Dec. 23, 2010

COMPUTER ENGINEERING DEPARTMENT

COE 561

Digital System Design and Synthesis

MAJOR EXAM II

(Open Book Exam)

First Semester (101)

Time: 1:00-4:00 PM

Student Name : _KEY____

Student ID.   : ____

Question	Max Points	Score
Q1	15	
Q2	20	
Q3	15	
Q4	14	
Q5	18	
Q6	18	
Total	100	

[15 Points]

(Q1) Consider the function F(A, B, C, D) with **ON-SET=Σm(0, 5, 7, 13, 15)** and **OFF-SET=Σm(8, 10, 11, 12)**. Note that you do not need to use the positional-cube notation in your solution.

- **(i) Expand** the minterm **A'B'C'D'** using ESPRESSO heuristics.
- **(ii)** A cover of the function is given by F = A' + BD. **Reduce** the cube **A'** using Theorem 7.4.1.
- **(iii)** Use Corollary 7.4.1 to check if the implicant **BD** is an **essential** prime implicant.

(i) Expand the minterm $\bar{A}\bar{B}\bar{C}\bar{D}$

Free set = {2, 4, 6, 8}

Column 2 can't be raised as it has distance 1 from the off-set.

The overexpanded cube = $\bar{A}$

Thus, we need to check the cubes $\bar{A}B\bar{C}D$ and $\bar{A}BCD$ for being feasibly covered.

$SC(\bar{A}\bar{B}\bar{C}\bar{D}, \bar{A}B\bar{C}D) = \bar{A}\bar{C}$ feasible

$SC(\bar{A}\bar{B}\bar{C}\bar{D}, \bar{A}BCD) = \bar{A}$ feasible

Thus, the minterm is expanded to $\bar{A}$ as it covers the other feasibly covered cubes. Since free set is empty, the expanded cube is $\bar{A}$.

(ii) Reduce the cube $\bar{A}$

$$Q = BD + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}BC\bar{D} + ABC\bar{D} + A\bar{B}\bar{C}D$$

$$= \bar{B}\bar{D}\,[\bar{A}C] + \bar{B}D\,[\bar{A}\bar{C} + \bar{A}C + A\bar{C}]$$
$$+ B\bar{D}\,[\bar{A}\bar{C} + \bar{A}C + AC] + BD\,[1]$$

$$= \bar{B}\bar{D}\,[\bar{A}C] + \bar{B}D\,[\bar{A} + \bar{C}]$$
$$+ B\bar{D}\,[\bar{A} + C] + BD\,[1]$$

$$\bar{Q} = \bar{B}\bar{D}\,[A + \bar{C}] + \bar{B}D\,[AC] + B\bar{D}\,[A\bar{C}] + BD[0]$$

$$\bar{Q}_{\bar{A}} = \bar{B}\bar{D}\,[\bar{C}] + \bar{B}D\,[0] + B\bar{D}\,[0] + BD[0] = \bar{B}\bar{D}\bar{C}$$

$$\Rightarrow \bar{A} \cap SC(\bar{Q}_{\bar{A}}) = \bar{A}\bar{B}\bar{C}\bar{D}$$

Thus, the cube $\bar{A}$ is reduced to $\bar{A}\bar{B}\bar{C}\bar{D}$.

(iii) <u>Check if BD is an essential prime implicant</u>

$\alpha = BD$

$G = \{\bar{A}, \bar{A}\bar{B}\bar{C}D, \bar{A}\bar{B}CD, \overline{AB}C\bar{D}, \bar{A}B\bar{C}\bar{D}, \bar{A}BC\bar{D}, ABC\bar{D}, A\bar{B}\bar{C}D\}$

$G \# \alpha = \{\bar{A}\bar{B}, \bar{A}\bar{D}, \bar{A}\bar{B}\bar{C}D, \bar{A}\bar{B}CD, \bar{A}\bar{B}C\bar{D}, \bar{A}B\bar{C}\bar{D}, \bar{A}BC\bar{D}, ABC\bar{D}, A\bar{B}\bar{C}D\}$

$H = \text{consensus}(G \# \alpha, \alpha) = \{\bar{A}D, \bar{A}B, \bar{A}\bar{C}D, \bar{A}CD, \bar{A}C\bar{D}, \bar{A}B\bar{C}, \bar{A}BC, ABC, A\bar{C}D\}$

$H \cup DC = \{\bar{A}D, \bar{A}B, \bar{A}C\bar{D}, ABC, A\bar{C}D\}$

$\{H \cup DC\}_{\alpha} = \{\bar{A}, \bar{A}, 0, AC, A\bar{C}\}$

$\Rightarrow$ Tautology

Thus, the prime implicant C is <u>not</u> an essential prime implicant.

[20 Points]

(Q2) Consider the following cover of a function F*(A,B,C,D)*

$$F = \overline{A}\overline{B}\overline{C} + AB\overline{C} + \overline{A}\overline{C}D + \overline{A}C\overline{D} + ACD + BD + \overline{A}BC$$

With $F^{DC} = \sum m(5, 11, 12)$

(i) Determine the relatively essential set of cubes, E^r.

(ii) Determine the totally redundant, R^t, and partially redundant, R^p, sets of cubes.

(iii) Find a subset of R^p that, together with E^r, covers the function by solving a covering problem.

(i) Relatively Essential Set E^r:

- Check $\overline{A}\overline{B}\overline{C}$:

$\{AB\overline{C}, \overline{A}\overline{C}D, \overline{A}C\overline{D}, ACD, BD, \overline{A}BC\}_{\overline{A}\overline{B}\overline{C}}$

$= \{0, D, 0, 0, 0, 0\}$ Not Tautology $\Rightarrow$ Relativ. Ess.

- Check $AB\overline{C}$:

$\{\overline{A}\overline{B}\overline{C}, \overline{A}\overline{C}D, \overline{A}C\overline{D}, ACD, BD, \overline{A}BC, AB\overline{C}\overline{D}\}_{AB\overline{C}}$

$= \{0, 0, 0, 0, D, 0, \overline{D}\}$ Tautology $\Rightarrow$ Not Rel. Ess.

- Check $\overline{A}\overline{C}D$:

$\{\overline{A}\overline{B}\overline{C}, AB\overline{C}, \overline{A}C\overline{D}, ACD, BD, \overline{A}BC\}_{\overline{A}\overline{C}D}$

$= \{\overline{B}, 0, 0, 0, B, 0\}$ Tautology $\Rightarrow$ Not Rel. Ess.

- Check $\overline{A}C\overline{D}$:

$\{\overline{A}\overline{B}\overline{C}, AB\overline{C}, \overline{A}\overline{C}D, ACD, BD, \overline{A}BC\}_{\overline{A}C\overline{D}}$

$= \{0, 0, 0, 0, 0, B\}$ Not Tautology $\Rightarrow$ Rel. Ess.

- Check ACD:

$\{\overline{A}\overline{B}\overline{C}, AB\overline{C}, \overline{A}\overline{C}D, \overline{A}C\overline{D}, BD, \overline{A}BC, A\overline{B}CD\}_{ACD}$

$= \{0, 0, 0, 0, B, 0, \overline{B}\}$ Tautology $\Rightarrow$ Not Rel. Ess.

- Check BD

$\{\bar{A}\bar{B}\bar{C},\ AB\bar{C},\ \bar{A}\bar{C}D,\ \bar{A}C\bar{D},\ ACD,\ \bar{A}BC\}_{BD}$

$= \{0,\ A\bar{C},\ \bar{A}\bar{C},\ 0,\ AC,\ \bar{A}C\}$ Tautology $\Rightarrow$ Not. Rel. Ess.

- Check $\bar{A}BC$

$\{\bar{A}\bar{B}\bar{C},\ AB\bar{C},\ \bar{A}\bar{C}D,\ \bar{A}C\bar{D},\ ACD,\ BD\}_{\bar{A}BC}$

$= \{0, 0, 0, \bar{D}, 0, D\}$ Tautology $\Rightarrow$ Not Rel. Ess.

Thus, $E^r = \{\bar{A}\bar{B}\bar{C},\ \bar{A}C\bar{D}\}$

(ii) Totally redundant set R^t:

- Check $AB\bar{C}$

$\{\bar{A}\bar{B}\bar{C},\ \bar{A}C\bar{D},\ \bar{A}B\bar{C}D,\ AB\bar{C}\bar{D},\ A\bar{B}CD\}_{AB\bar{C}}$

$= \{0, 0, 0, \bar{D}, 0\}$ Not Tautology $\Rightarrow$ Part. Red.

- Check $\bar{A}\bar{C}D$

$\{\bar{A}\bar{B}\bar{C},\ \bar{A}C\bar{D},\ \bar{A}B\bar{C}D,\ AB\bar{C}\bar{D},\ A\bar{B}CD\}_{\bar{A}\bar{C}D}$

$= \{\bar{B}, 0, B, 0, 0\}$ Tautology $\Rightarrow$ Tot. Red.

- Check ACD

$\{\bar{A}\bar{B}\bar{C},\ \bar{A}C\bar{D},\ \bar{A}B\bar{C}D,\ AB\bar{C}\bar{D},\ A\bar{B}CD\}_{ACD}$

$= \{0, 0, 0, 0, \bar{B}\}$ Not Tautology $\Rightarrow$ Part. Red.

- Check BD

$\{\bar{A}\bar{B}\bar{C},\ \bar{A}C\bar{D},\ \bar{A}B\bar{C}D,\ AB\bar{C}\bar{D},\ A\bar{B}CD\}_{BD}$

$= \{0, 0, \bar{A}\bar{C}, 0, 0\}$ Not Tautology $\Rightarrow$ Part. Red.

- Check $\bar{A}BC$

$\{\bar{A}\bar{B}\bar{C},\ \bar{A}C\bar{D},\ \bar{A}B\bar{C}D,\ AB\bar{C}\bar{D},\ A\bar{B}CD\}_{\bar{A}BC}$

$= \{0, \bar{D}, 0, 0, 0\}$ Not Tautology $\Rightarrow$ Part. Red.

Thus, $R^t = \{\bar A \bar C D\}$ and $R^p = \{AB\bar C, ACD, BD, \bar A BC\}$

(iii) First, we find coverage relations:

- $\underline{AB\bar C}$:

$\{\bar A \bar B \bar C, \bar A C \bar D, \bar A B \bar C D, AB\bar C \bar D, A\bar B CD, ACD, BD, \bar A BC\}_{AB\bar C}$

$= \{0, 0, 0, \bar D, 0, 0, D, 0\} \Rightarrow$ added row $(1,0,1,0)$

- $\underline{ACD}$:

$\{\bar A \bar B \bar C, \bar A C \bar D, \bar A B \bar C D, AB\bar C \bar D, A\bar B CD, AB\bar C, BD, \bar A BC\}_{ACD}$

$= \{0, 0, 0, 0, \bar B, 0, B, 0\} \Rightarrow$ added row $(0,1,1,0)$

- $\underline{BD}$:

$\{\bar A \bar B \bar C, \bar A C \bar D, \bar A B \bar C D, AB\bar C \bar D, A\bar B CD, AB\bar C, ACD, \bar A BC\}_{BD}$

$= \{0, 0, \bar A \bar C, 0, 0, A\bar C, AC, \bar A C\}$

- Expand on A: $\Rightarrow$ added row $(1,0,1,0)$
  - $A = 1$: $\{0, 0, 0, 0, 0, \bar C, C, 0\} \Rightarrow$ added row $(0,1,1,0)$
  - $A = 0$: $\{0, 0, \bar C, 0, 0, 0, 0, C\} \Rightarrow$ added row $(0,0,1,1)$

- $\underline{\bar A BC}$:

$\{\bar A \bar B \bar C, \bar A C \bar D, \bar A B \bar C D, AB\bar C \bar D, A\bar B CD, AB\bar C, ACD, BD\}_{\bar A BC}$

$= \{0, \bar D, 0, 0, 0, 0, 0, D\} \Rightarrow$ added row $(0,0,1,1)$

Covering Matrix:

	$AB\bar C$	ACD	BD	$\bar A BC$
$AB\bar C$	1	0	1	0
ACD	0	1	1	0
BD	1	0	1	0
	0	1	1	0
	0	0	1	1
$\bar A BC$	0	0	1	1

Thus, BD is selected and the minimal cover is $\{\bar A \bar B \bar C, \bar A C \bar D, BD\}$.

[15 Points]

(Q3) Consider the logic network defined by the following expression:

$$x = a\,c\,d + a\,e + a'\,b'\,f + a'\,b'\,g + b\,c\,d + b\,e$$

Using the recursive procedure **KERNELS**, compute all the kernels and co-kernels of x. Show all the steps of the algorithm. Assume the following lexicographic order: {a, a', b, b', c, d, e, f, g}.

- $i = 1$ (a):
  Cubes $(x, a) = \{acd, ae\} \geq 2$, $C = a$
  The kernel $cd + e$ will be returned.

- $i = 2$ $(\bar{a})$:
  Cubes $(x, \bar{a}) = \{\bar{a}\bar{b}f, \bar{a}\bar{b}g\} \geq 2$, $C = \bar{a}\bar{b}$
  The kernel $f + g$ will be returned.

- $i = 3$ (b):
  Cubes $(x, b) = \{bcd, be\} \geq 2$, $C = b$
  The kernel $cd + e$ will be returned.

- $i = 4$ $(\bar{b})$:
  Cubes $(x, \bar{b}) = \{\bar{a}\bar{b}f, \bar{a}\bar{b}g\} \geq 2$, $C = \bar{a}\bar{b}$
  since the cube contains literal $\bar{a} < 4$, no kernels will be found.

- $i = 5$ (c):
  Cubes $(x, c) = \{acd, bcd\} \geq 2$, $C = cd$
  The kernel $a + b$ will be returned.

- $i = 6$ (d):
  Cubes $(x, d) = \{acd, bcd\} \geq 2$, $C = cd$
  Since the cube contains literal $c < 6$, no kernels will be found.

- $i = 7$ (e):

  Cubes $(x, e) = \{ae, be\} \geq 2$, $C = e$

  The kernel $a+b$ will be returned.

- $i = 8$ (f):

  Cubes $(x, f) = \{\bar{a}\bar{b}f\} < 2 \Rightarrow$ No kernels found

- $i = 9$ (g):

  Cubes $(x, g) = \{\bar{a}\bar{b}g\} < 2 \Rightarrow$ No kernels found

Thus, the list of kernels and co-kernels of x are:

Kernel	Co-Kernel
$cd + e$	a, b
$f + g$	$\bar{a}\bar{b}$
$a + b$	cd, e
x	1

[14 Points]

(Q4) Consider the logic network defined by the following expression:

$$x = a\,b\,d + a\,b\,e + a\,b'\,d'\,e' + a'\,c\,d + a'\,c\,e + a'\,c'\,d'\,e'$$

Compute the weight of the double cube divisors $d_1 = a\,b + a'\,c$ and $d_2 = d + e$. Extract the double cube divisor with the highest weight and show the resulting network after extraction and the number of literals saved.

Double Cube Divisor	Base
$d1 = ab + \bar{a}c$	d, e
$\bar{d}1 = a\bar{b} + \bar{a}\bar{c}$	$\bar{d}\bar{e}$
$d2 = d + e$	$ab, \bar{a}c$

weight $(d1) = 3 * 4 - 3 - 4 + 1 + 1 + 2 = 9$

weight $(d2) = 2 * 2 - 2 - 2 + 2 + 2 + 2 = 6$

Since d1 has higher weight, it will be extracted.

The resulting network after extraction of d1 is:

$[1] = ab + \bar{a}c$

$x = [1]d + [1]e + \overline{[1]}\,\bar{d}\bar{e}$

11 literals

Original number of literals = 20 literals

Number of literals saved = 9 literals.

[18 Points]

(Q5) Consider the logic network defined by the following expressions with inputs *{a, b, c}* and output *{z}*:

$$x = a + b$$
$$y = x'c + ax'$$
$$z = y + c'$$

(i) Compute the CDC set for the cut at the inputs of circuit Y.

(ii) Compute the ODC set for node Y.

(iii) Simplify the function of Y using both its ODC and CDC.

(iv) Based on perturbation analysis starting with the original network, determine if it is possible to change the implementation of y to $y = x'$.

(i) The cut is $\{a, c, x\}$

$SDC_x = x \oplus (a+b) = x\bar{a}\bar{b} + \bar{x}a + \bar{x}b$

$CDC = x\bar{a}\bar{b} + \bar{x}a + \bar{x}b$

We need to remove b :

$(\bar{x}a + \bar{x})(x\bar{a} + \bar{x}a) = (\bar{x})(x\bar{a} + \bar{x}a) = \bar{x}a$

$\Rightarrow CDC_{cut} = \bar{x}a$

(ii) $ODC_y = \bar{c}$

(iii) Simplification of y using CDC_{cut} & ODC_y :

$x \backslash ac$	00	01	11	10
0	X	1	X	X
1	X	0	0	X

$\Rightarrow Y = \bar{x}$

(iv) $\delta = (\bar{x} c + a\bar{x}) \oplus \bar{x}$

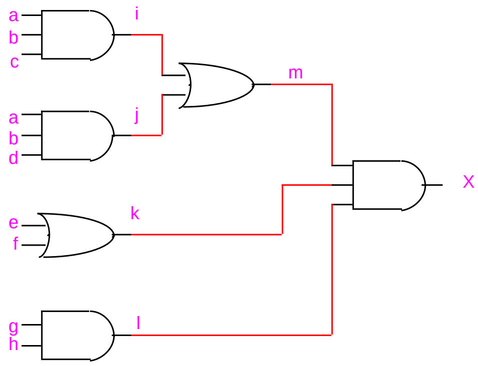
$= \bar{x}(a+c) \oplus \bar{x} = \bar{x}[(a+c) \oplus 1]$

$= \bar{x}\,\bar{a}\,\bar{c}$

Since $\delta \subseteq ODC_y = \bar{c}$, then it is possible to change the implementation of y to $y = \bar{x}$.

[18 Points]

(Q6) Consider the logic network below with inputs *{a, b, c, d, e, f, g, h}* and output *{X}*:



Assume that the delay of a gate is related to the number of its inputs i.e. the delay of a 2-input AND gate is 2. Also, assume that the input data-ready times are zero for all inputs.

(i) Compute the data ready times and slacks for all vertices in the network.

(ii) Determine the topological critical path.

(iii) Suggest an implementation of the function *X* to reduce the delay of the circuit to the minimum possible and determine the maximum propagation delay in the optimized circuit. Has the area been affected?

(i)

Data Ready Time	Required Time	Slack
$t_a = 0$	$\bar{t}_a = \min(0,0) = 0$	$S_a = 0$
$t_b = 0$	$\bar{t}_b = \min(0,0) = 0$	$S_b = 0$
$t_c = 0$	$\bar{t}_c = 0$	$S_c = 0$
$t_d = 0$	$\bar{t}_d = 0$	$S_d = 0$
$t_e = 0$	$\bar{t}_e = 3$	$S_e = 3 - 0 = 3$
$t_f = 0$	$\bar{t}_f = 3$	$S_f = 3 - 0 = 3$
$t_g = 0$	$\bar{t}_g = 3$	$S_g = 3 - 0 = 3$
$t_h = 0$	$\bar{t}_h = 3$	$S_h = 3 - 0 = 3$
$t_i = 3$	$\bar{t}_i = 3$	$S_i = 3 - 3 = 0$
$t_j = 3$	$\bar{t}_j = 3$	$S_j = 3 - 3 = 0$
$t_k = 2$	$\bar{t}_k = 5$	$S_k = 5 - 2 = 3$
$t_l = 2$	$\bar{t}_l = 5$	$S_l = 5 - 2 = 3$
$t_m = 5$	$\bar{t}_m = 5$	$S_m = 5 - 5 = 0$
$t_x = 8$	$\bar{t}_x = 8$	$S_x = 8 - 8 = 0$

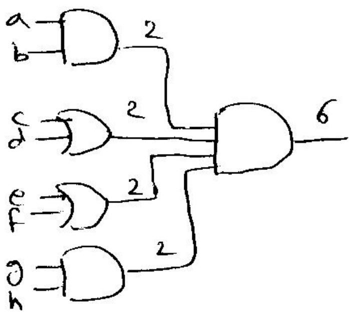
(ii) The topological critical paths are:

$\{a, i, m, x\}$, $\{b, i, m, x\}$, $\{c, i, m, x\}$,
$\{a, j, m, x\}$, $\{b, j, m, x\}$, $\{d, j, m, x\}$.

(iii) To optimize the delay of the network, we need to improve the delay of nodes in the critical paths.

We rewrite m as $ab(c+d)$ and then we have $x = m \times l = ab(c+d)(e+f)gh$ and we combine them into a 4-input AND gate to optimize both delay and area.

The resulting network is:



Resulting delay $= 6$

Number of literals $= 12$

Thus, we have improved both delay from 8 to 6 and area from 15 literals to 12.