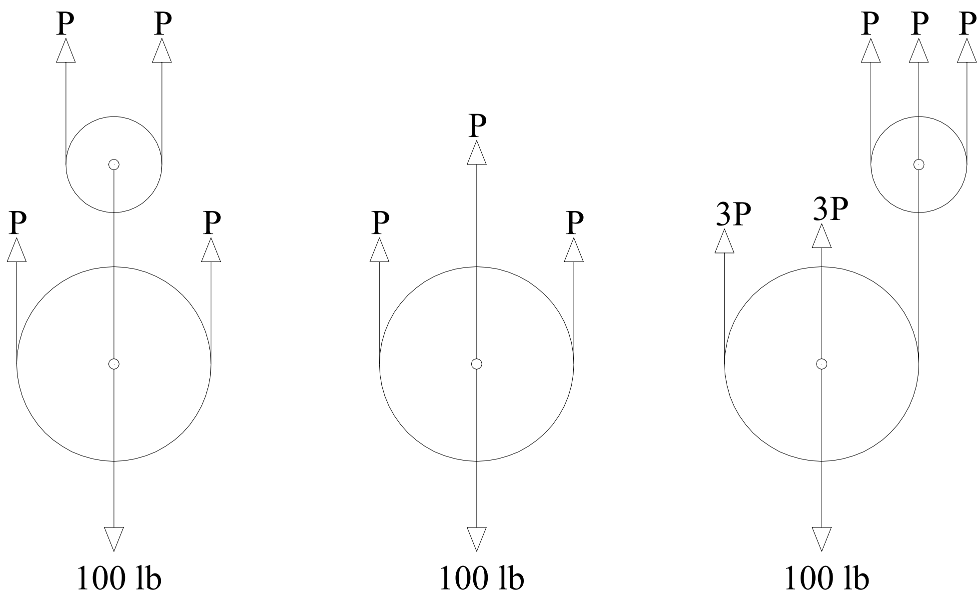
CE 202 HW #5 Solution

#1. Determine the force **P** required to maintain equilibrium in each case.



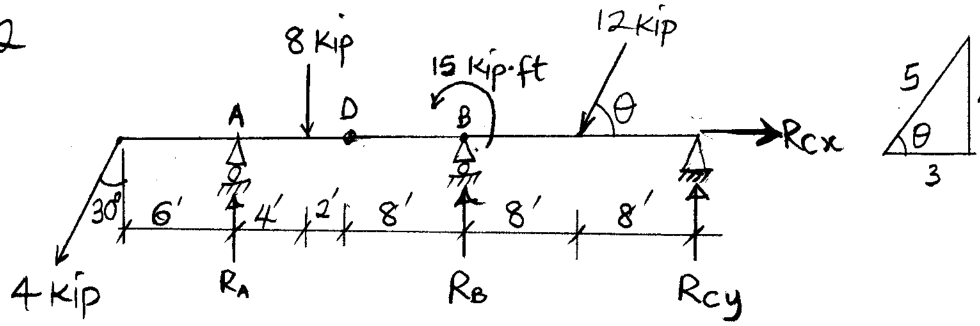
(a) (b) (c)

(a) $\sum Fy = 4P - 100 = 0 \implies P = 25$ lb

(b) $\sum Fy = 3P - 100 = 0 \implies P = 33.33$ lb

(c) $\sum Fy = 3P + 3P + 3P - 100 = 0 \implies P = 11.11$ lb

#2



It's required to determine the reactions at the supports

$\cos\theta = \frac{3}{5}$; $\sin\theta = \frac{4}{5}$

Since D is a hinge, moment at B varnishes. Thus

$M_D = 4\cos 30°(12) - 6R_A + 8(2) = 0$

$6R_A = 57.5692 \Rightarrow \underline{R_A = 9.595 \text{ kips}}$

Moment of all forces & reactions about B = 0

However for the left part of D, all forces & reactions can not transfer moment across the hinge but can transfer shear forces. So

shear at D $= -4\cos 30° + 9.595 - 8 = -1.869\,K = 1.869\,K \downarrow_D$

$\therefore M_B = 1.869(8) + 15 - 12\left(\frac{4}{5}\right)(8) + 16R_{cy} = 0$

$16R_{cy} = +46.848 \Rightarrow \underline{R_{cy} = 2.928 \text{ kips}}$

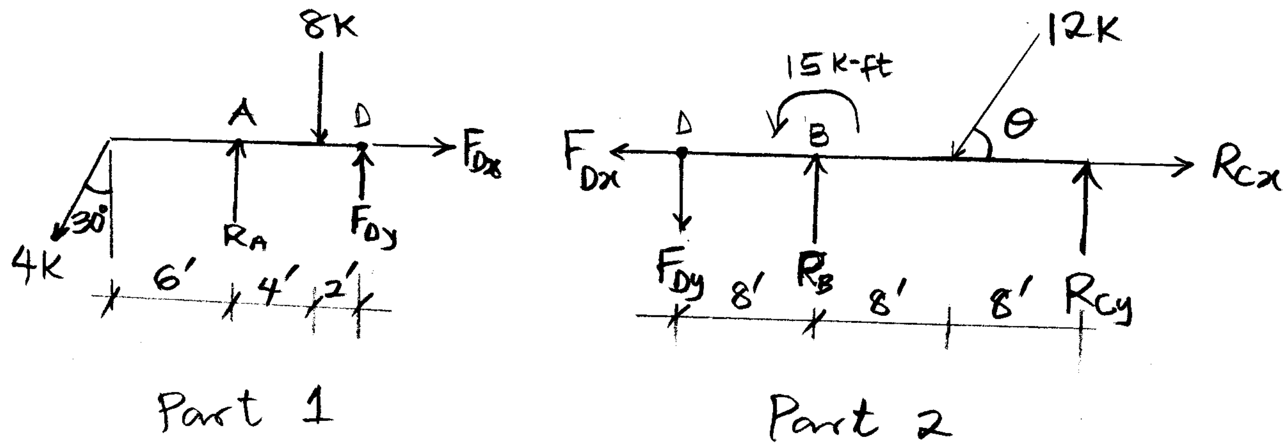
$\sum F_y = -1.869 + R_B - 12\sin\theta + R_c = 0$

$-1.869 + R_B - 12\left(\frac{4}{5}\right) + 2.928 = 0$

$\underline{R_B = 8.541 \text{ kips}}$

$\sum F_x = -4\sin 30° - 12\left(\frac{3}{5}\right) + R_{cx} = 0 \Rightarrow \underline{R_{cx} = 9.2 \text{ kips}}$

Alternatively we can separate the structure at the hinge D as



Equilibrium of part 1:

$\sum F_x = -4\sin 30° + F_{Dx} = 0 \Rightarrow \underline{F_{Dx} = 2 \text{ Kips}}$

$\sum M_D = 4\cos 30°(12) - 6R_A + 8(2) = 0$

$6R_A = 57.5692 \Rightarrow \underline{R_A = 9.595 \text{ Kips}}$

$\sum F_y = -4\cos 30° + R_A - 8 + F_{Dy} = 0$

$-4\cos 30° + 9.595 - 8 + F_{Dy} = 0$

$\underline{F_{Dy} = 1.869 \text{ Kip} \uparrow}$

Equilibrium of part 2:

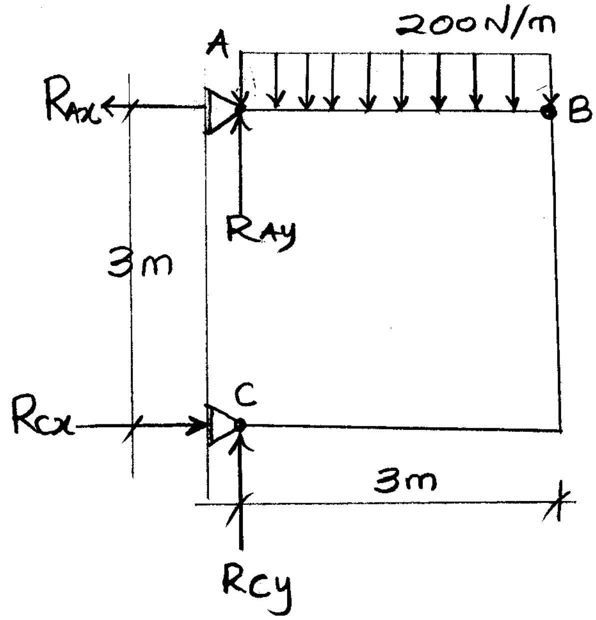
$\sum M_B = 1.869(8) + 15 - 12\left(\frac{4}{5}\right)(8) + 16R_{Cy} = 0$

$R_{Cy} = 2.928 \text{ Kips}$

$\sum F_y = -1.869 + R_B - 12\left(\frac{4}{5}\right) + 2.928 = 0 \Rightarrow \underline{R_B = 8.541 \text{ Kips}}$

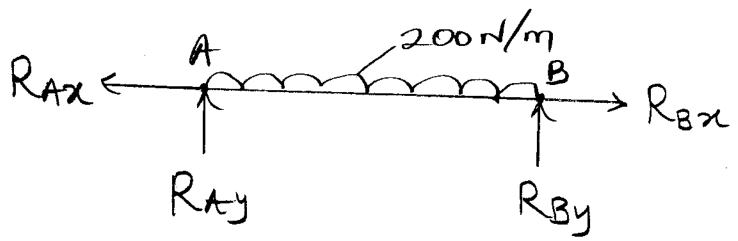
$\sum F_x = -2 - 12\left(\frac{3}{5}\right) + R_{Cx} = 0 \Rightarrow \underline{R_{Cx} = 9.2 \text{ Kip}}$

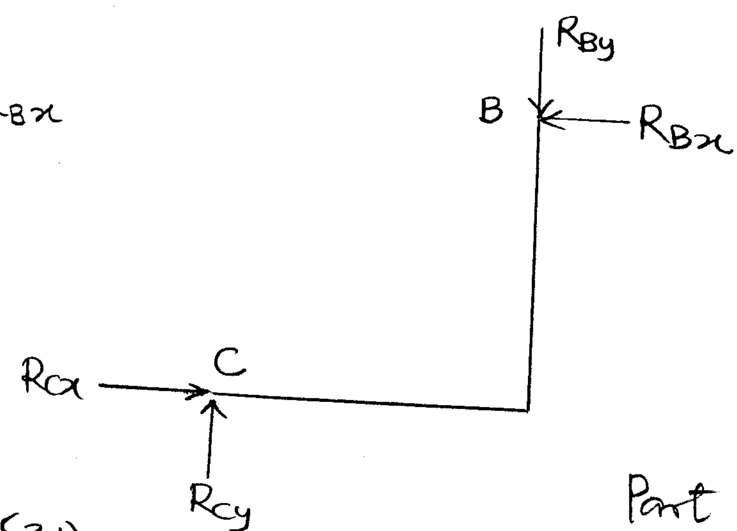
#3



It's required to determine the horizontal and vertical components of force at pins A and C of the frame

Solution: Divide the frame at hinge B like in problem 2





Equilibrium of part 1:

$\sum F_x = R_{Bx} - R_{Ax} = 0$ ---- (3.1)

$\sum M_B = -3R_{Ay} + 200(3)(\frac{3}{2}) = 0 \Rightarrow \underline{R_{Ay} = 300N}$

$\sum F_y = 300 - 200(3) + R_{By} = 0 \Rightarrow \underline{R_{By} = 300N}$

Equilibrium of part 2:

$\sum F_x = R_{Cx} - R_{Bx} = 0$ ---- (3.2)

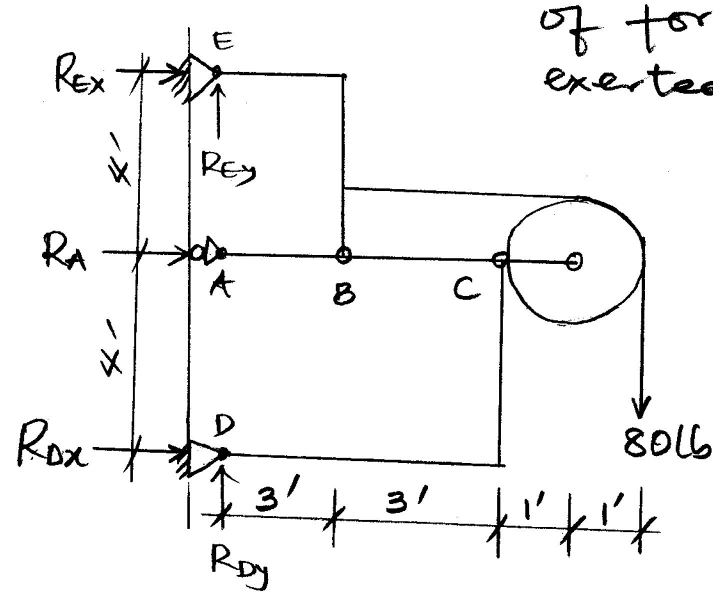
$\sum F_y = R_{Cy} - 300 = 0 \Rightarrow \underline{R_{Cy} = 300N}$

$\sum M_C = 3R_{Bx} - 3(300) = 0 \Rightarrow \underline{R_{Bx} = 300N}$

From (3.1), $R_{Ax} = R_{Bx} = 300N$, and from (3.2), $R_{Cx} = R_{Bx} = 300N$

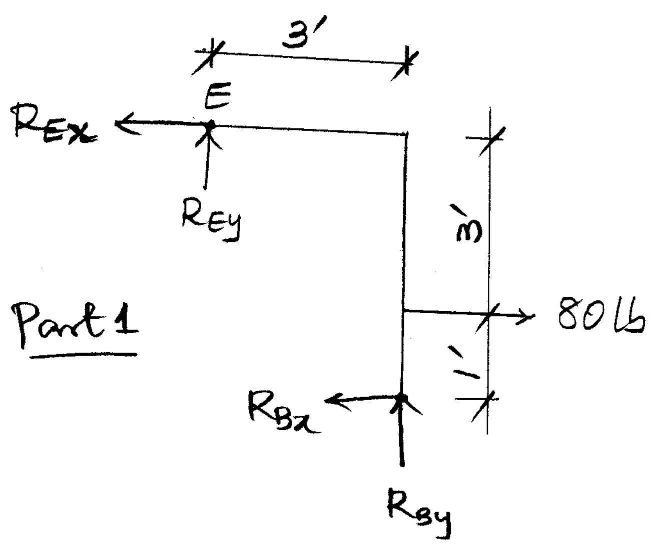
$\therefore$ At A, $\underline{R_{Ax} = R_{Ay} = 300N}$, and at C, $\underline{R_{Cx} = R_{Cy} = 300N}$

4- Determine the horizontal & vertical components of force at pins A, B, & C exerted on member ABC

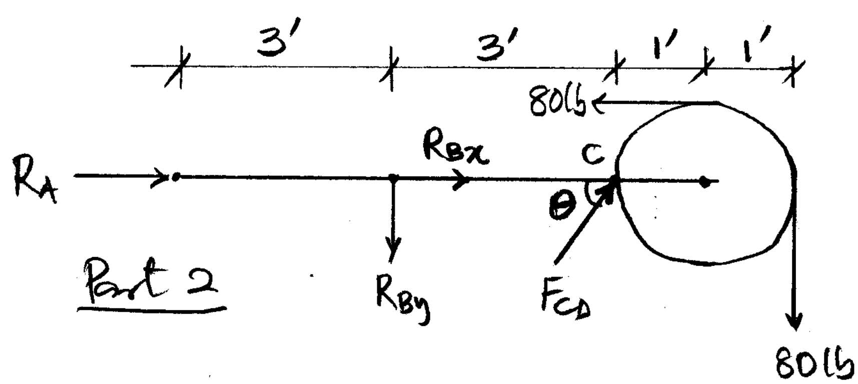


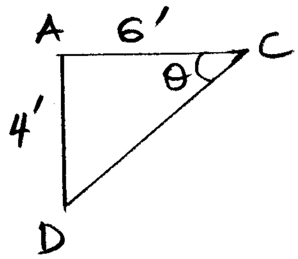
Solution:

The structure can be separated into 2 FBD parts and replacing member CD by a single axial component as:



Member CD runs between 2 hinges and so it cannot transmit moment but can transmit force. So it acts like an inclined axial member running straight from C to D in the frame





$$\cos\theta = \frac{6}{\sqrt{4^2+6^2}} = 0.8321$$

$$\sin\theta = \frac{4}{\sqrt{4^2+6^2}} = 0.5547$$

$$\tan\theta = \frac{0.5547}{0.8321} = \frac{2}{3}$$

Equilibrium of part 2:

$$\Sigma M_c = 3R_{By} + 80(1) - 80(2) = 0$$

$$3R_{By} = 80 \Rightarrow R_{By} = 26.67 \text{ lb}$$

$$\Sigma F_y = -R_{By} + F_{CD}\sin\theta - 80 = 0$$

$$-\frac{80}{3} + 0.5547F_{CD} - 80 = 0$$

$$0.5547F_{CD} = \frac{320}{3} \Rightarrow F_{CD} = 192.30 \text{ lb}$$

$R_{Cx} = F_{CD}\cos\theta = 160 \text{ lb} \rightarrow$; $R_{Cy} = F_{CD}\sin\theta = 106.667 \text{ lb} \uparrow$

$$\Sigma F_x = R_A + R_{Bx} + F_{CD}\cos\theta - 80 = 0 \text{ ------- } (4.1)$$

Equilibrium of part 1:

$$\Sigma M_E = -4R_{Bx} + 3R_{By} + 80(3) = 0$$

$$-4R_{Bx} + 3\left(\frac{80}{3}\right) + 80(3) = 0$$

$$4R_{Bx} = 320 \Rightarrow R_{Bx} = 80 \text{ lb}$$

Then from (4.1),

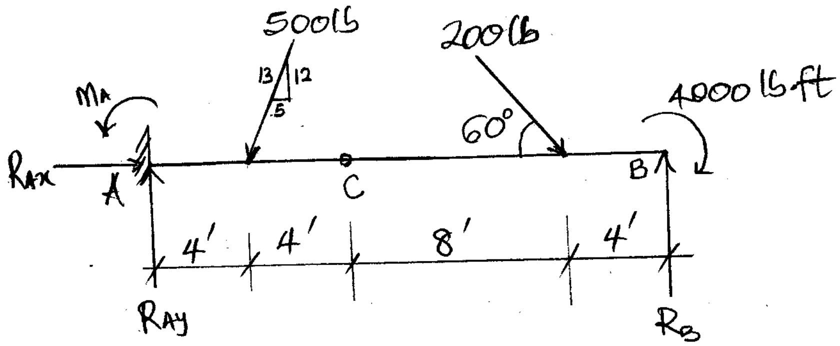
$$R_A + 80 + 192.30(0.8321) - 80 = 0 \Rightarrow R_A = -160 \text{ lb}$$

Hence at A, $R_A = 160 \text{ lb} \leftarrow$

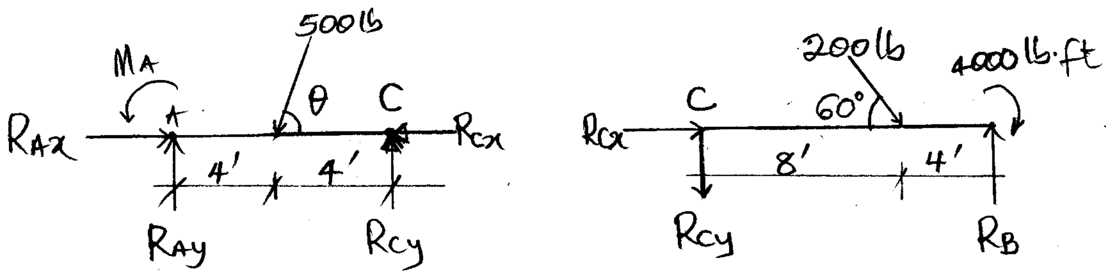
at B, $R_{Bx} = 80 \text{ lb} \rightarrow$, $R_{By} = 26.67 \text{ lb} \downarrow$

at C, $R_{Cx} = 160 \text{ lb} \rightarrow$, $R_{Cy} = 106.667 \text{ lb} \uparrow$

#5 Determine the reactions at the supports



Solution



$\cos\theta = \frac{5}{13}$

$\sin\theta = \frac{12}{13}$

Part 1 F.B.D Part 2

Equilibrium of part 2

$\Sigma M_C = 12R_B - 200(8)\sin 60° - 4000 = 0$

$12R_B = 5385.6406 \Rightarrow \underline{R_B = 448.80 \text{ lb}}$

$\Sigma F_y = -R_{Cy} - 200\sin 60° + 448.80$

$\underline{R_{Cy} = 275.60 \text{ lb}}$

$\Sigma F_x = R_{Cx} + 200\cos 60° = 0 \Rightarrow R_{Cx} = -100 \text{ lb}$

Equilibrium of part 1:

$\Sigma M_A = M_A - 500(\frac{12}{13})(4) + 275.60(8) = 0 \Rightarrow \underline{M_A = -358.65 \text{ lb·ft}}$

$\Sigma F_y = R_{Ay} - 500(\frac{12}{13}) + 275.60 = 0 \Rightarrow \underline{R_{Ay} = 185.94 \text{ lb}}$

$\Sigma F_x = R_{Ax} - 500(\frac{5}{13}) + 100 = 0 \Rightarrow \underline{R_{Ax} = 92.31 \text{ lb}}$