Decision Making with Utilities
• In some situations, there may be factors that make decision making using expected value unacceptable.

• This usually happens when
  – the amount of loss is so big that it will not be acceptable even at low probability (e.g. insurance)
  – the amount of profit is so big that it will be sought even at low probability (e.g. lottery)
Utility

Utility is a measure of the worth of an outcome to the decision maker. It reflects the decision maker’s attitude towards risk.
• Consider the 3 decision alternatives
d_1 = make investment A
d_2 = make investment B
d_3 = do not invest
• The states of nature are
  s_1 = prices go up \Rightarrow P(s_1)=0.30
  s_2 = prices remain stable \Rightarrow P(s_1)=0.50
  s_3 = prices go down \Rightarrow P(s_3)=0.20
## Payoff Table

<table>
<thead>
<tr>
<th>Decision</th>
<th>Prices up</th>
<th>Prices stable</th>
<th>Prices down</th>
</tr>
</thead>
<tbody>
<tr>
<td>d₁</td>
<td>30,000</td>
<td>20,000</td>
<td>-50,000</td>
</tr>
<tr>
<td>d₂</td>
<td>50,000</td>
<td>-20,000</td>
<td>-30,000</td>
</tr>
<tr>
<td>d₃</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Decision based on Expected Value

\[
\begin{align*}
EV(d_1) &= 0.3(30,000) + 0.5(20,000) + 0.2(-50,000) = 9,000 \\
EV(d_2) &= 0.3(50,000) + 0.5(-20,000) + 0.2(-30,000) = -1,000 \\
EV(d_3) &= 0.3(0) + 0.5(0) + 0.2(0) = 0
\end{align*}
\]

Therefore, SELECT \( d_1 \) with \( EV = 9,000 \)

- But, suppose that this company is in financial difficulty and the loss of 50,000 would be detrimental to it.
- In such a case, \( d_1 \) is not desirable
- To deal with this situation, we need to determine the utilities of the monetary values in the decision
Determining Utilities

• Let $U(M) = \text{Utility of } M$ [M=Monetary Amount]
• Assign (arbitrarily) utility to the highest monetary value in payoff table
  $\Rightarrow U(50,000) = 10$
• Assign (arbitrarily) utility to the lowest monetary value in payoff table
  $\Rightarrow U(-50,000) = 0$
  *Note: $U(50,000) > U(-50,000)$*
• Assign utilities to the other values as follows
Assigning Utilities to Middle Values

Consider the payoff of 30,000:

Decision maker is asked to select one of two choices:
1. a guaranteed amount of 30,000, or
2. participate in the following lottery
   DM get a payoff of 50,000 with probability \( p \), or a payoff of -50,000 with probability \((1-p)\)

a. If \( p \) is close to 0, DM will prefer the 30,000
b. If \( p \) is close to 1, DM will prefer lottery

c. As we increase \( p \) from 0, at what value does DM changes preference to the lottery?
d. At this value of \( p \), DM has no preference between the two choices (equivalent)
Assigning Utility to the Value 30,000

- Suppose the DM chooses $p=0.95$
- Now we can determine the $U(30,000)$ as follows:
  \[
  U(30,000) = pU(50,000) + (1-p)U(-50,000)
  \]
  \[
  = 0.95 \times 10 + 0.05 \times 0
  \]
  \[
  = 9.5
  \]
- Note that at $p=0.95$ the EV(lottery) is:
  \[
  EV(lottery) = 0.95(50,000)+0.05(-50,000)
  \]
  \[
  = 45,000
  \]
• DM is willing to accept a guaranteed amount of 30,000 rather than risk anything more than 5% chance of a loss of 50,000
• The difference between 45,000 and 30,000 is referred to as the risk premium.
• The DM is willing to pay 15,000 risk premium to avoid the 5% chance of losing 50,000.
Assigning Utility to the Value -20,000

- As before, DM is asked to select one of two choices:
  1. a guaranteed amount of -20,000, or
  2. participate in the lottery
     DM get a payoff of \(50,000\) \text{ with probability } p, or a payoff of \(-50,000\) \text{ with probability } \(1-p\)

- For example, we ask DM if \(p=0.90\), would he choose lottery or an assured loss of 20,000 (\(\rightarrow\)lottery)
- We lower \(p\) until point of indifference is reached.
- Suppose at this point \(p=0.55\)
- Now we can determine the \(U(-20,000)\) as follows:
  \[U(-20,000) = pU(50,000) + (1-p)U(-50,000)\]
  \[= 0.55 \times 10 + 0.45 \times 0\]
  \[= 5.5\]
Assigning Utility in General

• We follow the same procedure for all monetary values in the payoff table.

• DM is asked to select one of two choices:
  1. a guaranteed amount M, or
  2. participate in the lottery
     DM get a payoff of $50,000 with probability $p$, or a payoff of $-50,000$ with probability $(1-p)$

• We find the probability $p$ of indifference.

• Then, we can determine the U(M) as follows:
  \[ U(M) = pU(50,000) + (1-p)U(-50,000) \]
  \[ = p(10) + (1-p)(0) \]
  \[ = 10p \]
## Utility of Payoff Values

<table>
<thead>
<tr>
<th>Monetary Value</th>
<th>Indifference Value of p</th>
<th>Utility Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>50,000</td>
<td>NA</td>
<td>10.0</td>
</tr>
<tr>
<td>30,000</td>
<td>0.95</td>
<td>9.5</td>
</tr>
<tr>
<td>20,000</td>
<td>0.90</td>
<td>9.0</td>
</tr>
<tr>
<td>0</td>
<td>0.75</td>
<td>7.5</td>
</tr>
<tr>
<td>-20,000</td>
<td>0.55</td>
<td>5.5</td>
</tr>
<tr>
<td>-30,000</td>
<td>0.40</td>
<td>4.0</td>
</tr>
<tr>
<td>-50,000</td>
<td>NA</td>
<td>0</td>
</tr>
</tbody>
</table>
Utility Table

<table>
<thead>
<tr>
<th>Decision</th>
<th>Prices up $s_1$</th>
<th>Prices stable $s_2$</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>9.5</td>
<td>9.0</td>
<td>0</td>
</tr>
<tr>
<td>$d_2$</td>
<td>10</td>
<td>5.5</td>
<td>4.0</td>
</tr>
<tr>
<td>$d_3$</td>
<td>7.5</td>
<td>7.5</td>
<td>7.5</td>
</tr>
</tbody>
</table>
Decision based on Expected Utility

\[
\begin{align*}
EU(d_1) &= 0.3(9.5) + 0.5(9.0) + 0.2(0) = 7.35 \\
EU(d_2) &= 0.3(10) + 0.5(5.5) + 0.2(4.0) = 6.55 \\
EU(d_3) &= 0.3(7.5) + 0.5(7.5) + 0.2(7.5) = 7.50
\end{align*}
\]

Therefore, SELECT \( d_3 \) [Do Not Invest] with \( EU = 7.50 \)
### Ranking of Alternative with Expected Utility

<table>
<thead>
<tr>
<th>Rank of Alternative</th>
<th>Expected Utility</th>
<th>Expected Monetary Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do not Invest</td>
<td>7.50</td>
<td>0</td>
</tr>
<tr>
<td>Investment A</td>
<td>7.35</td>
<td>9,000</td>
</tr>
<tr>
<td>Investment B</td>
<td>6.55</td>
<td>-1,000</td>
</tr>
</tbody>
</table>

Investment A is rejected because the 20% of 50,000 loss was too RISKY for the DM. DM, in this case, is a Risk Avoider.
Decision Maker is a Risk Taker

- Suppose the DM was feeling comfortable about the financial status of his company
- DM was seeking investments that may lead to high payoff and was willing to take risk
- The DM will follow the same procedure for determination of utilities of the payoffs
- But he will take the choice of the Lottery at lower probabilities
### Utility of Payoff Values for a Risk Taker

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</thead>
<tbody>
<tr>
<td>50,000</td>
<td>NA</td>
<td>10.0</td>
</tr>
<tr>
<td>30,000</td>
<td>0.50</td>
<td>5.0</td>
</tr>
<tr>
<td>20,000</td>
<td>0.40</td>
<td>4.0</td>
</tr>
<tr>
<td>0</td>
<td>0.25</td>
<td>2.5</td>
</tr>
<tr>
<td>-20,000</td>
<td>0.15</td>
<td>1.5</td>
</tr>
<tr>
<td>-30,000</td>
<td>0.10</td>
<td>1.0</td>
</tr>
<tr>
<td>-50,000</td>
<td>NA</td>
<td>0</td>
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## Utility Table

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</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>5.0</td>
<td>4.0</td>
<td>0</td>
</tr>
<tr>
<td>$d_2$</td>
<td>10</td>
<td>1.5</td>
<td>1.0</td>
</tr>
<tr>
<td>$d_3$</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>
Decision based on Expected Utility

\[
EU(d_1) = 0.3(5.0) + 0.5(4.0) + 0.2(0) = 3.50 \\
EU(d_2) = 0.3(10) + 0.5(1.5) + 0.2(1.0) = 3.95 \\
EU(d_3) = 0.3(2.5) + 0.5(2.5) + 0.2(2.5) = 2.50
\]

Therefore, SELECT \(d_2\) [Investment B] with EU = 3.95
### Ranking of Alternative with Expected Utility

<table>
<thead>
<tr>
<th>Rank of Alternative</th>
<th>Expected Utility</th>
<th>Expected Monetary Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment B</td>
<td>3.95</td>
<td>-1000</td>
</tr>
<tr>
<td>Investment A</td>
<td>3.50</td>
<td>9,000</td>
</tr>
<tr>
<td>Do not Invest</td>
<td>2.50</td>
<td>0</td>
</tr>
</tbody>
</table>

- Investment B is selected even though EV is negative.
- DM, in this case, is a Risk Taker and willing to seek the opportunity of 50,000 payoff in Investment B.
Utility Function for Risk Avoider, Risk Taker, and Risk Neutral
EV vs. EU

• As can be seen from the graph above, EV and EU will result in the same recommendation for a risk neutral DM.
• There is a range of monetary values where the DM is neutral ➔ the EV approach should be selected because EV & EU will be in agreement.
• The range is where the payoffs (profit or loss) are not considered great.