ISE 307, Term 153

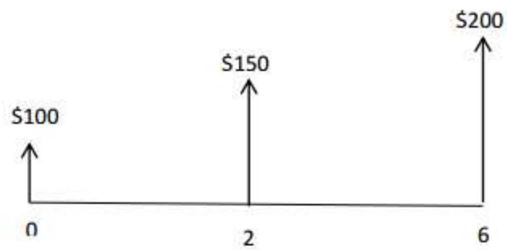
ENGINEERING ECONOMIC ANALYSIS

HW# 1 Solution

Due date: Wednesday, July 20

Q.1. Which of the following alternatives would you choose, assuming an interest rate of 10% compounded annually?

- Alternative 1: Receive $100 today;
- Alternative 2: Receive $150 two years from now.
- Alternative 3: Receive $200 six years from now.



Solution 1: (bring all values to the end of the period)

F1 = P1(F/P, 10%, 6) = 100(1.7716) = $177.16
F2 = P2(F/P, 10%, 4) = 150(1.4641) = $219.62
F3 = $200

Therefore, it is better to receive $150 two years from now.

Or

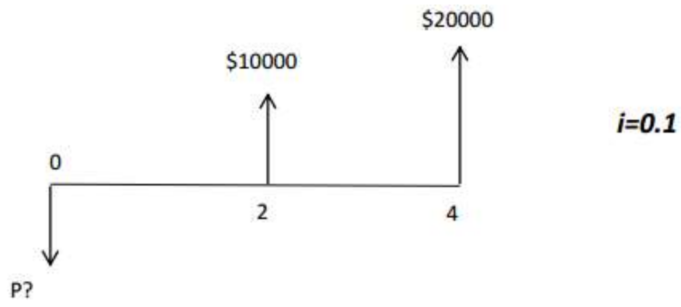
Solution 2: (bring all values to the beginning period now)

P1 =$100
P2 =150 (P / F, 10%, 2) = 150 (0.8264) = $123.96
P3 =200 (P / F, 10%, 6) = 200 (0.5645) = $112.9

Q.2. How many years will it take to triple your investment of $5,000 if it has an interest rate of 10% compounded annually?

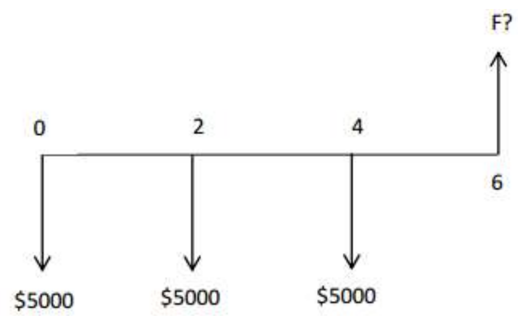
$3P = P\,(F/P, 10\%, N) = P\,(1.10)^N$
$3 = (1.10)^N$
$N = \log 3/\log 1.10 = 11.53 = 12$ years

Q.3. If you want to withdraw $10,000 at the end of two years and $20,000 at the end of four years, how much should you deposit now into an account that pays 10% interest compounded annually?



$P = 10000(1.1)^{-2} + 20000(1.1)^{-4} = \$21924.73$

Q.4. You deposit $5,000 today, $5,000 two years from now, and $5,000 fours years from now. How much money will you have at the end of year six if there are different annual compound-interest rates per period such that interest rate in the first two years is 5%, 7% in the third and fourth years and 8% in the fifth and sixth year?



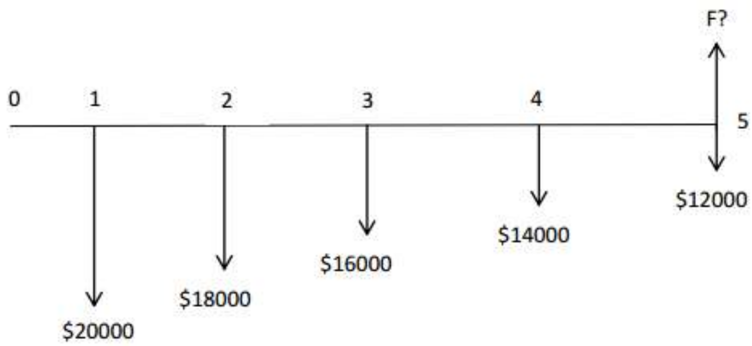
$F = 5000(1.05)^2 (1.07)^2 (1.08)^2 + 5000(1.07)^2 (1.08)^2 + 5000(1.08)^2 = \$19870.51$

Q.5. If $500 is deposited in a savings account at the beginning of each year for 12 years and the account earns 10% interest compounded annually, what will be the balance on the account the end of the 15 years (F)?

$F12 = A(F/A, i, N) = 500(F/A, 10\%, 13) = 500*24.5227 = 12,261.36$
OR
$F12 = A(F/A, i, N)*1.1+500 = 500(F/A, 10\%, 12)*1.1+500$
$= 500(21.3843)*1.1+500 = \$10,692.15 *1.1+500=12,261.36$
OR $F12 = 500(F/A, 10\%, 12) + 500(F/P, 10\%, 12) = \$10,692.15+\$1,569.2= \$12,261.36$

$F15 = 12,261.36 (1.1)^3 = \$16,319.87$

Q.6. Five annual deposits in the amounts of \$20,000, \$18,000, \$16,000, \$14,000, and \$12,000 are made into a fund that pays interest at a rate of 10% compounded annually. Determine the amount in the fund immediately after the fifth deposit.



$F = 20000(1.1)^4 + 18000(1.1)^3 + 16000(1.1)^2 + 14000(1.1)^1 + 12000 = \$100,000.$

Another solution using Linear Gradient Series:

F = 20000(F/A, 10%, 5) – 2000 (P/G, 10%, 5) (F/P, 10%, 5)
= 20000*6.1051 – 2000*6.8618*1.6105= \$100,000.

Q.7. Suppose that an oil well is expected to produce 1,200,000 barrels of oil during its first production year. However, its subsequent production (yield) is expected to decrease by 9% over the previous year's production.

(a) Suppose that the price of oil is expected to be \$120 per barrel for the next five years. What would be the present worth of the anticipated revenue trim at an interest rate of 10% compounded annually over the next five years?

$$g = -0.09 \qquad A_1 = 120 \times 1,200,000 = 144 \times 10^6 \qquad i = 0.1 \quad N = 5$$

$$P = A_1\left[\frac{(1-(1+g)^N(1+i)^{-N})}{(i-g)}\right] = 144 \times 10^6\left[\frac{(1-(0.91)^5(1.1)^{-5})}{0.19}\right] = \$464,229,575.9$$

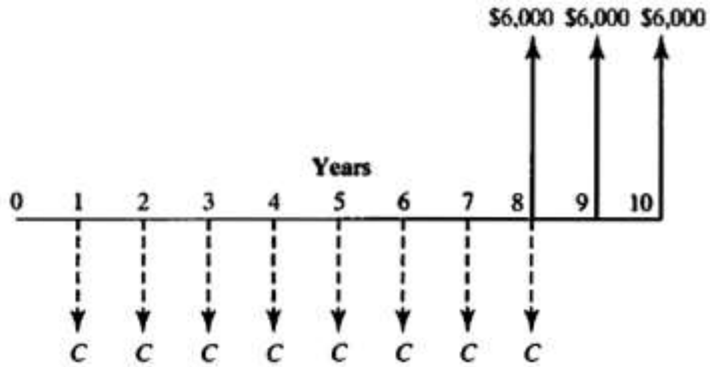
(b) Suppose that the price of oil is expected to start at \$120 per barrel during the first year, but to increase at the rate of 5% over the previous year's price. What would be the present worth of the anticipated revenue stream at an interest rate of 10% compounded annually over the next five years?

(1+g) =(1+g1)(1+g2) => g = g1 + g2 + g1*g2

g = -0.09 + 0.05 - 0.0045 = -0.0445

$$\mathbf{P = 144 \times 10^6 [1 - (1-0.0445)^5(1.1)^{-5}]/[0.1+0.0445] = \$503,723,932}$$

Q.8. From the following cash flow diagram, find the value of C that will establish economic equivalence between the deposit series and the withdrawal series at an interest rate of 8% compounded annually.



C (F/A, 8%, 8) = 6000 (P/A, 8%, 2) + 6000

C*10.6366 = 6000*1.7833 + 6000

C*10.6366 = 16699.8

C = 16699.8/10.6366 = $1570.03

Another solution:

$P8 = 6000 + 6000(1.08)^{-1} + 6000(1.08)^{-2} = \$16699.59$

$P8 = F \rightarrow C = A = F(A/F, i, N) = 16699.59(A/F, 8\%, 8) = 16699.59(0.0940) = \$1569.76$

Q.9. It is said that a lump-sum amount of $40,000 at the end of five years is equivalent to an equal-payment series of $4,000 per year for 10 years, where the first payment occurs at the end of year 1. What earning interest is assumed in this calculation?

$40000\ (1+i)^{5} = 4000\ [(1+i)^{10}-1]/i$

We can solve this by trial and error or using Goal Seek function in Excel and we get i=13.06%

Another Solution:

$P1 = 40000\ (P/F, i, 5) = 40000\ (1+i)^{-5}$

$P2 = 4000\ (P/A, i, 10) = 4000\ [(1+i)^{10}-1]/[i*(1+i)^{10}]$

$40000\ (1+i)^{-5} = 4000\ [(1+i)^{10}-1]/[i*(1+i)^{10}]$

We can solve this by trial and error or using Goal Seek function in Excel and we get i=13.06%.