMENT

COE-541 – Performance Analysis of LANs
December 2nd, 2003 – Major Exam #1

Student Name:
Student Number:
Exam Time: 120 mins

- Do not open the exam book until instructed
- The use of programmable calculators and cell phone calculators is not allowed – only basic calculators are permitted
- Answer all questions
- All steps must be shown
- Any assumptions made must be clearly stated

Question No.	Max Points	
1	30	
2	30	
3	30	
4	20	

Total: 110

Q.1) (30 points) A Markov model for packet speech assumes that if the n^{th} packet contains silence then the probability of silence in the next packet is 1- a and the probability of speech activity is a. Similarly if the n^{th} packet contains speech activity, then the probability of speech activity in next packet is 1- b and the probability of silence is b.

a) **(6 points)** Draw the state diagram and write the state transition matrix for this process?

b) **(10 points)** Find the steady state probability mass function for this function?

c) **(6 points)** What is the distribution of the length of time (in packets times) the process spends in the silent state and in the active speech state? Write the distributions mathematically.

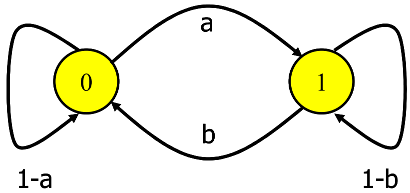
d) **(2 points)** What is the mean of the length of time (in packet times) the process spends in the silent state?

e) **(2 points)** What is the mean of the length of time (in packet times) the process spends in the active speech state?

f) **(4 points)** For a = 1/10 and b = 1/5, If the speech packet length is 360 bits and a packet is produced every 20 msec when the source is active, what is the average output bit rate of this speech encoder?

[See identical solved example in notes slides 27-31 in the Markov Process package]

a) $P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}$



b) The steady state pmf $\prod = [\pi_0 \quad \pi_1]$:

The vector $\prod = [\pi_0 \quad \pi_1]$ satisfies: $\prod = \prod P$

➔ $\pi_0 = (1-a)\pi_0 + a\pi_1$ and $\pi_1 = b\pi_0 + (1-b)\pi_1$

We also need the condition: $\pi_0 + \pi_1 = 1$

➔ $\pi_0 = \dfrac{b}{a+b}$ and $\pi_1 = \dfrac{a}{a+b}$

c) The distribution of the length of time (in packet times), Ls, the process spends in the silent state is geometric – Prob[Ls = k] = $a(1 - a)^k$

The distribution of the length of time (in packet times), La, the process spends in the active state is geometric – Prob[La = k] = $b(1 - b)^k$

d) mean of Ls = 1/a packet times

e) mean of La = 1/b packet times

f) a = 1/10, b = 1/5, packet size = 360 bits, packet time = 20 msec

Ron = 360/0.02 = 18 kb/s, Roff = 0

π_1 = a/(a+b) = 1/3 ➔ average bit rate = π_1 X Ron = 6 kb/s

Q.2) (30 point) A company has a system with four private telephone lines connecting two of its sites. Suppose that requests for these lines arrive according to a Poisson process at rate of one call every 2 minutes, and suppose that call durations are exponentially distributed with mean 4 minutes. When all lines are busy, the system delays (i.e. queues) call requests until a line becomes available.

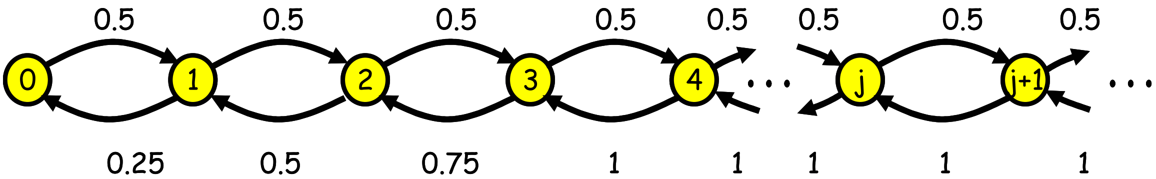
a. **(4 points)** What queueing model applies to this problem?
b. **(6 points)** What is the average number of simultaneous calls?
c. **(4 points)** Draw the transition rate diagram for this Markov chain? Specify the transition rates values and the state indices (i.e. do not just use labels like μ or λ)
d. **(6 points)** Find the probability that a caller has to wait for a line?
e. **(10 points)** What is the average waiting time for a caller?

[See identical solved example in notes slides 37-38 in the Queueing Models package]

a) Queueing model: M/M/c

b) Average number of simultaneous calls: a = ½ X 4 = 2 simultaneous calls

c)



d) $\lambda = \frac{1}{2}$, $1/\mu = 4$, $c = 4$ ➔ $a = \lambda/\mu = 2$

➔ $\rho = a/c = \frac{1}{2}$

$p_0 = \{1+2+2^2/2!+2^3/3!+2^4/4!\ (1/(1-\rho))\}^{-1}$
$= 3/23$

$p_c = a^c/c!\ p_0$
$= 2^4/4! \times 3/23 = 2/23$

$\text{Prob}[W > 0] = p_c/(1-\rho)$
$= 2/23 \times 1/(1-1/2)$
$= 4/23$
≈ 0.17

e) The average waiting time, E[*W*]:

$E[W] = E[Nq]/\lambda$

But $E[Nq] = \frac{\rho}{1-\rho}\Pr ob[W > 0]$ = 0.17 customer

Therefore, E[W] = 0.17/ (1/2) = 8/23 ≈ 0.35 minutes

{For parts **d** and **e** you are required to derive the relations if you do not memories them – for derivation refer to class notes}

Q.3) (30 points) A centralized network providing a maximum of 10 Mbps and services a large set of user terminal with slotted ALOHA protocol.

a. **(5 points)** What is the maximum normalized throughput for network? What is the maximum throughput for network in Mb/s?
b. **(5 points)** At the maximum throughput point, what is the offered traffic (in bits per second) in the medium and how is it composed?
c. **(5 points)** If a packet length is 64KBytes, what is the average waiting time in terminal before a packet is transmitted in milliseconds? Assume average backoff time = 1000 millisecond.
d. **(5 points)** What is the average number of transmissions of a packet? What is the average number of retransmissions of a packet?
e. **(5 points)** Plot/sketch the throughput versus offered load curve for an ideal network access (or resource sharing) scheme? Clearly specify the axes and their units.
f. **(5 points)** Plot/sketch the total packet delay versus throughput curve for slotted ALOHA? Clearly specify the axes and their units.

[See identical solved example in notes slides 26-27 – but for pure ALOHA rather than slotted - in the LAN Performance package]

a) S = G * exp(-G) ➔ maximum normalized throughput S = 0.36 packet / packet time for G = 1.0 packet / packet transmission time

Maximum throughput = 0.36 X 10 Mb/s = 3.6 Mb/s

b) The offered load (G) = 1.0 X 10 Mb/s = 10 Mb/s

Composition: Total = 10 Mb/s
Useful = 3.6 Mb/s
Collisions/retransmissions = 6.4 Mb/s

c) packet length = 64 Kbyte = 64 X 1024 X 8 = 524288 bits, Average backoff, B = 1000 millisecond

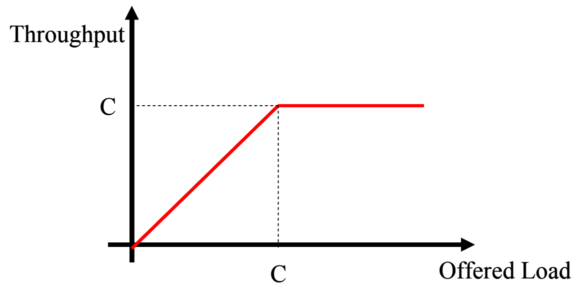
Packet time = 52.4 msec

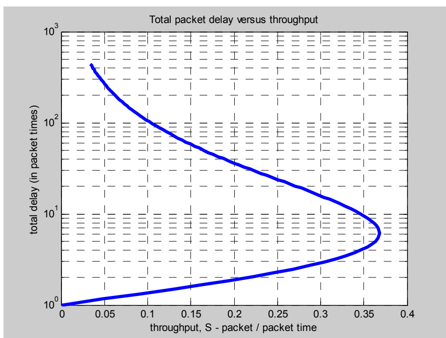
W = (exp(G) – 1) X (P + B) = (exp(1.0) – 1) X (52.4 + 1000)
= 1808 millisecond

d) Average number of transmissions per packet = G/S = 2.7 times

Average number of transmissions per packet = G/S – 1 = 1.7 times

e) Ideal Load-Throughput relation: (Throughput – bits/ second or packets/packet time Offered Load - bits/ second or packets/packet time C – link capacity limit bits/ second or packets/packet time)	**f)** The total packet delay versus throughput for slotted ALOHA: (Throughput – bits/ second or packets/packet time Total delay – time units (seconds, or packet time
(figure below)	(figure below)





Q.4) (20 points) To minimize bandwidth resources, a Raised Cosine Pulse is used as a baseband signal. The pulse, $p(t)$, and its Fourier Transform, $P(f)$, are given below:

$$p(t) = \frac{(2A)}{T}\frac{\cos(2\pi\alpha t)}{1-(4\alpha t)^2}\frac{\sin(2\pi t/T)}{2\pi t/T}$$

$$P(f) = \begin{cases} A & |f| > \frac{1}{T} - \alpha \\ A\cos^2\left(\frac{\pi}{4\alpha}\left(|f| - \frac{1}{T} + \alpha\right)\right) & \frac{1}{T} - \alpha < |f| < \frac{1}{T} + \alpha \\ 0 & |f| > \frac{1}{T} + \alpha \end{cases}$$

a. **(4 points)** What is the range of the parameter α? What is the role of this parameter?
b. **(4 points)** Briefly explain the advantages of using the Raised Cosine Pulse?
c. **(4 points)** What is the minimum and maximum bandwidth?
d. **(8 points)** A communication link design uses a value of α equal to 3/(4T) where T is equal to 5 microseconds. The estimated link SNR is 20 dBs, what is the maximum data rate that can be transmitted over the channel using these pulses – assuming channel bandwidth is identical to that for the pulses. (**Hint**: use Shannon's capacity equation).

[See notes slide 7 in the Digital Communications package for properties of the Raised Cosine Filter – parts a, b, and c of the question

See notes slide 20 in the Digital Communications package for application of Shannon's formula – part d of the question]

a) Range for α: 0 < α < 1/T – controls bandwidth of pulse
α = 0 ➔ BW = 1/T Hz, α = 1/T ➔ BW = 2/T Hz

b) zero ISI – ideal for pulse shaping – limited bandwidth

c) α = 0 ➔ BW = 1/T Hz, α = 1/T ➔ BW = 2/T Hz
d) α = ¾ /T ➔ BW = 1.75 / T, since T = 5 microseconds ➔ BW = 350 KHz

SNR = 20 dBs = $10^{(20/10)}$ = 100 (on the linear scale)
Using Shannon's formula
R = BW X $\log_2$(1 + SNR) = 350 X $\log_2$(101) = 2.33 Mb/s

Identities you MIGHT need:

$\sum_{n=1}^{M} n = \frac{1}{2} M(M+1)$	$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}; \lvert r \rvert < 1$
$\sum_{n=0}^{M} \binom{M}{n} r^n = (1+r)^M ; \lvert r \rvert < 1$	$\sum_{n=0}^{M} r^n = \frac{1-r^{M+1}}{1-r}; \lvert r \rvert < 1, M = 1,2,\ldots$
$\sum_{n=0}^{\infty} n r^{n-1} = \frac{1}{(1-r)^2}; \lvert r \rvert < 1$	$\sum_{n=0}^{M} n r^{n-1} = \frac{1+(Mr-M-1)r^M}{(1-r)^2}; \lvert r \rvert < 1$