

**COE 561, Term 091**  
**Digital System Design and Synthesis**

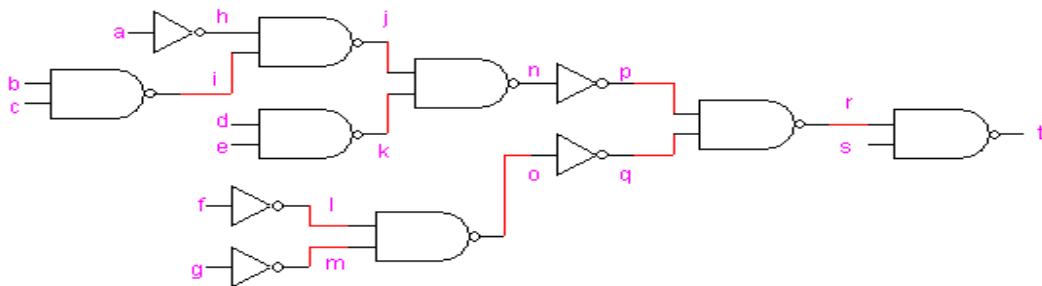
**HW# 4 Solution**

**Due date: Tuesday, Jan. 19**

**Q.1.** Consider a technology library containing the following cells:

Cell	Area Cost
$INV(x1) = x1'$	1
$NAND2(x1, x2) = (x1 \ x2)'$	2
$NAND3(x1, x2, x3) = (x1 \ x2 \ x3)'$	3
$NOR2(x1, x2) = (x1 + x2)'$	2
$AOI21(x1, x2, x3) = ((x1 \ x2) + x3)'$	3
$OAI21(x1, x2, x3) = ((x1+x2) \ x3)'$	3
$AOI22(x1, x2, x3, x4) = (x1 \ x2 + x3 \ x4)'$	4
$OAI22(x1, x2, x3, x4) = ((x1+x2) \ (x3+x4))'$	4

- (i) Show the **pattern trees** of the library cells using **NAND2** and **INV** as base functions. Assume that symmetric representations do not need to be stored.
- (ii) Using the dynamic programming approach, **map** the circuit given below using the given library into the **minimum area** cost solution. Inputs are  $\{a, b, c, d, e, f, g, s\}$  and output is  $\{t\}$ .
- (iii) Using the given library, use the SIS command *read\_libray q1.lib* to read the library. Then, map the circuit to the library using the sis command *map -s -m 0*. Compare your solution to the solution obtained in (iii) and comment on any differences. You can save the mapped circuit using the sis command *write\_blif -n*.



**Q.2.** Assuming **Boolean matching**, determine the number of ROBDD's that need to be stored in the cell library for each of the following cells. Justify your answer.

(i)  $f = a \oplus b \oplus c$





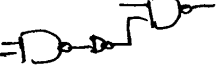
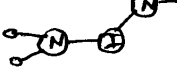




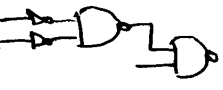

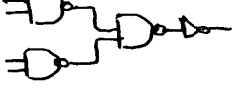

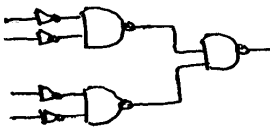
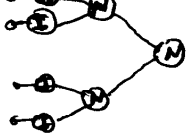
(ii)  $f = a b + a c + b c$

(iii)  $f = a b + a' b' + a c + b c$

HW#4 Solution

Q1.

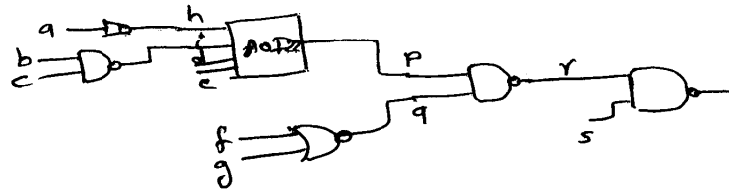
(1) Library Pattern Trees:

	cell	gate	pattern tree	cost
t1	INV			1
t2	NAND2			2
t3	NAND3			3
t4	NOR2			2
t5	AOI21			3
t6	OAI21			3
t7	AOI22			4
t8	OAI22			4

(ii) Mapping for minimum area:

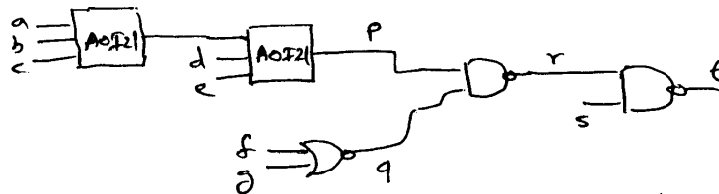
vertex	Match	gate	cost
h	t <sub>1</sub>	INV(a)	1
i	t <sub>2</sub>	NAND <sub>2</sub> (b,c)	2
l	t <sub>1</sub>	INV(f)	1
m	t <sub>1</sub>	INV(g)	1
k	t <sub>2</sub>	NAND <sub>2</sub> (d,e)	2
j	t <sub>2</sub>	NAND <sub>2</sub> (h,i)	2+1+2=5
o	t <sub>2</sub>	NAND <sub>2</sub> (l,m)	2+1+1=4
n	t <sub>2</sub>	NAND <sub>2</sub> (j,k)	2+5+2=9
p	t <sub>1</sub>	INV(n)	9+1=10
	t <sub>7</sub>	AOI <sub>22</sub> (h,i,d,e)	4+1+2=7
q	t <sub>1</sub>	INV(o)	4+1=5
	t <sub>4</sub>	NOR <sub>2</sub> (f,g)	2
r	t <sub>2</sub>	NAND <sub>2</sub> (p,q)	2+7+2=11
	t <sub>3</sub>	NAND <sub>3</sub> (j,k,q)	3+5+2+2=12
	t <sub>3</sub>	NAND <sub>3</sub> (p,l,m)	3+7+1+1=12
s	t <sub>2</sub>	NAND <sub>2</sub> (r,s)	2+11=13
	t <sub>6</sub>	OAIF <sub>21</sub> (n,o,s)	3+9+4=16

Thus, the minimum cost is 13 and the mapped solution is shown below:



(iii) Mapping using SIS:

The solution obtained by SIS has an area cost of 12 as follows:



This optimization is done based on inserting pairs of inverters at each line in the subject graph and then finding an optimal mapping of the subject graph. This is followed by removing any cascaded inverters and removing their cost.

The solution obtained by SIS resulted from adding two inverters after node  $j$ .

Q2. Number of ROBDD's for Boolean Matching

$$(i) f = a \oplus b \oplus c$$

since  $a, b$  and  $c$  are all symmetric

$$C_3 = \{(a, b, c)\} \Rightarrow \# \text{ROBDD's} = 1$$

$$(ii) f = ab + ac + bc$$

since  $a, b$  and  $c$  are all symmetric

$$C_3 = \{(a, b, c)\} \Rightarrow \# \text{ROBDD's} = 1$$

$$(iii) f = ab + \bar{a}\bar{b} + ac + bc$$

variables  $a, b$  are symmetric

$$C_2 = \{(a, b)\}$$

$$\Rightarrow \# \text{ROBDD's} = 1$$