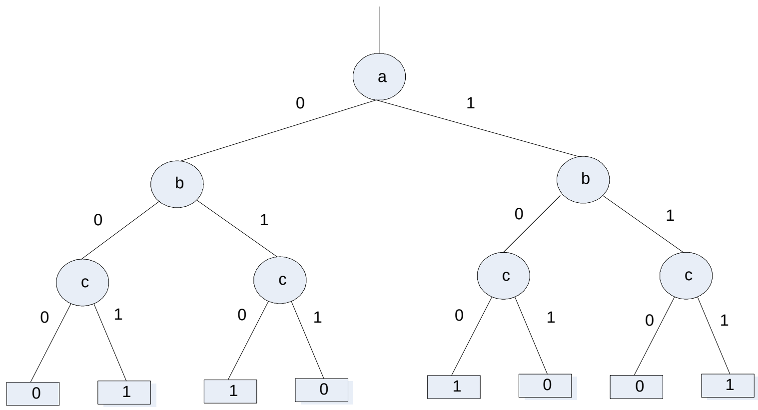
COE 561, Term 111

Digital System Design and Synthesis

HW# 1 Solution

Due date: Saturday, Oct. 15

Q.1. Consider the following OBDD with the variable ordering {a, b, c}. Reduce it based on **Reduce** function to obtain the ROBDD. Show the details of your work.



Q.2. Consider the function f= a (b+c)(d+e):

(i) Draw the **ROBDD** for the function using the variable order {a, b, c, d, e}.

(ii) Draw the **ROBDD** for the function using the variable order {b, d, a, c, e}.

(iii) Comment on the difference between the two obtained ROBDDS and what heuristic do you suggest one should choose in selecting a variable order.

Q.3. Consider the two functions $f=a\oplus b\oplus c$ and $g=b\oplus c'\oplus d$:

(i) Compute the function f.g based on orthonormal basis expansion.

(ii) Draw the **ITE DAG** for the function $f\oplus g$. Show the details of the ITE algorithm step by step. Use the variable order {a, b, c, d}

Q.4. Consider the following given matrix representing a covering problem:

$$A=\begin{bmatrix}1&1&0&0&0&0\\0&1&1&1&0&0\\1&0&0&1&0&1\\1&0&1&0&0&1\\0&0&0&0&1&0\\0&0&0&1&1&0\\1&1&1&0&0&0\end{bmatrix}$$

Find a **minimum cover** using **EXACT_COVER** procedure. Show all the details of the algorithm. Assume the following order in branching selection when needed: C_1, C_2, C_3, C_4, C_5, C_6.

Q.5. Consider the function $F(A,B,C,D) = \overline{A}\overline{C} + CD + AB + BC + \overline{B}\overline{D} + AD$. Using recursive paradigm, determine if the function F is **tautology** or not. You need to choose the right variable for expansion to minimize computations.

Q.6. Consider the function $F(A,B,C,D) = \overline{B}\overline{D} + ACD + B\overline{C} + BC\overline{D}$

(i) Compute the **complement** of the function using the recursive complementation procedure outlined in section 7.3.4. You need to choose the right variable for expansion to minimize computations.

(ii) Compute all the **prime implicants** of the function using the method outlined in section 7.3.4. You need to choose the right variable for expansion to minimize computations.

COE 561 : 111

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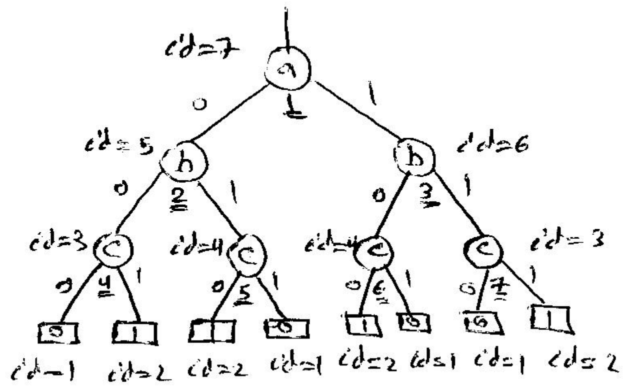
HW#1

Q1.



First, we set $id(v)=1$ for all leaf vertices with value 0 and $id(v)=2$ for all leaf vertices with value 1.

We initialize ROBDD with two leaf vertices for 0 and 1.

Then, we process vertices at level 3, i.e. nodes with index = c. $V = \{4, 5, 6, 7\}$

None of the vertices is removed since $id(low(v)) \neq id(high(v))$.

We assign keys to all vertices $\in V$.

$Key(4) = (1,2)$, $Key(5) = (2,1)$, $Key(6) = (2,1)$, $Key(7) = (1,2)$.

$oldKey = (0,0)$.

We next sort the vertices in V according to their keys. Thus, $V = \{4, 7, 5, 6\}$.

$v = \{4\}$: since $Key(4) \neq oldKey$, $nextid = 3$,

$id(4) = 3$, $oldKey = (1,2)$.

We add $v = \{4\}$ to the ROBDD.

$v = \{7\}$: since $Key(7) = oldKey$, $id(7) = 3$.

$v = \{5\}$: since $Key(5) \neq oldKey$, $nextid = 4$,
$id(5) = 4$, $oldKey = (2,1)$.
We add $v = \{5\}$ to the ROBDD.

$v = \{6\}$: since $Key(6) = oldKey$, $id(6) = 4$.

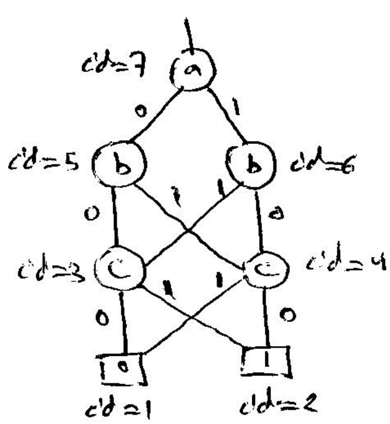
Next, we process vertices at level 2 with index $= b$.

$V = \{2, 3\}$.
None of the vertices is removed.
$Key(2) = (3,4)$, $Key(3) = (4,3)$, $oldKey = (0,0)$.

$v = \{2\}$: since $Key(2) \neq oldKey$, $nextid = 5$,
$id(2) = 5$, $oldKey = (3,4)$.
We add $v = \{2\}$ to the ROBDD.

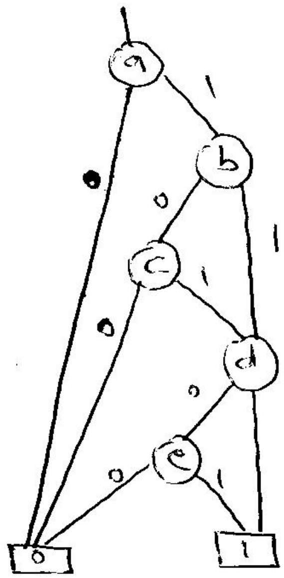
$v = \{3\}$: since $Key(3) \neq oldKey$, $nextid = 6$,
$id(3) = 6$, $oldKey = (4,3)$.
We add $v = \{3\}$ to the ROBDD.

Finally, we process vertices at level 1 with index $= a$. $V = \{1\}$.
The vertex is not removed.
Since $id(low(1)) \neq id(high(1))$, the vertex is not removed. Since $Key(1) \neq oldKey$,
$Key(1) = (5,6)$, $oldKey = (0,0)$. We add $v = \{1\}$ to the ROBDD.
$nextid = 7$, $id(1) = 7$.
Thus, the formed ROBDD is:

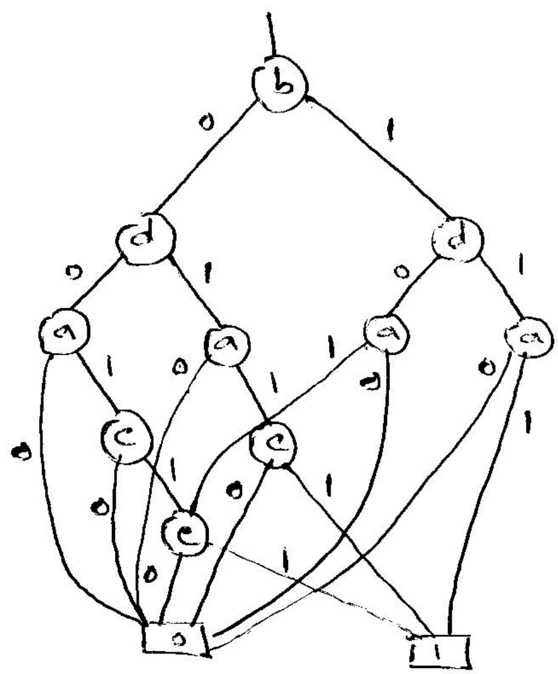


Q2, $f = a(b+c)(d+e)$

(i) variable order $\{a, b, c, d, e\}$



(ii) variable order $\{b, d, a, c, e\}$



(iii) We can see that the variable order in (i) produces a smaller size ROBDD than (ii). As a general heuristic, we should choose the variable that eliminates the largest number of terms in the expression.

Q3. $f = a \oplus b \oplus c$, $g = b \oplus \bar{c} \oplus d$

(i) $f \cdot g$

$$f = \bar{b}\bar{c}\,[a] + \bar{b}c\,[\bar{a}] + b\bar{c}\,[\bar{a}] + bc\,[a]$$

$$g = \bar{b}\bar{c}\,[\bar{d}] + \bar{b}c\,[d] + b\bar{c}\,[d] + bc\,[\bar{d}]$$

$$f \cdot g = \bar{b}\bar{c}\,[a\bar{d}] + \bar{b}c\,[\bar{a}d] + b\bar{c}\,[\bar{a}d] + bc\,[a\bar{d}]$$

(ii) ITE diagram for the function $f \oplus g$

$$f \oplus g = ITE(f, \bar{g}, g) \qquad \{a, b, c, d\}$$

$$= ITE(a \oplus b \oplus c,\ b \oplus c \oplus d,\ b \oplus \bar{c} \oplus d)$$

- $x = a$

  $t = ITE(\overline{b \oplus c},\ b \oplus c \oplus d,\ b \oplus \bar{c} \oplus d)$

  - $x = b$

    $t = ITE(c,\ \overline{c \oplus d},\ c \oplus d)$

    - $x = c$

      $t = ITE(1, d, \bar{d}) = d$ (trivial case)

      we assign $id = 3 \Rightarrow t = 3$

      $e = ITE(0, \bar{d}, d) = d$ (trivial case)

      $\Rightarrow e = 3$

since $t = e$, we return 3

$\Rightarrow t = 3$

$e = ITE(\bar{c}, c \oplus d, \bar{c} \oplus d)$

- $x = c$

$t = ITE(0, \bar{d}, d) = d$ (trivial case)

$\Rightarrow t = 3$

$e = ITE(1, d, \bar{d}) = d$ (trivial case)

$\Rightarrow e = 3$

since $t = e$, we return 3

$\Rightarrow e = 3$

since $t = e$, we return 3

$\Rightarrow t = 3$

$e = ITE(b \oplus c, b \oplus c \oplus d, b \oplus \bar{c} \oplus d)$

- $x = b$

$t = ITE(\bar{c}, \overline{c \oplus d}, c \oplus d)$

- $x = c$

$t = ITE(0, d, \bar{d}) = \bar{d}$ (trivial case)

we assign $id = 4 \Rightarrow t = 4$

$e = ITE(1, \bar{d}, d) = \bar{d}$ (trivial case)

$\Rightarrow e = 4$

since $t = e$, we return 4

$\Rightarrow t = 4$

$e = ITE(c, c \oplus d, \bar{c} \oplus d)$

- $x = c$

  $t = ITE(1, \bar{d}, d) = \bar{d}$ (trivial case)

  $\Rightarrow t = 4$

  $e = ITE(0, d, \bar{d}) = \bar{d}$ (trivial case)

  $\Rightarrow e = 4$

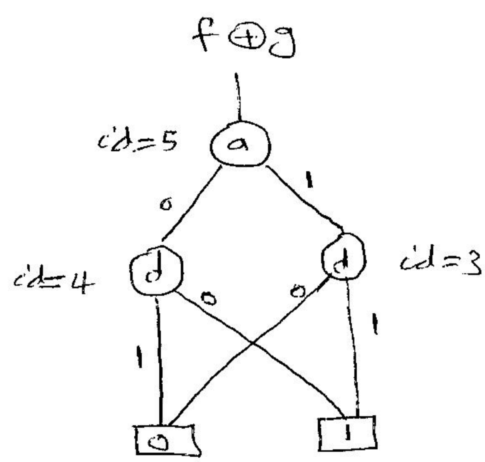
  Since $t = e$, we return 4

$\Rightarrow e = 4$

Since $t \neq e$, we add the entry $(a, 3, 4)$ in the unique table with $id = 5$

Unique Table:

id	var	H	L
3	d	2	1
4	d	1	2
5	a	3	4



Computed Table:

f	g	h	id
$a \oplus b \oplus c$	$b \oplus c \oplus d$	$b \oplus \bar{c} \oplus d$	5

Q4. The matrix to be covered:

	c_1	c_2	c_3	c_4	c_5	c_6
r_1	1	1	0	0	0	0
r_2	0	1	1	1	0	0
r_3	1	0	0	1	0	1
r_4	1	0	1	0	0	1
r_5	0	0	0	0	1	0
r_6	0	0	0	1	1	0
r_7	1	1	1	0	0	0

c_5 is essential and is selected and rows r_6 & r_5 are covered and removed.

r_7 dominates r_1 and is removed.

c_6 is dominated by c_1 and is removed

Thus, the resulting matrix is:

	c_1	c_2	c_3	c_4
r_1	1	1	0	0
r_2	0	1	1	1
r_3	1	0	0	1
r_4	1	0	1	0

$x = (0, 0, 0, 0, 1, 0)$.

Next, we select c_1 and call exact-cover with $x = (1, 0, 0, 0, 1, 0)$ and $b = (1, 1, 1, 1, 1, 1)$ and the matrix:

	c_2	c_3	c_4
r_2	1	1	1

Since c_2 dominates all other columns, they get removed and c_2 becomes essential and is selected. Since matrix has no rows, then $x = (1,1,0,0,1,0)$ and $b = (1,1,0,0,1,0)$.

Next, exact-cover is called with c_1 not selected with $x = (0,0,0,0,1,0)$ and $b = (1,1,0,0,1,0)$ and the matrix:

	c_2	c_3	c_4
r_1	1	0	0
r_2	1	1	1
r_3	0	0	1
r_4	0	1	0

All columns are essential and are selected

$\Rightarrow x = (0,1,1,1,1,0)$

Since current estimate $= 4 \geq |b|$, the solution $(1,1,0,0,1,0)$ is returned.

Since the returned solution is the same as the best, it will finally be returned.

Thus, the exact minimum cover is $(1,1,0,0,1,0)$.

Q 5. $F = \overline{A}\,\overline{C} + CD + AB + BC + \overline{B}\,\overline{D} + AD$

Since all variables are binate, we can expand on any variable

$F = \overline{A}\,[\,\overline{C} + CD + BC + \overline{B}\,\overline{D}\,]$

$+ A\,[\,CD + B + BC + \overline{B}\,\overline{D} + D\,]$

We need to show that both $F_{\overline{A}}$ and F_A are tautology.

$F_{\overline{A}} = \overline{C} + CD + BC + \overline{B}\,\overline{D}$

Since all variables are binate, we can expand on any variable; we expand on C

$F_{\overline{A}\,\overline{C}} = 1$

$F_{\overline{A}C} = D + B + \overline{B}\,\overline{D}$

We expand next on B

$\Rightarrow F_{\overline{A}C\overline{B}} = D + \overline{D} = 1$

$F_{\overline{A}CB} = 1$

Thus, $F_{\overline{A}C} = 1$

$\Rightarrow F_{\overline{A}} = 1$

$F_A = CD + B + BC + \overline{B}\,\overline{D} + D$

Since C is positive unate, it is sufficient to show that $F_{A\overline{C}}$ is tautology.

$F_{A\bar{C}} = B + \bar{B}\bar{D} + D$

we next expand on B.

$F_{A\bar{C}B} = 1$

$F_{A\bar{C}\bar{B}} = \bar{D} + D = 1$

$\Rightarrow F_{A\bar{C}} = 1$

$\Rightarrow F_A = 1$

$\Rightarrow$ F is tautology.

Q6. $F = \bar{B}\bar{D} + ACD + B\bar{C} + BC\bar{D}$

(i) $F = \bar{B}\,[\,\bar{D} + ACD\,]$
$\quad + B\,[\,ACD + \bar{C} + C\bar{D}\,]$

$= \bar{B}\,[\,\bar{D}[1] + D\,[AC]\,]$
$\quad + B\,[\,\bar{C}[1] + C\,[\,AD + \bar{D}\,]\,]$

$= \bar{B}\,[\,\bar{D}[1] + D\,[AC]\,]$
$\quad + B\,[\,\bar{C}[1] + C\,[\,\bar{D}[1] + D\,[A]\,]\,]$

$\Rightarrow \bar{F} = \bar{B}\,[\,\bar{D}[0] + D\,[\bar{A}+\bar{C}]\,]$
$\quad + B\,[\,\bar{C}[0] + C\,[\,\bar{D}[0] + D\,[\bar{A}]\,]\,]$

$= \bar{B}\bar{A}D + \bar{B}\bar{C}D + BCD\bar{A}$

(ii) Based on expansion in (i) we have

$$F = \bar{B}\,[\,\bar{D}\,[1] + D\,[AC]\,]$$
$$+ B\,[\,\bar{C}\,[1] + C\,[\,\bar{D}\,[1] + D\,[A]\,]\,]$$

Prime implicants of $f_{\bar{B}} = scc\{\bar{D}, ACD, AC\}$
$= \{\bar{D}, AC\}$

prime implicants of $f_{B\bar{C}} = \{1\}$

prime implicants of $f_{BC} = scc\{\bar{D}, DA, A\}$
$= \{\bar{D}, A\}$

$\Rightarrow$ prime implicants of $f_B = scc\{\bar{C}, C\bar{D}, CA, \bar{D}, A\}$
$= \{\bar{C}, \bar{D}, A\}$

$\Rightarrow$ prime implicants of $f = scc\{\bar{B}\bar{D}, \bar{B}AC, B\bar{C}, B\bar{D}, BA, \bar{C}\bar{D}, \bar{D}, A\bar{D}, AC\bar{D}, AC\}$

$= \{B\bar{C}, AB, \bar{D}, AC\}$