HW#1

Q1. $S = (ab + cd)(\bar{c} + \bar{d} + \bar{e})(efh)$

The set of all test vectors that detect the fault h stuck-at-1 is defined by

$\bar{h} \cdot (S_h \oplus S_{\bar{h}}) = 1$

$S_h = (ab + cd)(\bar{c} + \bar{d} + \bar{e})(ef)$

$S_{\bar{h}} = 0$

$$S_h \oplus S_{\bar{h}} = (ab + cd)(\bar{c} + \bar{d} + \bar{e})(ef)$$
$$= (ab + cd)(\bar{c}ef + \bar{d}ef)$$
$$= (ab\bar{c}ef + ab\bar{d}ef)$$

Thus, the set of test vectors that detect the fault h stuck-at-1 are:

$ab\bar{c}ef\bar{h} + ab\bar{d}ef\bar{h} = 1$

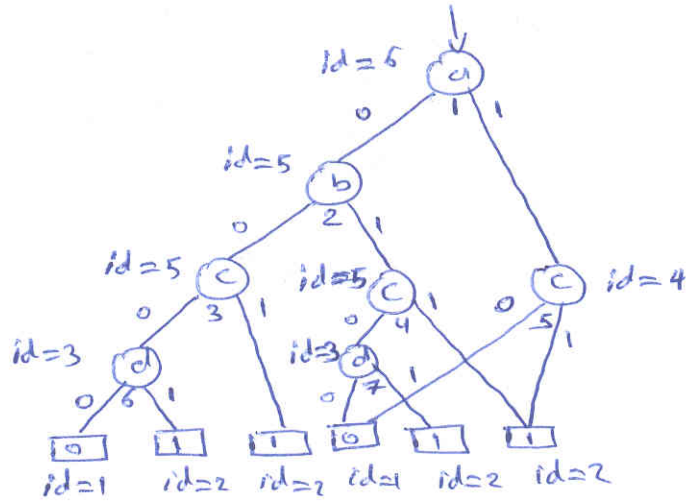
which can be represented as:

$\{a, b, c, d, e, f, h\} = \{110x110, 11x0110\}$

$= \{1100110, 1101110, 1110110\}$

Thus, there are only 3 test vectors that can detect the fault.

Q2.



First, we set $id(v) = 1$ for all leave vertices with value 0 and $id(v) = 2$ for all leave vertices with value 1. We initialize ROBDD with two vertices for 0 and 1.

Then, we process vertices at level 4, i.e. nodes with index = d.

$V = \{6, 7\}$

Since none of these vertices has $id(low(v)) = id(high(v))$ none of these vertices is dropped. Keys are assigned to vertices; $Key(6) = (1,2)$, $Key(7) = (1,2)$.

$oldKey = (0,0)$.

We process again the set $V = \{6, 7\}$.

$v = \{6\}$
$nextid = 3$, $id(6) = 3$, $oldKey = (1,2)$
we add $v = \{6\}$ to the ROBDD.

$v = \{7\}$
Since $Key(7) = oldKey$, $id(7) = 3$

Next, we process vertices at level 3, i.e. nodes with index = c.

$V = \{3, 4, 5\}$.
None of the nodes is dropped.
$Key(3) = (3,2)$, $Key(4) = (3,2)$, $Key(5) = (1,2)$
$oldKey = (0,0)$

$v = \{5\}$

$nextid = 4$, $oldKey = (1, 2)$, $id(5) = 4$

$v = \{5\}$ is added to the ROBDD.

$v = \{3\}$

$nextid = 5$, $id(3) = 5$, $oldKey = (3, 2)$

$v = \{3\}$ is added to the ROBDD.

$v = \{4\}$

Since $Key(4) = oldKey$, $id(4) = 5$

Then, we process vertices at level 2, i.e., nodes with index $= b$.

$V = \{2\}$

Since $id(low(2)) = id(high(2)) = 5$, $id(2) = 5$

and the vertex is dropped from V.

Finally, we process vertices at level 1, i.e. nodes with index $= a$.

$V = \{1\}$

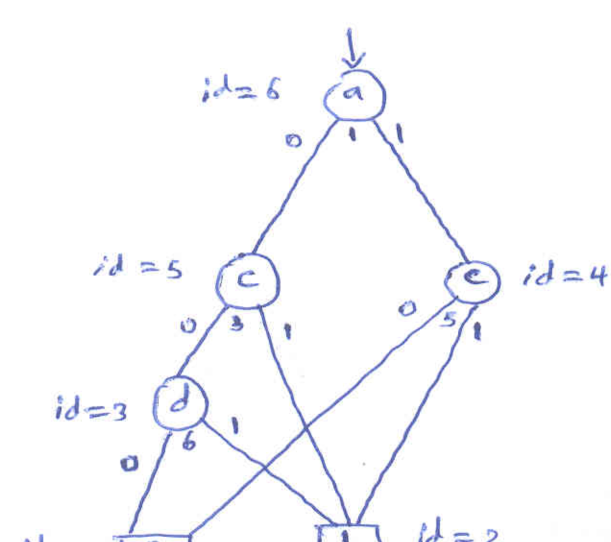
$Key(1) = (5, 4)$.

$oldKey = (0, 0)$

$nextid = 6$, $id(1) = 6$, $oldKey = (5, 4)$.

$v = (1)$ is added to the ROBDD.

Thus, the reduced OBDD is as follows:



Q3. $f = a_3\bar{b}_3 + (\bar{a}_3 \oplus b_3)\, a_2\bar{b}_2 + (\bar{a}_3 \oplus b_3)(\bar{a}_2 \oplus b_2)\, a_1\bar{b}_1$

(i) implementation using 2X1 MUXs.

We perform Shannon expansion on variable a_3.

$$f = \bar{a}_3\,[\,\bar{b}_3\, a_2\bar{b}_2 + \bar{b}_3\,(\bar{a}_2 \oplus b_2)\, a_1\bar{b}_1\,] + a_3\,[\,\bar{b}_3 + b_3\, a_2\bar{b}_2 + b_3\,(\bar{a}_2 \oplus b_2)\, a_1\bar{b}_1\,]$$

Then, we expand on variable b_3.

$$f = \bar{a}_3\,[\,\bar{b}_3\,[a_2\bar{b}_2 + (\bar{a}_2 \oplus b_2)\, a_1\bar{b}_1] + b_3\,[0]\,] + a_3\,[\,\bar{b}_3\,[1] + b_3\,[a_2\bar{b}_2 + (\bar{a}_2 \oplus b_2)\, a_1\bar{b}_1]\,]$$

Note that the cofactor $a_2\bar{b}_2 + (\bar{a}_2 \oplus b_2)\, a_1\bar{b}_1$ is shared and is implemented once.

We next expand on a2

$$a_2\bar{b}_2 + (\bar{a}_2 \oplus b_2)\, a_1\bar{b}_1 = \bar{a}_2\,[\,\bar{b}_2\, a_1\bar{b}_1\,] + a_2\,[\,\bar{b}_2 + b_2\, a_1\bar{b}_1\,]$$

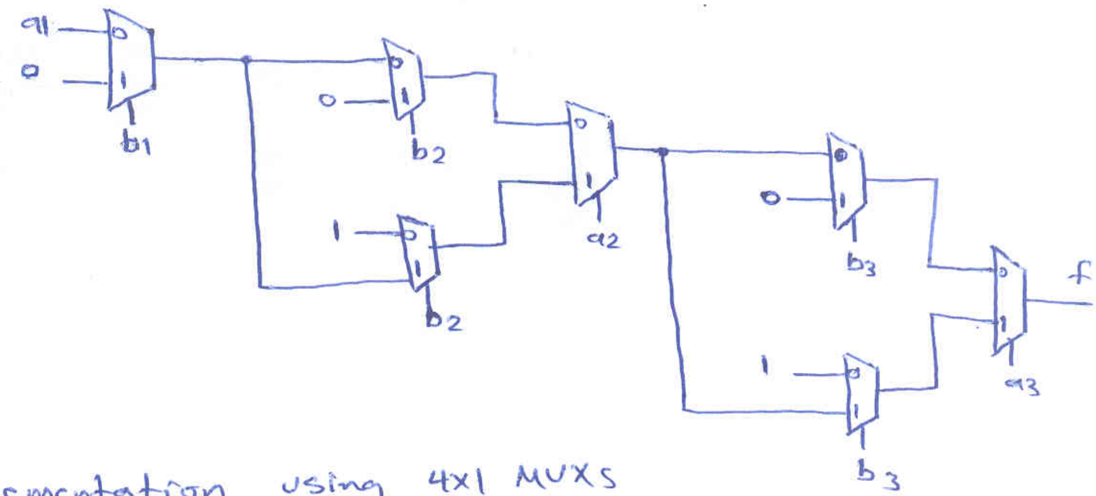
Next, we expand on b2

$$= \bar{a}_2\,[\,\bar{b}_2\,[a_1\bar{b}_1] + b_2\,[0]\,] + a_2\,[\,\bar{b}_2\,[1] + b_2\,[a_1\bar{b}_1]\,]$$

Also, $a_1\bar{b}_1$ is shared and is implemended once.

We expand on b1.

$$a_1\bar{b}_1 = \bar{b}_1\,[a] + b_1\,[0]$$



(ii) implementation using 4x1 MUXS

we perform expansion on variables a3 & b3

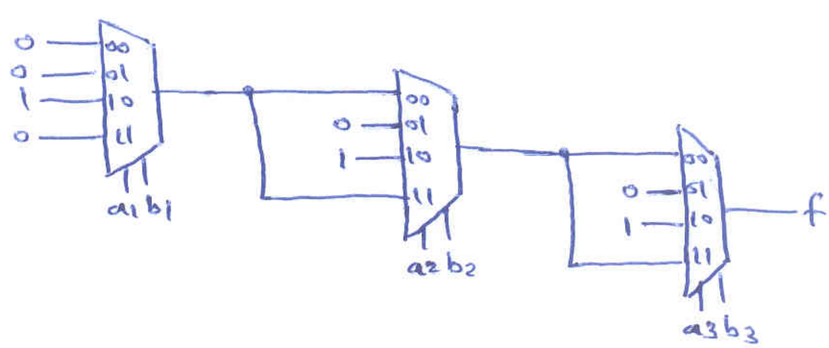
$$f = \bar{a}_3\bar{b}_3 \; [\; a_2\bar{b}_2 + (\bar{a}_2 \oplus b_2)\, a_1\bar{b}_1 \;]$$
$$+ \bar{a}_3 b_3 \; [\; 0 \;] \; + a_3\bar{b}_3 \; [\, 1 \,]$$
$$+ a_3 b_3 \; [\; a_2\bar{b}_2 + (\bar{a}_2 \oplus b_2)\, a_1\bar{b}_1 \;]$$

The cofactor $a_2\bar{b}_2 + (\bar{a}_2 \oplus b_2)\, a_1\bar{b}_1$ is shared and implemented once.

We next expand on variables a2 & b2.

$$a_2\bar{b}_2 + (\bar{a}_2 \oplus b_2)\, a_1\bar{b}_1 = \bar{a}_2\bar{b}_2 \, [a_1\bar{b}_1] + \bar{a}_2 b_2 \, [0] + a_2\bar{b}_2 \, [\; 1 \;] + a_2 b_2 \, [a_1\bar{b}_1]$$

The cofactor $a_1\bar{b}_1$ is shared and implemented once.



(iii)

$$f = \bar{a}_1 \left[a_3\bar{b}_3 + (\bar{a}_3 \oplus b_3)\, a_2\bar{b}_2 \right]$$
$$+ a_1 \left[a_3\bar{b}_3 + (\bar{a}_3 \oplus b_3)\, a_2\bar{b}_2 + (\bar{a}_3 \oplus b_3)(\bar{a}_2 \oplus b_2)\bar{b}_1 \right]$$

$$= \bar{a}_1 \left[a_3\bar{b}_3 + (\bar{a}_3 \oplus b_3)\, a_2\bar{b}_2 \right]$$
$$+ a_1 \Big[\bar{b}_1 \left[a_3\bar{b}_3 + (\bar{a}_3 \oplus b_3)\, a_2\bar{b}_2 + (\bar{a}_3 \oplus b_3)(\bar{a}_2 \oplus b_2) \right]$$
$$+ b_1 \left[a_3\bar{b}_3 + (\bar{a}_3 \oplus b_3)\, a_2\bar{b}_2 \right] \Big]$$

$$= \bar{a}_1 \left[\bar{a}_2 \left[a_3\bar{b}_3 \right] + a_2 \left[a_3\bar{b}_3 + (\bar{a}_3 \oplus b_3)\bar{b}_2 \right] \right]$$
$$+ a_1 \Big[\bar{b}_1 \Big[\bar{a}_2 \left[a_3\bar{b}_3 + (\bar{a}_3 \oplus b_3)\bar{b}_2 \right]$$
$$+ a_2 \left[a_3\bar{b}_3 + (\bar{a}_3 \oplus b_3)\bar{b}_2 + (\bar{a}_3 \oplus b_3)\, b_2 \right] \Big]$$
$$+ b_1 \left[\bar{a}_2 \left[a_3\bar{b}_3 \right] + a_2 \left[a_3\bar{b}_3 + (\bar{a}_3 \oplus b_3)\bar{b}_2 \right] \right] \Big]$$

$$= \bar{a}_1 \left[\bar{a}_2 \left[a_3\bar{b}_3 \right] + a_2 \left[a_3\bar{b}_3 + (\bar{a}_3 \oplus b_3)\bar{b}_2 \right] \right]$$
$$+ a_1 \Big[\bar{b}_1 \Big[\bar{a}_2 \left[a_3\bar{b}_3 + (\bar{a}_3 \oplus b_3)\bar{b}_2 \right]$$
$$+ a_2 \left[a_3\bar{b}_3 + (\bar{a}_3 \oplus b_3) \right] \Big]$$
$$+ b_1 \left[\bar{a}_2 \left[a_3\bar{b}_3 \right] + a_2 \left[a_3\bar{b}_3 + (\bar{a}_3 \oplus b_3)\bar{b}_2 \right] \right] \Big]$$

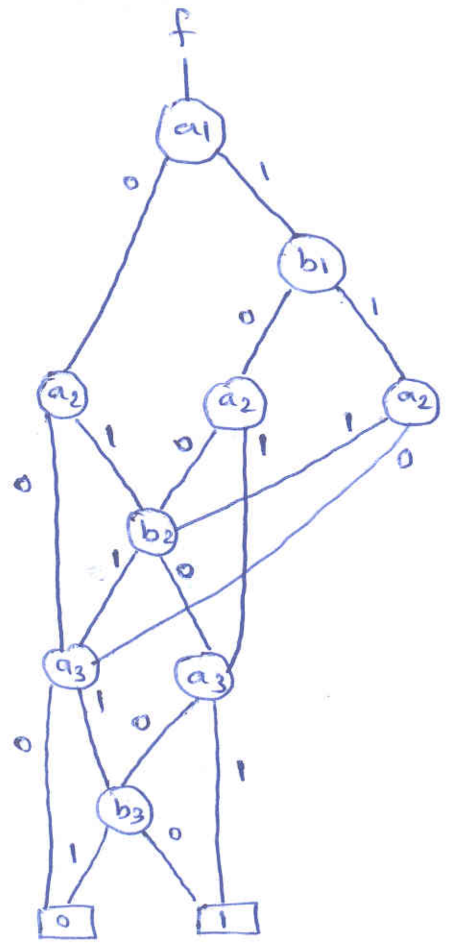
The common cofactors will be shared.

$$a_3\bar{b}_3 + (\bar{a}_3 \oplus b_3)\bar{b}_2 = \bar{b}_2 \left[a_3\bar{b}_3 + (\bar{a}_3 \oplus b_3) \right] + b_2 \left[a_3\bar{b}_3 \right]$$

$$a_3\bar{b}_3 + (\bar{a}_3 \oplus b_3) = \bar{a}_3 \left[\bar{b}_3 \right] + a_3 \left[1 \right]$$

$$a_3\bar{b}_3 = \bar{a}_3 \left[0 \right] + a_3 \left[\bar{b}_3 \right]$$

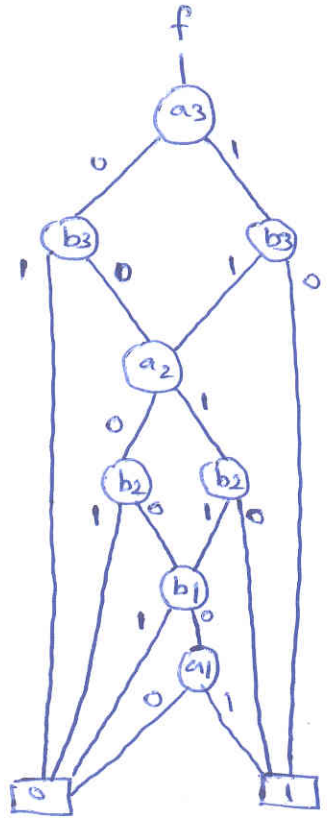
So, the ROBDD using the variable ordering $\{a_1, b_1, a_2, b_2, a_3, b_3\}$ is :



This variable ordering has resulted in an ROBDD with 9 vertices.

Based on the variable ordering $\{a_3, b_3, a_2, b_2, a_1, b_1\}$ is used, we will get an ROBDD with 8 vertices as can be seen from the solution in (1).

The ROBDD based on the variable ordering $\{a_3, b_3, a_2, b_2, a_1, b_1\}$ is shown below:



This ROBDD has 8 vertices. So, this is a better variable ordering.

Q4. $f = a \oplus b \oplus c$

$g = ac + \bar{a}\bar{b} + b\bar{c}$

(i) We first expand both functions on the variables a & b and then we perform the required operations

$$f = \bar{a}\bar{b}\,[c] + \bar{a}b\,[\bar{c}] + a\bar{b}\,[\bar{c}] + ab\,[c]$$

$$g = \bar{a}\bar{b}\,[1] + \bar{a}b\,[\bar{c}] + a\bar{b}\,[c] + ab\,[1]$$

$$f \cdot g = \bar{a}\bar{b}\,[c] + \bar{a}b\,[\bar{c}] + a\bar{b}\,[0] + ab\,[c]$$

$$f + g = \bar{a}\bar{b}\,[1] + \bar{a}b\,[\bar{c}] + a\bar{b}\,[1] + ab\,[1]$$

$$= \bar{a}\bar{b} + \bar{a}b\bar{c} + a\bar{b} + ab$$

$$= \bar{a}\bar{b} + \bar{a}b\bar{c} + a$$

$$= \bar{b} + b\bar{c} + a$$

$$= \bar{b} + \bar{c} + a$$

$$f \oplus g = \bar{a}\bar{b}\,[\bar{c}] + \bar{a}b\,[0] + a\bar{b}\,[1] + ab\,[\bar{c}]$$

$$= \bar{a}\bar{b}\bar{c} + a\bar{b} + ab\bar{c}$$

$$= \bar{b}\bar{c} + a\bar{b} + a\bar{c}$$

(ii) ITE DAG for $f \oplus g$

$$f \oplus g = \text{ITE}(f, \bar{g}, g)$$

$$\bar{g} = \bar{a}\bar{b}\,[0] + \bar{a}b\,[c] + a\bar{b}\,[\bar{c}] + ab\,[0]$$

$$= \bar{a}bc + a\bar{b}\bar{c}$$

$$f \oplus g = \text{ITE}(a \oplus b \oplus c,\ \bar{a}bc + a\bar{b}\bar{c},\ ac + \bar{a}\bar{b} + b\bar{c})$$

We assume the variable ordering $\{a, b, c\}$.

- $x = a$

  $t = ITE(\bar{b} \oplus c, \bar{b}\bar{c}, c + b\bar{c})$

  $e = ITE(b \oplus c, bc, \bar{b} + b\bar{c})$

- $ITE(\bar{b} \oplus c, \bar{b}\bar{c}, c + b\bar{c})$

  - $x = b$

    $t = ITE(c, 0, 1) \Rightarrow$ trivial case $= \bar{c}$

    $\Rightarrow t = 3$

    $e = ITE(\bar{c}, \bar{c}, c) \Rightarrow$ trivial case $= 1$

    $\Rightarrow e = 2$

    since $t \neq e$, identifier 4 will be returned for $ITE(\bar{b} \oplus c, \bar{b}\bar{c}, c + b\bar{c})$

- $ITE(b \oplus c, bc, \bar{b} + b\bar{c})$

  - $x = b$

    $t = ITE(\bar{c}, c, \bar{c}) \Rightarrow$ trivial case $= 0$

    $\Rightarrow t = 1$

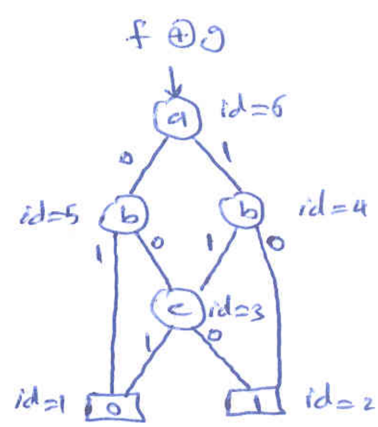
    $e = ITE(c, 0, 1) \Rightarrow$ trivial case $= \bar{c}$

    $\Rightarrow e = 3$

    since $t \neq e$, identifier 5 will be assigned to $ITE(b \oplus c, bc, \bar{b} + b\bar{c})$.

Since $t \neq e$, identifier 6 will be assigned.
The constructed unique table and the corresponding ITE DAG is shown below:

id	var	left child	right child
3	c	2	1
4	b	2	3
5	b	3	1
6	a	5	4



Q5. Minimum cover using Exact_Cover procedure.

$$A = \begin{array}{c} \\ r1 \\ r2 \\ r3 \\ r4 \\ r5 \\ r6 \\ r7 \\ r8 \\ r9 \\ r10 \end{array} \begin{array}{c} \begin{array}{ccccccc} c1 & c2 & c3 & c4 & c5 & c6 & c7 \end{array} \\ \left[\begin{array}{ccccccc} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \end{array}$$

First, we check if there are essential columns. we can see that c7 is essential and hence it is selected and rows r6, r7, and r10 are removed.
Thus, $x = [0\ 0\ 0\ 0\ 0\ 0\ 1]$ and the reduced matrix is :

$$A = \begin{array}{c} \\ r1 \\ r2 \\ r3 \\ r4 \\ r5 \\ r8 \\ r9 \end{array} \begin{array}{c} \begin{array}{cccccc} c1 & c2 & c3 & c4 & c5 & c6 \end{array} \\ \left[\begin{array}{cccccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \end{array}\right] \end{array}$$

Next, we check row dominance and column dominance.

- c1 dominates c6, so c6 is removed
- c4 dominates c5, so c5 is removed
- r8 dominates r1 and r3, so r8 is removed
- r9 dominates r4, so r9 is removed

Thus, the reduced matrix is :

$$A = \begin{array}{c} \\ r1 \\ r2 \\ r3 \\ r4 \\ r5 \end{array} \begin{array}{c} \begin{array}{cccc} c1 & c2 & c3 & c4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

The matrix cannot be reduced further.

Let us assume a branching column c1.

If we select c1, then $x = [1\ 0\ 0\ 0\ 0\ 0\ 1]$, and the matrix $\tilde{A}$ becomes

$$\tilde{A} = \begin{array}{c} \\ r4 \\ r5 \end{array} \begin{array}{c} \begin{array}{ccc} c2 & c3 & c4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

- c4 dominates c2, so c2 is removed
- c4 dominates c3, so c3 is removed

Then, c4 becomes essential and is selected

$x = [1\ 0\ 0\ 1\ 0\ 0\ 1]$
$b = [1\ 0\ 0\ 1\ 0\ 0\ 1]$

if c1 is not selected, then $x = [0\ 0\ 0\ 0\ 0\ 0\ 1]$ and the matrix $\tilde{A}$ becomes

$$\tilde{A} = \begin{array}{c} \\ r1 \\ r2 \\ r3 \\ r4 \\ r5 \end{array} \begin{array}{c} \begin{array}{ccc} c2 & c3 & c4 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

Then, c2, c3 and c4 become essential columns and are selected.

Thus, $x = [0\ 1\ 1\ 1\ 0\ 0\ 1]$

Since the cost of this solution is larger than the best solution so far, the returned best solution is $x = [1\ 0\ 0\ 1\ 0\ 0\ 1]$.