Flow through Packed Beds

The mechanical energy balance can be applied for flow across a packed bed:

\[ \frac{\Delta P}{\rho} + \Delta \left( \frac{u_0^2}{2} \right) + g \Delta z + \mathcal{Z} + w_s = 0 \]  

(1)

If the cross-sectional area of the bed \( A \) is constant and there is no shaft work between the bed’s inlet and outlet, the mechanical energy balance can be simplified:

\[ -\frac{\Delta P}{\rho} = g \Delta z + \mathcal{Z} \]  

(2)

The pressure drop across the bed is due gravity and friction. For packed beds the frictional losses is given by the following equation:

\[ \mathcal{Z} = 3 f_F \left( \frac{1 - \varepsilon}{\varepsilon^3} - \frac{u_0^2}{D_p} \right) \frac{L}{D_p} \]  

(3)

where \( \varepsilon \) is the fraction of void not occupied by particles (void fraction), \( L \) is the total length of the bed and \( D_p \) is the effective particle diameter defined as follows:

\[ D_p = \frac{6}{a_v} \]  

(4)
and $a_v$ is the total external surface area of the particle divided by particle volume. For spherical particles, $D_p$ is simply the particle diameter. $f_f$ appearing in equation (3) is the friction factor for packed beds which accounts for both laminar and turbulent flow regimes:

$$f_f = \frac{1}{3} \left[ \frac{150}{\text{Laminar Contribution}} \left( \frac{1}{\text{Re}} \right) + \frac{1}{\text{Turbulent Contribution}} \left( \frac{1.75}{\text{Re}} \right) \right]$$  \hspace{1cm} (5)$$

and $Re$ is the Reynolds number for packed beds defined as:

$$Re = \frac{\rho u_0 D_p}{(1 - \varepsilon) \mu}$$  \hspace{1cm} (6)$$

where $\rho$ and $\mu$ are the fluid density and viscosity, respectively. $u_0$ is the fluid superficial velocity defined as the ratio of fluid volumetric flow rate by the cross-sectional area of the bed:

$$u_0 = \frac{Q}{A}$$  \hspace{1cm} (7)$$

Substituting equations (3-5) into the mechanical energy balance equation (2) and rearranging, the following equation can be derived for flow through packed beds known as Ergun equation:

$$-\frac{\Delta P}{\rho u_0^2} \frac{D_p}{L} \varepsilon^3 = \left[ \frac{150}{\text{Re}} + 1.75 \right] + \frac{1}{u_0^2} \frac{D_p}{L} \varepsilon^3 g \Delta z$$  \hspace{1cm} (8)$$

The above equation is similar to equation [4.26] of the textbook, however, it is more general to account for non-horizontal beds. Hence, equation [4.26] is for the special case when $\Delta z = 0$.

Rearranging equation (8), the following equation can be written for the pressure drop across packed beds:

$$-\frac{\Delta P}{\rho} = \sqrt{\left[ \frac{150}{\text{Re}} + 1.75 \right] u_0^2 \frac{L}{D_p} \frac{1 - \varepsilon}{\varepsilon^3} + g \Delta z}$$  \hspace{1cm} (9)$$

Once again the above equation is similar to equation [4.29] of the textbook, however, it is more general to account for non-horizontal beds. Hence, equation [4.29] is for the special case when $\Delta z = 0$. 