Name: KEY **Id#**

COE 405, Term 122

Design & Modeling of Digital Systems

Quiz# 1

Date: Monday, Feb. 18, 2013

Q.1. Consider the two functions: $F_1(A,B,C,D) = AD + BC + A\overline{C} + \overline{A}\,\overline{D}$ and $F_2(A,B,C,D) = ABC + \overline{B}\,\overline{C} + \overline{A}\,\overline{C} + \overline{A}\,\overline{D}$

(i) Compute the expansion of F_1 and F_2 using the **Orthonormal Basis** $\{\varnothing_1 = \overline{A}\,\overline{B}$, $\varnothing_2 = \overline{A}B$, $\varnothing_3 = A\overline{B}$, $\varnothing_4 = AB\}$.

(ii) Compute the function $F_1 \oplus F_2$.

(I)

$$F_1 = \overline{A}\,\overline{B}\,[\overline{D}] + \overline{A}B\,[C + \overline{D}] + A\overline{B}\,[\overline{C} + D] + AB\,[1]$$

$$F_2 = \overline{A}\,\overline{B}\,[\overline{C} + \overline{D}] + \overline{A}B\,[\overline{C} + \overline{D}] + A\overline{B}\,[\overline{C}] + AB\,[C]$$

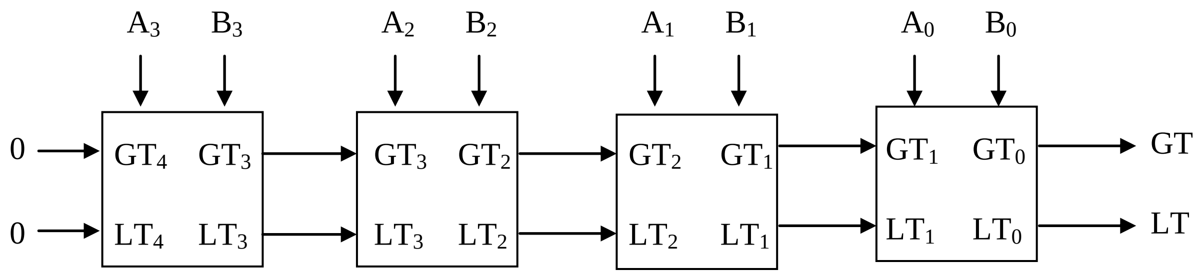
(ii)

$$F_1 \oplus F_2 = \overline{A}\,\overline{B}\,[\overline{D} \oplus (\overline{C} + \overline{D})] + \overline{A}B\,[(C + \overline{D})(\overline{C} + \overline{D})] + A\overline{B}\,[\overline{C} \oplus (\overline{C} + D)] + AB\,[C \oplus 1]$$

$$= \overline{A}\,\overline{B}\,[\overline{D} \cdot CD + D(\overline{C} + \overline{D})] + \overline{A}B\,[(C + \overline{D})CD + \overline{C}D(\overline{C} + \overline{D})] + A\overline{B}\,[\overline{C} \cdot C\overline{D} + C(\overline{C} + D)] + AB\,[\overline{C}]$$

$$= \overline{A}\,\overline{B}\,[\overline{C}D] + \overline{A}B\,[D] + A\overline{B}\,[CD] + AB\,[\overline{C}]$$

Q.2. It is required to model a 4-bit comparator that compares two 4-bit numbers $A=A_3A_2A_1A_0$ and $B=B_3B_2B_1B_0$, and produces two outputs GT and LT. If A>B, then the output signal GT is set to 1 and LT is set to 0. If A<B, then the output signal LT is set to 1, and GT is set to 0. Otherwise both signals will be set to 0, which indicates that the two numbers are equal (i.e. A=B). The 4-bit comparator circuit can be designed in a modular way as shown below:



(i) Derive the truth table for a 1-bit magnitude comparator.

(ii) Find the simplified equations for the outputs of a 1-bit magnitude comparator.

(i) Truth Table:

GT_{i+1}	LT_{i+1}	A_i	B_i	GT_i	LT_i
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	0
0	1	x	x	0	1
1	0	x	x	1	0
1	1	x	x	x	x

(ii)

$GT_{i+1}LT_{i+1}$ \ A_iB_i	00	01	11	10
00	0	0	0	1
01	0	0	0	0
11	x	x	x	x
10	1	1	1	1

$GT_{i+1}LT_{i+1}$ \ A_iB_i	00	01	11	10
00	0	1	0	0
01	1	1	1	1
11	x	x	x	x
10	0	0	0	0

$$GT_i = GT_{i+1} + \overline{LT_{i+1}} \cdot A_i \overline{B_i}$$

$$LT_i = LT_{i+1} + \overline{GT_{i+1}} \cdot \overline{A_i} B_i$$