

August 13, 2007

COMPUTER ENGINEERING DEPARTMENT

ICS 233

COMPUTER ARCHITECTURE & ASSEMBLY LANGUAGE

Major Exam II

Summer Semester (063)

Time: 7:00-9:30 PM

Student Name : _KEY_____

Student ID. : _____

Question	Max Points	Score
Q1	20	
Q2	16	
Q3	16	
Q4	14	
Q5	8	
Q6	16	
Q7	10	
Total	100	

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[20 Points]

(Q1) Suppose that you are given a positive integer. You can add individual digits of this number to get another integer. If we repeat this procedure, eventually we will end up with a single digit. Here is an example:

$$7391928 = 7+3+9+1+9+2+8 = 39$$

$$39 = 3+9 = 12$$

$$12 = 1+2 = 3$$

Write a procedure, **ToSDigit**, that receives a positive integer in \$a0 and returns a single digit in register \$v0 according to the method described above. It is required that the procedure **preserves the content of all used registers** by saving and restoring them on the stack. Then, write a program to read a positive integer from the user and display the **single digit** obtained by the above procedure.

A **sample execution** of the program is:

```
Enter a number: 7391928
Result is: 3
```

```
##### Data segment #####
.data
MSG1: .asciiz "Enter a number: "
MSG2: .asciiz "Result is: "

##### Code segment #####
.text
.globl main
main: # main program entry

# Getting the number
    li $v0, 4
    la $a0, MSG1
    syscall
    li $v0, 5
    syscall

# Calling the procedure
    move $a0, $v0
    jal ToSDigit

# Displaying the result
    move $t0, $v0
    li $v0, 4
    la $a0, MSG2
    syscall
```

```

        li $v0, 1
        move $a0, $t0
        syscall

```

```

# Exit program
    li $v0, 10
    syscall

```

The procedure receives a positive integer in \$a0 and returns a single digit in register \$v0
ToSDigit:

```

# Saving used registers on the stack
    addi $sp, $sp, -4    #saving register $t0
    sw $t0, ($sp)
    addi $sp, $sp, -4    #saving register $t1
    sw $t1, ($sp)
    addi $sp, $sp, -4    #saving register $t2
    sw $t2, ($sp)
    addi $sp, $sp, -4    #saving register $t3
    sw $t3, ($sp)

```

```

    li $t0, 10
    move $t1, $a0

```

Next:

```

    xor $t3, $t3, $t3    # Holds the sum of the digits

```

Compute the sum of the digits

Again:

```

    divu $t1, $t0
    mflo $t1                # $t1 = quotient
    mfhi $t2                # $t2 = remainder
    addu $t3, $t3, $t2
    bnez $t1, Again

```

Check if the sum is greater than 9

```

    move $t1, $t3
    bge $t1, $t0, Next
    move $v0, $t1

```

Restoring registers from the stack

```

    lw $t3, ($sp)
    addi $sp, $sp, 4    #restoring register $t3
    lw $t2, ($sp)
    addi $sp, $sp, 4    #restoring register $t2
    lw $t1, ($sp)
    addi $sp, $sp, 4    #restoring register $t1
    lw $t0, ($sp)
    addi $sp, $sp, 4    #restoring register $t0

```

```

    jr $ra

```

(Q2) Given that **Multiplicand=1010** and **Multiplier=1001**.

- (i) Using the **refined unsigned multiplication** hardware, show the **unsigned** multiplication of **Multiplicand** by **Multiplier**. The result of the multiplication should be an 8 bit **unsigned** number in HI and LO registers. Show the steps of your work.

Iteration	Multiplicand	Carry	Product = HI,LO
0	Initialize (LO = Multiplier)	1010	0000 1001
1	LO[0] = 1 => ADD	0	1010 1001
	Shift Product = (HI, LO) right 1 bit	1010	0101 0100
2	LO[0] = 0 => Do Nothing	0	0101 0100
	Shift Product = (HI, LO) right 1 bit	1010	0010 1010
3	LO[0] = 0 => Do Nothing	0	0010 1010
	Shift Product = (HI, LO) right 1 bit	1010	0001 0101
4	LO[0] = 1 => ADD	0	1011 0101
	Shift Product = (HI, LO) right 1 bit		0101 1010

- (ii) Using the **refined signed multiplication** hardware, show the **signed** multiplication of **Multiplicand** by **Multiplier**. The result of the multiplication should be an 8 bit **signed** number in HI and LO registers. Show the steps of your work.

Iteration	Multiplicand	Sign	Product = HI,LO
0	Initialize (LO = Multiplier)	1010	0000 1001
1	LO[0] = 1 => ADD	1	1010 1001
	Shift Product = (HI, LO) right 1 bit	1010	1101 0100
2	LO[0] = 0 => Do Nothing	1	1101 0100
	Shift Product = (HI, LO) right 1 bit	1010	1110 1010
3	LO[0] = 0 => Do Nothing	1	1110 1010
	Shift Product = (HI, LO) right 1 bit	1010	1111 0101
4	LO[0] = 1 => SUB (ADD 2's comp.)	0	0101 0101
	Shift Product = (HI, LO) right 1 bit		0010 1010

[
16 Points]**(Q3)** Given that **Dividend=1010** and **Divisor=0100**.

- (i) Using the **refined unsigned division** hardware, show the **unsigned** division of **Dividend** by **Divisor**. The result of division should be stored in the Remainder and Quotient registers. Show the steps of your work.

Iteration		Remainder	Quotient	Divisor	Difference
0	Initialize	0000	1010	0100	
1	1: SLL, Difference	0001	0100	0100	1101
	2: Diff < 0 => Do Nothing				
2	1: SLL, Difference	0010	1000	0100	1110
	2: Diff < 0 => Do Nothing				
3	1: SLL, Difference	0101	0000	0100	0001
	2: Rem = Diff, set lsb Quotient	0001	0001		
4	1: SLL, Difference	0010	0010	0100	1110
	2: Diff < 0 => Do Nothing				

- (ii) Using the **refined unsigned division** hardware, show the **signed** division of **Dividend** by **Divisor**. The result of division should be stored in the Remainder and Quotient registers. Show the steps of your work.

First, we convert the dividend into a positive number by taking its 2's complement. Thus, the dividend becomes 0110. Then, we perform unsigned division as shown below.

Iteration		Remainder	Quotient	Divisor	Difference
0	Initialize	0000	0110	0100	
1	1: SLL, Difference	0000	1100	0100	1100
	2: Diff < 0 => Do Nothing				
2	1: SLL, Difference	0001	1000	0100	1101
	2: Diff < 0 => Do Nothing				
3	1: SLL, Difference	0011	0000	0100	1111
	2: Diff < 0 => Do Nothing				
4	1: SLL, Difference	0110	0000	0100	0010
	2: Rem = Diff, set lsb Quotient	0010	0001		

Since Quotient sign should be negative we take its 2's complement. Thus, Quotient=1111.

Also, since remainder sign is negative, we take its 2's complement. Thus, Remainder=1110.

(Q4) Given the following two floating-point numbers in single-precision format:

$$X = 1100\ 0011\ 1100\ 0000\ 0000\ 0000\ 0000\ 0001$$

$$Y = 0011\ 1110\ 1000\ 0000\ 0000\ 0010\ 0011\ 0001$$

- (i) Perform the floating-point operation $X - Y$ rounding the result to the nearest even, using guard, round and sticky bits. Represent the result in single-precision format.

Since the sign of X is negative and the operation is subtraction, then the effective operation is addition and the sign of the result is negative.

$$\begin{array}{r}
 \\
 + \quad 1.100\ 0000\ 0000\ 0000\ 0000\ 0001\ 000 \quad \times 2^8 \\
 \\
 + \quad 1.000\ 0000\ 0000\ 0010\ 0011\ 0001\ 000 \quad \times 2^{-2} \\
 \hline
 = \quad 1.100\ 0000\ 0000\ 0000\ 0000\ 0001\ 000 \quad \times 2^8 \\
 + \quad 0.000\ 0000\ 0010\ 0000\ 0000\ 0000\ 101 \quad \times 2^8 \\
 \hline
 = \quad -1.100\ 0000\ 0010\ 0000\ 0000\ 0001\ 101 \quad \times 2^8
 \end{array}$$

Then, we round to the nearest even and we add a 1. Thus, the result will be:

$$-1.100\ 0000\ 0010\ 0000\ 0000\ 0010 \quad \times 2^8$$

The result represented in single-precision format is given below:

$$1100\ 0011\ 1100\ 0000\ 0010\ 0000\ 0000\ 0010$$

- (ii) Perform the floating-point operation $X * Y$ rounding the result to the nearest even. Represent the result in single-precision format.

$$\begin{array}{r}
 \\
 * \quad -1.100\ 0000\ 0000\ 0000\ 0000\ 0001 \quad \times 2^8 \\
 \\
 * \quad +1.000\ 0000\ 0000\ 0010\ 0011\ 0001 \quad \times 2^{-2} \\
 \hline
 = \quad +1.000\ 0000\ 0000\ 0010\ 0011\ 0001 \quad \times 2^{-2} \\
 * \quad -1.100\ 0000\ 0000\ 0000\ 0000\ 0001 \quad \times 2^8 \\
 \hline

 \end{array}$$

The result for the exponent will be $2^{8-2} = 2^6$

To obtain the result of the fraction, we multiply the two fractions as shown below:

$$\begin{array}{r}
 \\
 \\
 + \quad 1000110001 \\
 + \quad 1000110001 \\
 \hline
 11000000000001101001010100000000001000110001
 \end{array}$$

Thus, the fraction result will be equal to the following since we will have 46 bits in the fraction part:

1.10000000000011010010101000000000001000110001

Rounding the result to the nearest even makes us add 1 since the bit next to the least significant bit is 1 and the remaining bits are not 0. Thus, the result of multiplication will be:

$$-1.100\ 0000\ 0000\ 0011\ 0100\ 1011 \quad \times 2^6$$

The result expressed in single-precision format is:

1100 0010 1100 0000 0000 0011 0100 1011

[8 Points]

(Q5) Suppose that a program runs in 150 seconds on a machine, with multiply operations responsible for 40 seconds of this time, divide operations responsible for 60 seconds of this time. The remaining time is taken by the remaining operations. Suppose that a new implementation of the machine has improved the execution time of the multiplier by a factor of 3 and the execution time of the divider by a factor of 2. Determine the new execution time and the speedup of the program on the new implementation.

Execution time of new implementation = $40/3 + 60/2 + 50 = 93.33$ seconds

Speedup = $150/93.33 = 1.607$

[16 Points]**(Q6)** Given the following instruction mix of a program on a RISC processor:

Class	CPI	Frequency
ALU	3	20%
Branch	2	30%
Jump	1	25%
Load	5	15%
Store	4	10%

(i) What is the average CPI?

$$\text{Average CPI} = 3 \times 0.2 + 2 \times 0.3 + 1 \times 0.25 + 5 \times 0.15 + 4 \times 0.10 = 2.6$$

(ii) Assuming that the processor has a clock rate of 3 GHz, determine MIPS.

$$\text{MIPS} = \text{Clock Rate} / (\text{CPI} \times 10^6) = 3 \times 10^9 / 2.6 \times 10^6 = 1.154 \times 10^3$$

(iii) What is the percent of time used by each instruction class?

Class	Percentage of Time
ALU	$3 \times 0.2 / 2.6 = 0.6 / 2.6 = 23.07\%$
Branch	$2 \times 0.3 = 0.6 / 2.6 = 23.07\%$
Jump	$1 \times 0.25 / 2.6 = 0.25 / 2.6 = 9.62\%$
Load	$5 \times 0.15 / 2.6 = 0.75 / 2.6 = 28.85\%$
Store	$4 \times 0.10 / 2.6 = 0.4 / 2.6 = 15.38\%$

(iv) How much faster would the program run if load and store time are reduced to 3 cycles, and two ALU instructions could be executed at once, assuming that the cycle time has increased by 10% and the instruction count has increased by 15%?

$$\text{New Average CPI} = 1.5 \times 0.2 + 2 \times 0.3 + 1 \times 0.25 + 3 \times 0.15 + 3 \times 0.10 = 1.9$$

$$\begin{aligned} \text{Speedup} &= \text{Old Execution Time} / \text{New Execution Time} \\ &= \text{IC} \times 2.6 \times \text{Cycle Time} / 1.1 \times \text{IC} \times 1.9 \times 1.15 \times \text{Cycle Time} \\ &= 2.6 / 1.1 \times 1.9 \times 1.15 = 2.6 / 2.4035 = 1.082 \end{aligned}$$

[10 Points]

(Q7) Assume that a processor has four 8-bit registers: R0, R1, R2, and R3. You are required to design a register file that allows reading the value of one of the registers specified by the field RS[1:0] and writing into one of the registers specified by the field RD[1:0]. Show all required control signals for the register file.

