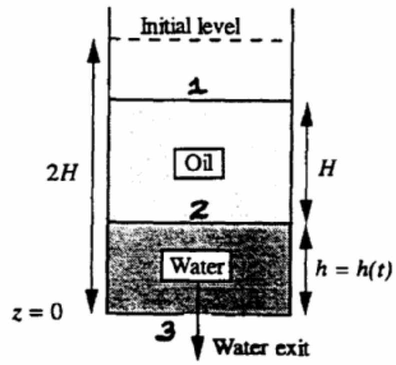
2.15

Draining Immiscible Liquids from a Tank



From hydrostatics,

$$p_2 = \rho_o g H$$

Bernoulli ② → ③

$$\frac{p_2}{\rho_w} + g h = \frac{u_3^2}{2}$$

Hence

$$u_3 = \sqrt{2g\left(\frac{\rho_o}{\rho_w} H + h\right)}$$

Unsteady-State Mass Balance on Tank (Rate balance)

$$-0.62 a\, u_3 \rho_w = -0.62 a \rho_w \sqrt{2g\left(\frac{\rho_o}{\rho_w} H + h\right)} = A \rho_w \frac{dh}{dt}$$

Leaving through orifice

Rate of accumulation of water in tank (will be negative, but don't anticipate this with a minus sign)

$$\int_0^t dt = -\frac{A}{0.62 a \sqrt{2g}} \int_H^0 \frac{dh}{\sqrt{\frac{\rho_o}{\rho_w} H + h}}$$

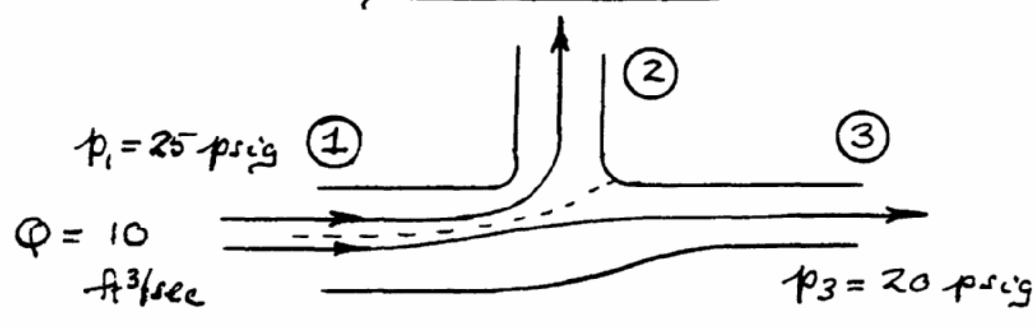
$k = 1$

$$t = \frac{2A}{\sqrt{2g}\, 0.62 a} \left[\sqrt{\frac{\rho_o}{\rho_w} H + h}\,\right]_0^H$$

$$t = \frac{2A}{0.62 a \sqrt{2g}} \left[\sqrt{H\left(\frac{\rho_o}{\rho_w} + 1\right)} - \sqrt{\frac{\rho_o}{\rho_w} H}\,\right]$$

2.21

Branch Pipe



Areas $A_1 = 0.1364$, $A_2 = 0.0873$, $A_3 = 0.0491$ ft²

$$u_1 = \frac{10}{0.1364} = 73.3 \text{ ft/sec}, \quad \rho = 0.8 \times 62.4 = 49.9 \frac{\text{lbm}}{\text{ft}^3}$$

Bernoulli ① → ③ $\quad \dfrac{u_1^2}{2} + \dfrac{p_1}{\rho} = \dfrac{u_3^2}{2} + \dfrac{p_3}{\rho}$

$$\frac{73.3^2}{2} + \frac{25 \times 32.2 \times 144}{49.9} = \frac{u_3^2}{2} + \frac{20 \times 32.2 \times 144}{49.9}$$

Hence $u_3 = 79.4$ ft/sec

Continuity (mass Balance) (ρ constant)

$$10 = 0.0873\, u_2 + 0.0491 \times 79.4$$

$$u_2 = 69.9 \text{ ft/sec}$$

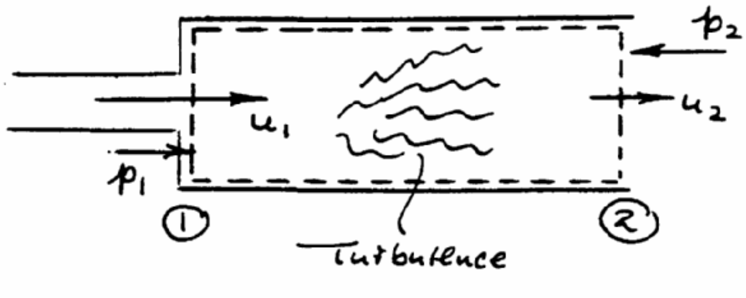
Bernoulli ① → ②

$$\frac{73.3^2}{2} + \frac{25 \times 32.2 \times 144}{49.9} = \frac{69.9^2}{2} + \frac{p_2 \times 32.2 \times 144}{49.9}$$

Hence $p_2 = 27.6$ psig.

2.22

Sudden Expansion in a Pipe



Bernoulli cannot be applied between 1 and 2 because of the turbulence and energy losses that are characteristic of an expanding jet.

Continuity ① → ② $A_1 u_1 = A_2 u_2$ (1)

Momentum ① → ②

$$p_1 A_2 - p_2 A_2 + \rho A_1 u_1^2 - \rho A_2 u_2^2 = 0 \qquad (2)$$

note

Substitute $u_1 = u_2 \dfrac{A_2}{A_1}$ from (1) into (2)

$$p_2 - p_1 = \rho u_2^2 \left(\frac{A_2}{A_1} - 1 \right), \qquad (3)$$

which is always positive, so pressure increases. Increase in pressure energy is at the expense of kinetic energy

Overall Energy Balance

$$\Delta\left(\frac{u^2}{2}\right) + \frac{\Delta p}{\rho} + \overset{\text{zero}}{\cancel{g\Delta z}} + \overset{\text{zero}}{\cancel{w}} + \mathcal{F} = 0$$

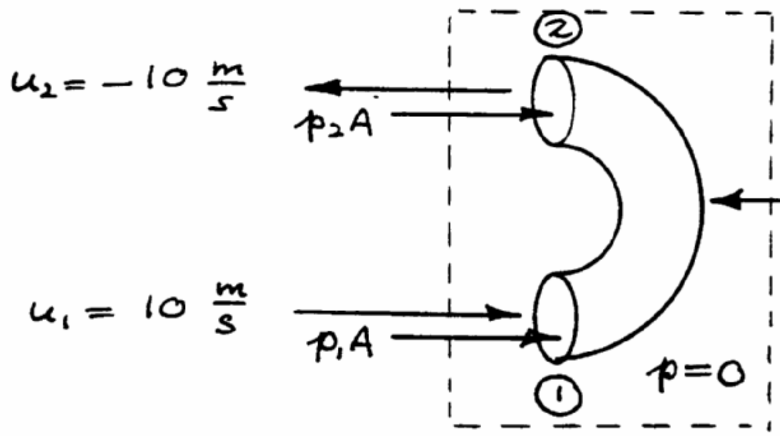
$$\mathcal{F} = \frac{u_1^2}{2} - \frac{u_2^2}{2} + \frac{p_1 - p_2}{\rho} = \frac{u_1^2}{2} - \frac{u_2^2}{2} + \rho u_2^2\left(1 - \frac{A_2}{A_1}\right)$$

$$= \frac{u_2^2}{2}\left(\frac{A_2^2}{A_1^2} - 1 + 2 - 2\frac{A_2}{A_1}\right) = \frac{u_2^2}{2}\left(\frac{A_2}{A_1} - 1\right)^2,$$

which is always positive.

2.23

Force on Return Elbow



$D = 0.1\ m$

$p_1 = 3.0$, $p_2 = 2.5$ } bar (gauge)

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.1)^2}{4} = 0.00785\ m^2$$

$$\dot{m} = \rho A u_1 = \underset{\frac{kg}{m^3}}{1{,}000} \times \underset{m^2}{0.00785} \times \underset{\frac{m}{s}}{10} = 78.54\ \frac{kg}{s}$$

Momentum Balance → on Return Elbow

$$\underbrace{+\ \dot{m}u_1 \ -\ \dot{m}u_2}_{\text{Addition Loss By flow}} + \underbrace{p_1 A + p_2 A}_{\text{Both } p_1 \text{ and } p_2 \text{ tend to push elbow to right}} - F = 0$$

Elbow Stays Put.

$$F = (p_1 + p_2) A + 2\dot{m}u_1$$

$$= (3.0 + 2.5) \times 10^5\ \tfrac{N/m^2}{bar} \times 0.00785 + 2 \times 78.54 \times 10$$

$$= 4{,}317 + 1571 = \underline{\underline{5{,}888\ N}}$$