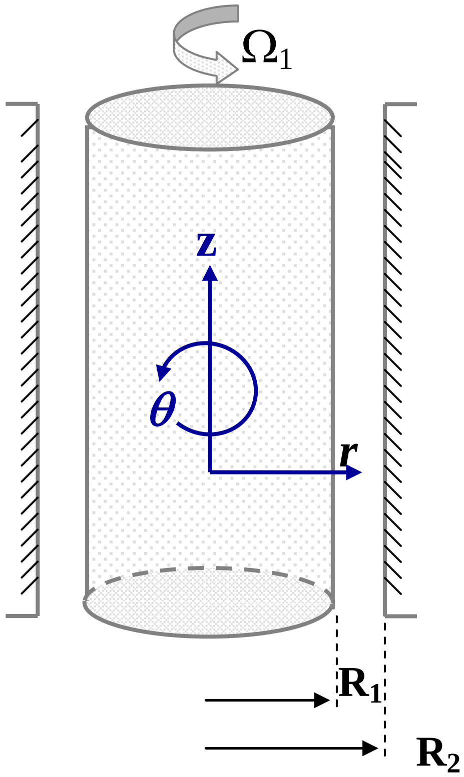
King Fahd University of Petroleum & Minerals
Chemical Engineering Department
CHE 560 –Numerical Methods in Chemical Engineering
2010 - 2011 (102)

HW#6
Due: Sunday: 1-May-2011

Taylor-Couette flow refers to the fluid motion in the annulus between two infinitely long, coaxial, and independently rotating cylinders with radii R_1 and R_2 and angular speeds of rotation Ω_1 and Ω_2 (s^{-1}) where the 1 and 2 refer to the inner and the outer cylinder, respectively.



For isothermal steady state conditions, the differential equation describing the flow is given as follows:

$$\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}\left(rv_\theta\right)\right)=0$$

and the boundary conditions:

$$r=R_1 \qquad v_\theta=R_1\Omega_1$$

$$r=R_2 \qquad v_\theta=R_2\Omega_2$$

Perform the following tasks:

(a) Show that the differential equation can be written as:

$$r^2 \frac{d^2 v_\theta}{dr^2} + r \frac{dv_\theta}{dr} - v_\theta = 0$$

(b) Put the differential equation and BC's in dimensionless form using $R_1\Omega_1$ as a scale for the velocity and R_2-R_1 as a scale for the length.

(c) Solve the dimensionless BVP analytically for $R_1/R_2 = 0.7$ and $\Omega_2/\Omega_1=0$.

(d) Provide an algorithm that allows the solution using Chebyshev-Collocation method showing all required details (mapping, residual equations, resulting matrix A and vector B ... etc.).

(e) Use the sample programs Code-5.f provided to you to solve this problem using $N = 10$ and compare your numerical answer with the analytical solution. Send your program by e-mail through WebCT as yourname-hw6.f