

## COE 405, Term 152

## Design &amp; Modeling of Digital Systems

## Quiz# 2

Date: Sunday, Feb. 28, 2016

- Q.1.** It is required to design a sequential circuit that has a single input X representing a signed 2's complement number and a single output Y. The circuit receives the number serially through the input X from the least significant bit (LSB) to the most significant bit (MSB), and computes the equation  $Y=X-3$  and generates the output serially from the least significant bit to the most significant bit. The circuit has an additional asynchronous reset input R that resets the circuit into an initial state. The following are examples of input and output data:

Examples:

		LSB			MSB		
Input	X	0	1	1	0	0	Input=6
Output	Y	1	1	0	0	0	Output=3

		LSB			MSB		
Input	X	0	0	1	1	0	Input=12
Output	Y	1	0	0	1	0	Output=9

Draw a state diagram or show the state table of the circuit with minimum number of states assuming a **Mealy** model. You are not required to implement the circuit.

Present State	Next State, Y	
	X=0	X=1
S0 (B=3)	S1, 1	S2, 0
S1 (B=2)	S2, 0	S2, 1
S2 (B=1)	S2, 1	S3, 0
S3 (B=0)	S3, 0	S3, 1

**Q.2.** Consider the given FSM that has 4states, one input X and one output Z, represented by the following state table:

Present State	Next State, Z	
	X=0	X=1
S0	S1, 0	S2, 0
S1	S0, 0	S3, 0
S2	S2, 0	S3, 1
S3	S3, 0	S2, 1

(i) Determine the equivalent states.

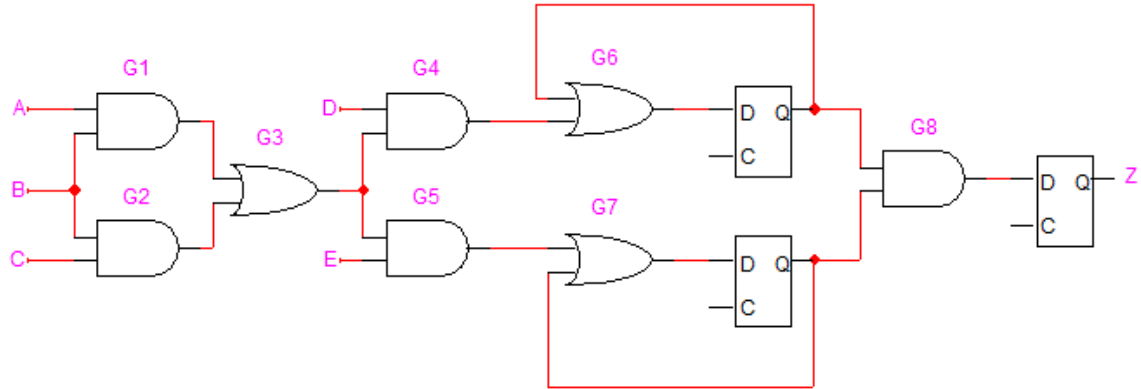
S1	(S2, S3)		
S2			
S3			√
	S0	S1	S2

Thus, the equivalent states are (S0, S1) and (S2, S3).

(ii) Reduce the state table into the minimum number of states and show the reduced state table.

Present State	Next State, Z	
	X=0	X=1
S0	S0, 0	S2, 0
S2	S2, 0	S2, 1

**Q.3.** Consider the sequential circuit given below having 5 inputs {A, B, C, D, E} and one output {Z}. Assume that the delay of a gate is related to the number of inputs i.e. the delay of a 2-input AND gate is 2 unit delays and the delay of a 2-input OR gate is 2 unit delays.



(i) Determine the critical path of this circuit and the maximum propagation delay.

(i) The maximum propagation delay is 8 and there are 8 critical paths as follows:

$\{A, G1, G3, G4, G6\}$ ,  $\{A, G1, G3, G5, G7\}$   
 $\{B, G1, G3, G4, G6\}$ ,  $\{B, G1, G3, G5, G7\}$   
 $\{B, G2, G3, G4, G6\}$ ,  $\{B, G2, G3, G5, G7\}$   
 $\{C, G2, G3, G4, G6\}$ ,  $\{C, G2, G3, G5, G7\}$

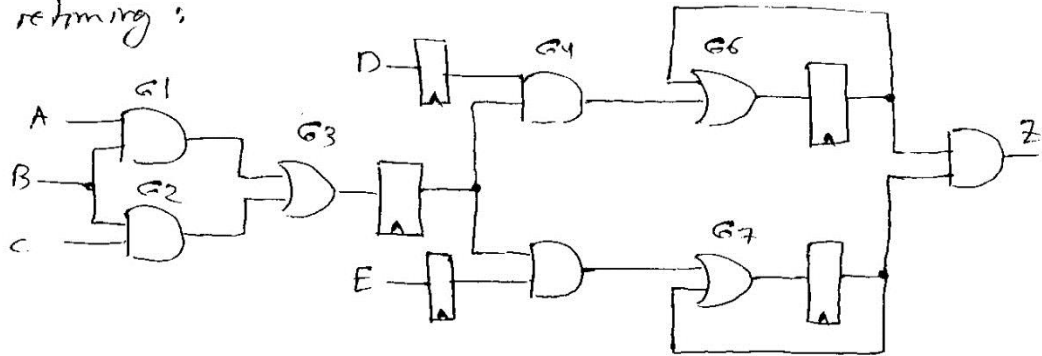
(ii) Using only the **Retiming** transformation, reduce the critical path of this circuit with the minimum number of flip-flops possible. Determine the maximum propagation delay after retiming.

(ii) we can apply the following retiming transform to reduce the critical path:

- retime G6 by +1
- retime G7 by +1
- retime G8 by +1
- retime the stem on fanout of G6 by +1
- retime the stem on fanout of G7 by +1

- return  $G_4$  by +1
- return  $G_5$  by +1
- return the stem on the fanout of  $G_3$  by +1

This results in the following circuit after retiming:



The maximum propagation delay in the resulting circuit is 4. The number of flip-flops has increased from 3 to 5.