

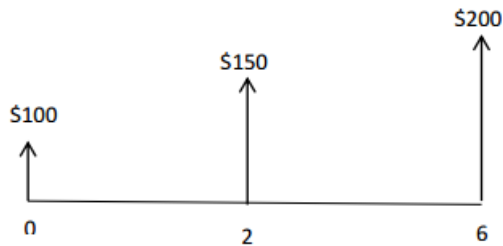
ENGINEERING ECONOMIC ANALYSIS

HW# 1 Solution

Due date: Wednesday, July 20

**Q.1.** Which of the following alternatives would you choose, assuming an interest rate of 10% compounded annually?

- Alternative 1: Receive \$100 today;
- Alternative 2: Receive \$150 two years from now.
- Alternative 3: Receive \$200 six years from now.



Solution 1: (bring all values to the end of the period)

$$F1 = P1(F/P, 10\%, 6) = 100(1.7716) = \$177.16$$

$$F2 = P2(F/P, 10\%, 4) = 150(1.4641) = \$219.62$$

$$F3 = \$200$$

Therefore, it is better to receive \$150 two years from now.

Or

Solution 2: (bring all values to the beginning period now)

$$P1 = \$100$$

$$P2 = 150 (P / F, 10\%, 2) = 150 (0.8264) = \$123.96$$

$$P3 = 200 (P / F, 10\%, 6) = 200 (0.5645) = \$112.9$$

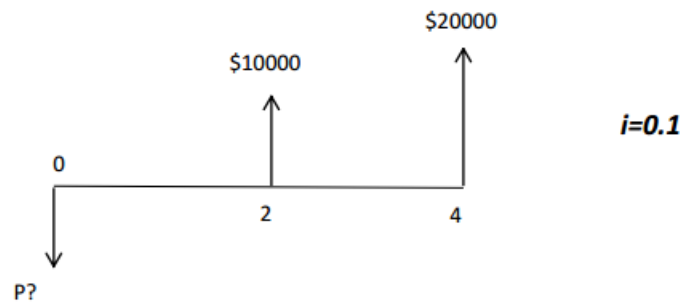
**Q.2.** How many years will it take to triple your investment of \$5,000 if it has an interest rate of 10% compounded annually?

$$3P = P (F/P, 10\%, N) = P (1.10)^N$$

$$3 = (1.10)^N$$

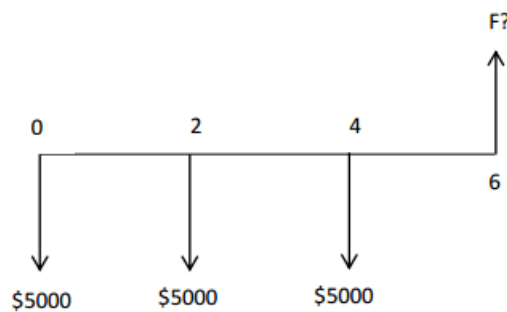
$$N = \log 3 / \log 1.10 = 11.53 = 12 \text{ years}$$

- Q.3.** If you want to withdraw \$10,000 at the end of two years and \$20,000 at the end of four years, how much should you deposit now into an account that pays 10% interest compounded annually?



$$P = 10000(1.1)^{-2} + 20000(1.1)^{-4} = \$21924.73$$

- Q.4.** You deposit \$5,000 today, \$5,000 two years from now, and \$5,000 four years from now. How much money will you have at the end of year six if there are different annual compound-interest rates per period such that interest rate in the first two years is 5%, 7% in the third and fourth years and 8% in the fifth and sixth year?



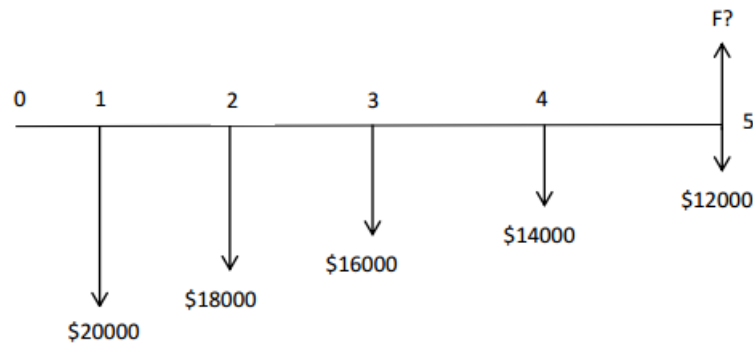
$$F = 5000(1.05)^2 (1.07)^2 (1.08)^2 + 5000(1.07)^2 (1.08)^2 + 5000(1.08)^2 = \$19870.51$$

- Q.5.** If \$500 is deposited in a savings account at the beginning of each year for 12 years and the account earns 10% interest compounded annually, what will be the balance on the account the end of the 15 years (F)?

$$F_{12} = A(F/A, i, N) \cdot 1.1 = 500(F/A, 10\%, 12) \cdot 1.1 \\ = 500(21.3843) \cdot 1.1 = \$10,692.15 \cdot 1.1 = 11761.37$$

$$F_{15} = 11761.37 (1.1)^3 = \$15654.42$$

- Q.6.** Five annual deposits in the amounts of \$20,000, \$18,000, \$16,000, \$14,000, and \$12,000 are made into a fund that pays interest at a rate of 10% compounded annually. Determine the amount in the fund immediately after the fifth deposit.



$$F = 20000(1.1)^4 + 18000(1.1)^3 + 16000(1.1)^2 + 14000(1.1)^1 + 12000 = \$100,000.$$

Another solution using Linear Gradient Series:

$$\begin{aligned} F &= 20000(F/A, 10\%, 5) - 2000(P/G, 10\%, 5)(F/P, 10\%, 5) \\ &= 20000 * 6.1051 - 2000 * 6.8618 * 1.6105 = \$100,000. \end{aligned}$$

- Q.7.** Suppose that an oil well is expected to produce 1,200,000 barrels of oil during its first production year. However, its subsequent production (yield) is expected to decrease by 9% over the previous year's production.

- (a) Suppose that the price of oil is expected to be \$120 per barrel for the next five years. What would be the present worth of the anticipated revenue stream at an interest rate of 10% compounded annually over the next five years?

$$g = -0.09 \quad A_1 = 120 \times 1,200,000 = 144 \times 10^6 \quad i = 0.1 \quad N = 5$$

$$\begin{aligned} P &= A_1 \left[ \frac{(1 - (1 + g)^N (1 + i)^{-N})}{(i - g)} \right] = 144 \times 10^6 \left[ \frac{(1 - (0.91)^5 (1.1)^{-5})}{0.19} \right] \\ &= \$464,229,575.9 \end{aligned}$$

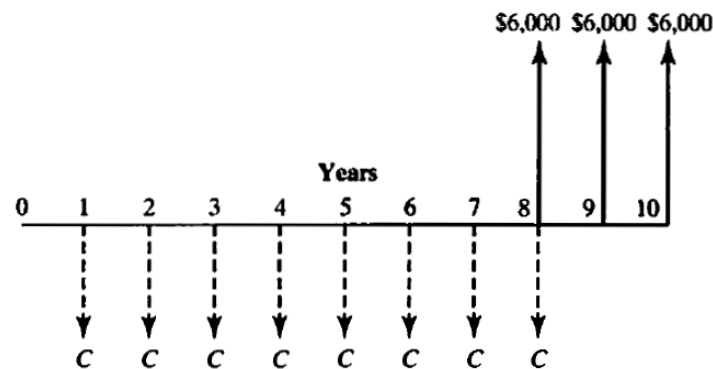
- (b) Suppose that the price of oil is expected to start at \$120 per barrel during the first year, but to increase at the rate of 5% over the previous year's price. What would be the present worth of the anticipated revenue stream at an interest rate of 10% compounded annually over the next five years?

$$(1+g) = (1+g_1)(1+g_2) \Rightarrow g = g_1 + g_2 + g_1 * g_2$$

$$g = -0.09 + 0.05 - 0.0045 = -0.0445$$

$$P = 144 \times 10^6 [1 - (1 - 0.0445)^5 (1.1)^{-5}] / [0.1 + 0.0445] = \$503,723,932$$

- Q.8.** From the following cash flow diagram, find the value of C that will establish economic equivalence between the deposit series and the withdrawal series at an interest rate of 8% compounded annually.



$$C (F/A, 8\%, 8) = 6000 (P/A, 8\%, 2) + 6000$$

$$C * 10.6366 = 6000 * 1.7833 + 6000$$

$$C * 10.6366 = 16699.8$$

$$C = 16699.8 / 10.6366 = \$1570.03$$

Another solution:

$$P_8 = 6000 + 6000(1.08)^{-1} + 6000(1.08)^{-2} = \$16699.59$$

$$P_8 = F \rightarrow C = A = F(A/F, i, N) = 16699.59(A/F, 8\%, 8) = 16699.59(0.0940) = \$1569.76$$

- Q.9.** It is said that a lump-sum amount of \$40,000 at the end of five years is equivalent to an equal-payment series of \$4,000 per year for 10 years, where the first payment occurs at the end of year 1. What earning interest is assumed in this calculation?

$$40000 (1+i)^5 = 4000 [(1+i)^{10}-1]/i$$

We can solve this by trial and error or using Goal Seek function in Excel and we get  $i=13.06\%$

Another Solution:

$$P_1 = 40000 (P/F, i, 5) = 40000 (1+i)^{-5}$$

$$P_2 = 4000 (P/A, i, 10) = 4000 [(1+i)^{10}-1]/[i*(1+i)^{10}]$$

$$40000 (1+i)^{-5} = 4000 [(1+i)^{10}-1]/[i*(1+i)^{10}]$$

We can solve this by trial and error or using Goal Seek function in Excel and we get  $i=13.06\%$ .