Problem # 1:

Given:

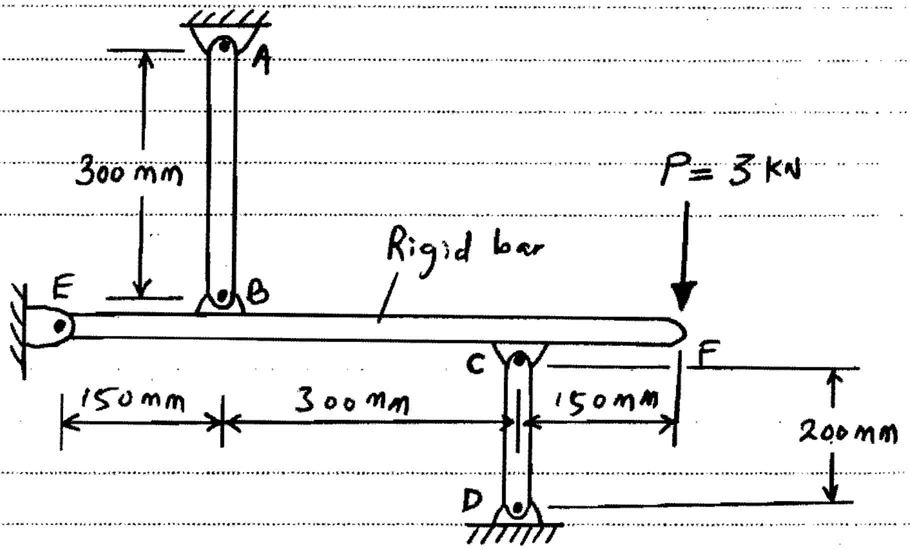
The figure shown

AB : $A = 80\ mm^2$

$E = 70\ GPa$

CD : $A = 30\ mm^2$

$E = 210\ GPa$



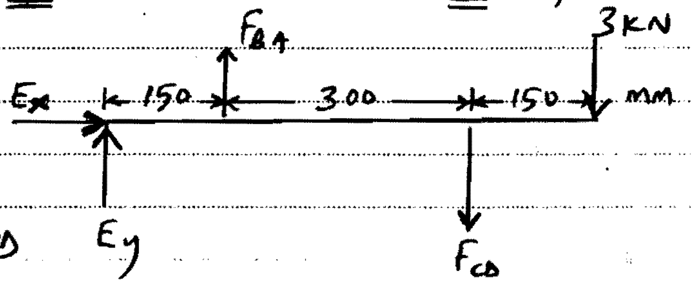
Required:

The stresses in AB and CD

Solution:

To find σ_{AB} and σ_{CD}, we need to find the forces in these members.

In the FBD shown, there are 4 unknowns and only 3 equilibrium equations. Thus the problem is statically indeterminate.



Note that AB and CD are two-force members (why?!)

Also note that both BA and CD are assumed "T" (why?!)

$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow E_x = 0$ "not needed!"

① Equilibrium

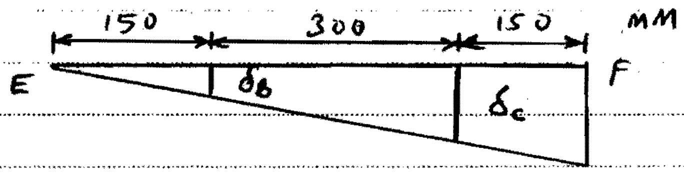
$+\circlearrowleft \sum M_E = 0$ (Why point E ?!)

$\Rightarrow 150 (F_{BA}) - 450 (F_{CD}) - 600 (3) = 0 \Rightarrow$

$F_{BA} - 3 F_{CD} - 12 = 0$ ①

② Geometric Compatibility:

From similar triangles,



$\dfrac{\delta_C}{450} = \dfrac{\delta_B}{150} \Rightarrow$

$\delta_C = 3 \delta_B$ ②

$$\delta_B = \ell_{BA} = \left(\frac{PL}{AE}\right)_{BA} = \left[\frac{F_{BA}(0.3)}{80(10)^{-6}(70)(10)^9}\right]$$

$$= 5.35714(10)^{-8} F_{BA} \quad (3)$$

$$\delta_C = -\ell_{CD} = -\left(\frac{PL}{AE}\right)_{CD} = -\left[\frac{F_{CD}(0.2)}{30(10)^{-6}(210)(10)^9}\right]$$

$$= -3.17460(10)^{-8} F_{CD} \quad (4)$$

Note that (–) sign is used as CD is shortened.

From (3) and (4) into (2)

$$-3.1746 F_{CD} = 3(5.35714) F_{AB} \Rightarrow$$

$$F_{CD} = -5.0625 F_{AB} \quad (5)$$

from (5) into (1), $\Rightarrow F_{BA} = 0.74131 \text{ KN}$ "T"

$\Rightarrow F_{CD} = -3.7529 \text{ KN} \Rightarrow F_{CD} = 3.7529 \text{ KN}$ "C"

Thus, $\sigma_{AB} = \dfrac{F_{BA}}{A_{BA}} = \dfrac{0.74131(10)^3}{80(10)^{-6}}$

$$\Rightarrow \boxed{\sigma_{AB} = 9.266 \text{ MPa} \quad \text{"T"}}$$

$$\sigma_{CD} = \frac{-3.7529(10)^3}{30(10)^{-6}}$$

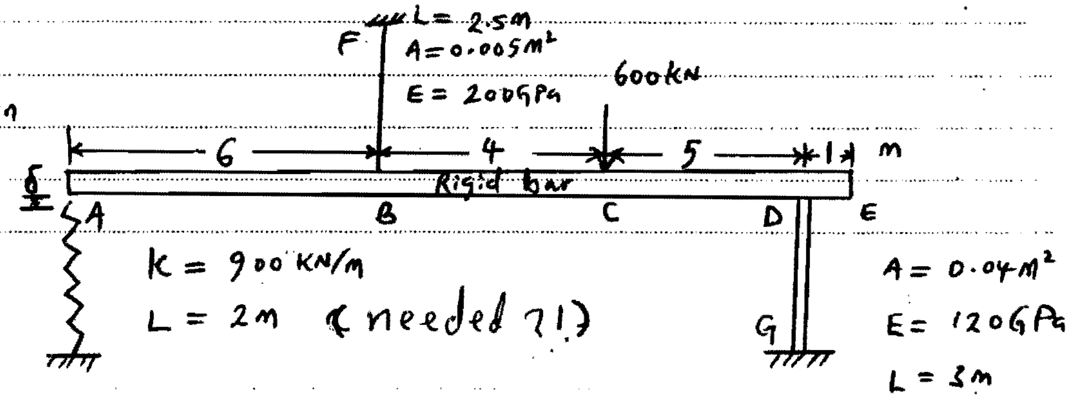
$$\Rightarrow \boxed{\sigma_{CD} = 125.1 \text{ MPa} \quad \text{"C"}}$$

Solution of HW # 5

Problem # 2:

Given:

The figure shown



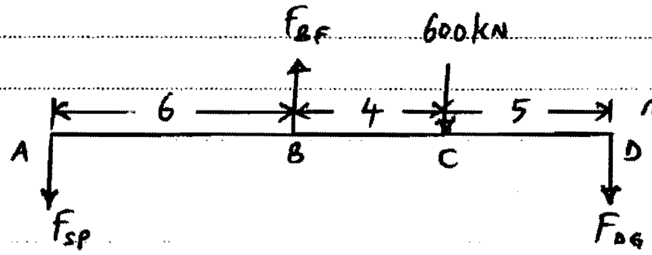
Required:

σ_{BF} , σ_{DG} , F_{spring}

where a) $\delta = 0$

b) $\delta = 0.5$ mm

c) $\delta = 1.5$ mm



Solution:

a) $\delta = 0$

From the FBD shown, there are 3 unknowns and 2 "usable" equilibrium equations.

* Note that all are assumed "T" (Why?!) You may assume some "C"

Thus the problem is Statically indeterminate (SI). Therefore, we need to use the geometric compatibility equation.

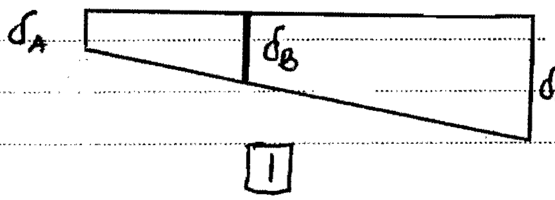
① Equilibrium

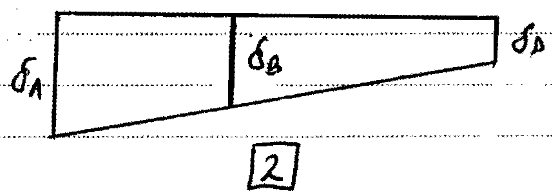
$+\uparrow \Sigma F_y = 0 \Rightarrow F_{BF} - F_{sp} - F_{DG} - 600(10)^3 = 0$ ①

$+\circlearrowleft \Sigma M_A = 0 \Rightarrow 6F_{BF} - 15F_{DG} - 6(10)^6 = 0$ ②

② Geom. Compat.

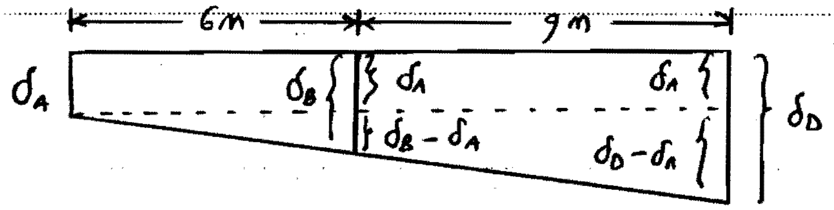
Depending on the values of the elongations in the different members, the deformation of the rigid bar may be as shown in 1 or 2





The values we got for δ's will "tell" us which one is the "actual".

Let's assume $\boxed{1}$ $\Rightarrow$



$$\frac{\delta_B - \delta_A}{6} = \frac{\delta_D - \delta_A}{15} \Rightarrow$$

$$2.5\,\delta_B - \delta_D - 1.5\,\delta_A = 0 \qquad (3)$$

In case deformation $\boxed{2}$ is assumed, we get

$$\frac{\delta_B - \delta_D}{9} = \frac{\delta_A - \delta_D}{15} \Rightarrow$$

$$\tfrac{5}{3}(\delta_B - \delta_D) = \delta_A - \delta_D$$

$$5\delta_B - 5\delta_D = 3\delta_A - 3\delta_D \Rightarrow$$

$$5\delta_B - 2\delta_D - 3\delta_A = 0$$

Dividing by 2 $\Rightarrow$

$$2.5\,\delta_B - \delta_D - 1.5\,\delta_A = 0$$

which is the same as (3) !

(3) Material behaviour

$$\delta_A = -e_{sp} = -\frac{F_{sp}}{k} \qquad (4) \quad \text{"got shorter" (from geometry)}$$

$$\delta_B = e_{BF} = \left(\frac{FL}{AE}\right)_{BF} \qquad (5) \quad \text{"got longer"}$$

$$\delta_D = -e_{DG} = -\left(\frac{FL}{AE}\right)_{DG} \qquad (6) \quad \text{"got shorter"}$$

Be careful about $(-)$ sign if the member is shortened !!!

from equations (4) – (6) into equation (3),

$$2.5\left[\frac{2.5\,F_{BF}}{0.005(200)(10)^9}\right] - \left[\frac{-3\,F_{DG}}{0.04(120)(10)^9}\right] - 1.5\left[\frac{-F_{sp}}{900(10)^3}\right] = 0$$

$6.25(10)^{-9} F_{BA} + 6.25(10)^{-10} F_{DG} + \frac{5}{3}(10)^{-6} F_{sp} = 0$ (7)

Solving eqs. (1), (2) and (7) yields

$F_{BF} = 331.43$ KN "T"

$F_{DG} = -267.43$ KN $= 267.43$ KN "C"

$F_{sp} = -1.143$ KN $\Rightarrow$ $\boxed{F_{sp} = 1.143 \text{ KN "C"}}$

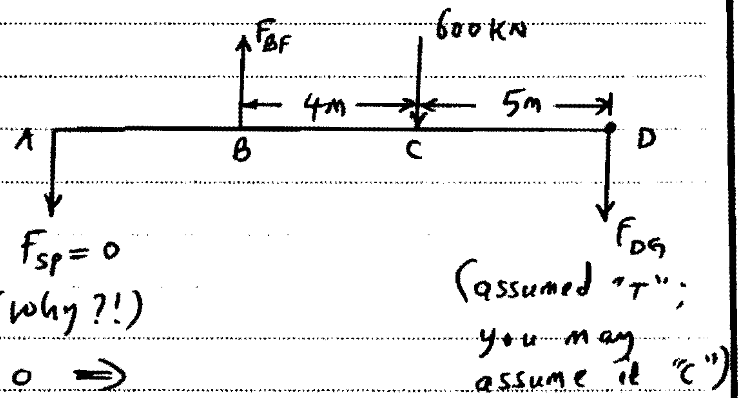
$\sigma = \frac{F}{A} \Rightarrow \sigma_{BF} = \frac{331.43(10)^3}{0.005} \Rightarrow \boxed{\sigma_{BF} = 66.29 \text{ MPa "T"}}$

$\sigma_{DG} = \frac{-267.43(10)^3}{0.04} \Rightarrow \boxed{\sigma_{DG} = 6.686 \text{ MPa "C"}}$

b- $\delta = 0.5$ mm

First, we need to check whether the gap (δ) closes or not. Thus, we ignore the Spring first (i.e, the gap does <u>not</u> close) and then calculate F_{BF} and F_{DG}. (Why ?!). After that, we calculate δ_1 and compare it with δ_{gap}. $\Rightarrow$

In the FBD,



$+\circlearrowleft \sum M_A = 0 \Rightarrow$

$-600(4) - F_{DG}(9) = 0$

$\Rightarrow F_{DG} = -266.667$ kN (C)

$+\uparrow \sum F_y = 0 \Rightarrow$

$F_{BF} - 600 - (-266.667) = 0 \Rightarrow$

$F_{BF} = 333.333$ KN (T)

[Note that the problem is statically determinate]

Now, we need to calculate the elongations $\Rightarrow$

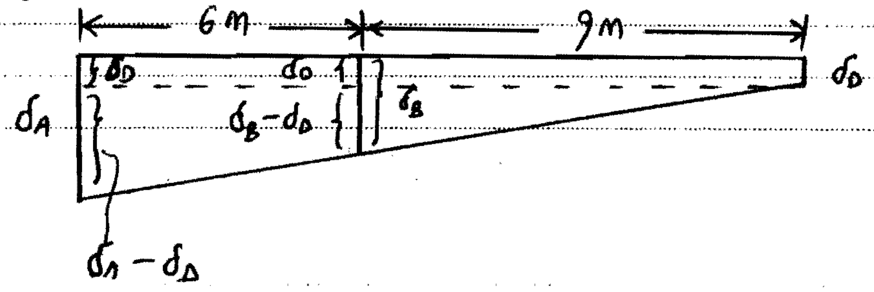
$e_{BF} = \frac{333.333(10)^3 (2.5)}{0.005(200)(10)^9} = 8.33333(10)^{-4}$ m

$\Rightarrow \delta_B = 8.33333(10)^{-4}$ m (down)

$e_{DG} = \frac{-266.667(10)^3(3)}{0.04(120)(10)^9} = -1.66667(10)^{-4}$ m

$\Rightarrow \delta_D = 1.66667(10)^{-4}$ m (down)

The geometry is drawn next.



$$\frac{\delta_B - \delta_D}{9} = \frac{\delta_A - \delta_D}{15} \Rightarrow$$

$$\frac{(8.33333 - 1.66667)(10)^{-4}}{9} = \frac{\delta_A - 1.66667(10)^{-4}}{15}$$

$\Rightarrow \delta_A = 1.2778(10)^{-3}m = 1.278\,mm > 0.5\,mm$

The gap closes, and the problem becomes SI. Thus, procedures similar to that of Part (a) will be followed.

Eqs. 1 to 6 are still valid except eq. 4 which becomes

$$\delta_A = -e_{sp} + \delta_{gap} = \frac{-f_{sp}}{R} + 0.5(10)^{-3} \qquad (4')$$

Thus eq. 7 becomes

$$6.25(10)^{-9} F_{BD} + 6.25(10)^{-10} f_{DG} + \frac{5}{3}(10)^{-6} f_{sp} - 0.75(10)^{-3} = 0 \qquad (7')$$

Then solving eqs. 1, 2, and 7' yields

$f_{BF} = 332.17\ kN$ "T"

$f_{DG} = -267.13\ kN = 267.13\ kN$ "C"

$f_{sp} = -0.6955\ kN \Rightarrow \boxed{F_{sp} = 0.6955\ kN \text{ "C"}}$

<u>As expected</u>, $|F_{sp}|$ decreased and the other two increased. The values of the forces are between $f_{no\ gap}$ and $f_{gap\ not\ closed}$ (Reasonable ?!)

$\Rightarrow \sigma_{BF} = \frac{332.17(10)^3}{0.005} \Rightarrow \boxed{\sigma_{BF} = 66.43\ MPa \text{ "T"}}$

$$\sigma_{DG} = \frac{-267.13\,(10)^3}{0.04} \Rightarrow \boxed{\sigma_{DG} = 6.678\ MPa\ \text{"C"}}$$

C- $\delta = 1.5\ mm$

Since $\delta_A = 1.278\ mm < gap = 1.5\ mm$ [from part (b)]

The gap does not close. $\Rightarrow$

The assumption at the beginning of Part (b) is valid $\Rightarrow$

$\boxed{F_{sp} = 0}$

$F_{BF} = 333.333\ KN \Rightarrow \boxed{\sigma_{BF} = 66.67\ MPa\ \text{"T"}}$

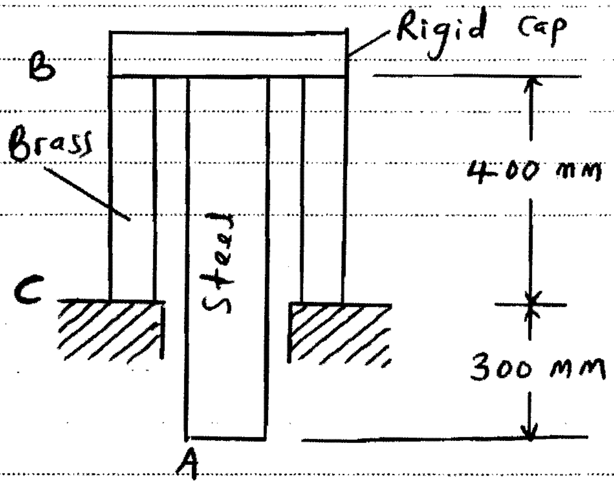
$F_{DG} = -266.667\ KN \Rightarrow \boxed{\sigma_{DG} = 6.667\ MPa\ \text{"C"}}$

Reasonable answers ?!

Problem #3:

Given:

The figure shown



$D_{st} = 12\,mm$

$D_{out\ br} = 30\,mm$

$D_{in\ br} = 20\,mm$

$\Delta T_{st} = +50°c$

$E_{st} = 210\,GPa$

$\alpha_{st} = 12(10)^{-6}/°c$

$\Delta T_{br} = -40°c$

$E_{br} = 105\,GPa$

$\alpha_{br} = 18(10)^{-6}/°c$

Required:

a- The displacement of Point A

b- The stresses in the steel and brass

Solution:

a- $\delta^T = \alpha \Delta T L$

Steel expanded by $\delta_{st} = (12)(10)^{-6}(50)(700)$

$= 0.42\,mm$

brass contracted by $\delta_{br} = (18)(10)^{-6}(40)(400)$

$= 0.288\,mm$

The steel expansion moved point A down, and the brass contraction moved it down also! Thus,

$\delta_A = 0.42 + 0.288 \Rightarrow \boxed{\delta_A = 0.708\,mm \text{ down}}$

b- Since the problem is statically determinate, the materials can expand freely, and thus are stress-free. (why?!)

$\Rightarrow \boxed{\sigma_{st} = \sigma_{br} = 0}$

Solution of HW #5

Problem #4:

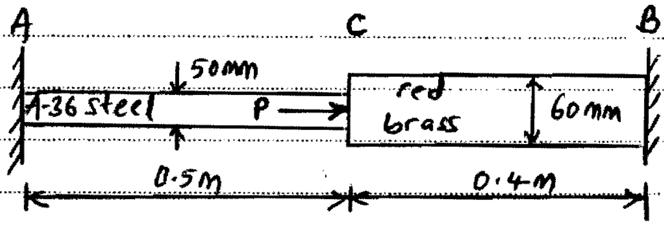
Given:

The figure shown

$T_1 = 80°c$

$T_2 = 20°c$

$P = 200\,KN$



Required:

The reactions at A and B by

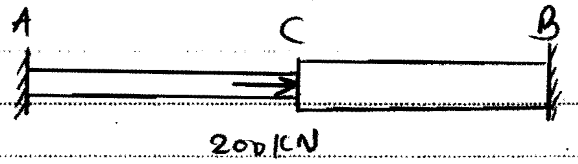
a- applying load, then temperature

b- applying temperature, then load

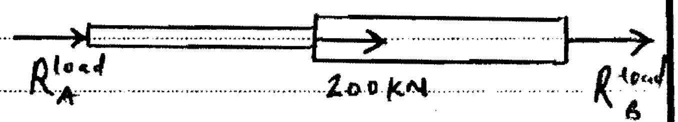
c- applying load and temperature simultaneously

Solution:

a- First apply P:



The problem is statically indeterminate as there are two reactions and only one equilibrium e.g. $(\Sigma F_x = 0)$



In the FBD,

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$

$R_A^{load} + R_B^{load} + 200(10)^3 = 0$ (1)

Note that both reactions are assumed positive (i.e →)

(why?!)

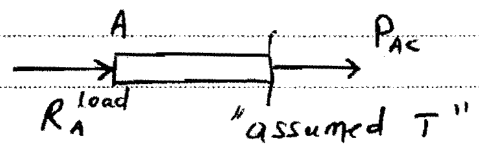
You may assume "something **else**" (e.g., both to the left as they are expected to be).

Geometric Compatibility:

Σ elongations $= 0 \Rightarrow$

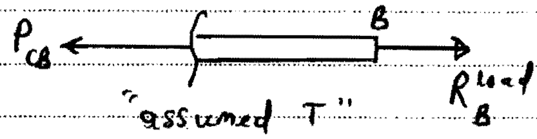
$\delta_{B/A} = 0 = e_{AC} + e_{CB} \Rightarrow$

$P_{AC} = -R_A^{load}$



Solution of HW #5

$P_{CB} = R_B^{load}$



$$\Rightarrow \left(\frac{PL}{AE}\right)_{AC} + \left(\frac{PL}{AE}\right)_{BC} = 0$$

$$\left[\frac{-R_A^{load}(0.5)}{\pi/4(0.05)^2(200)(10)^9}\right] + \left[\frac{R_B^{load}\ (0.4)}{\pi/4\,(0.06)^2(101)(10)^9}\right] = 0 \Rightarrow$$

$$-\frac{4}{\pi}(10)^{-9} R_A^{load} + \frac{4.40044(10)^{-9}}{\pi} R_B^{load} = 0 \qquad (2)$$

From (2), $R_A^{load} = 1.10011\, R_B^{load}$ (3)

From (3) into (1),

$2.10011 R_B^{load} + 200(10)^3 = 0 \Rightarrow$

$R_B^{load} = -95.233\ KN$ — $\underline{\underline{95.233\ KN}} \leftarrow$

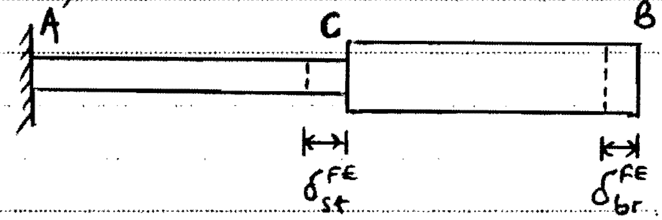
$\Rightarrow \underline{\underline{R_A^{load} = 104.767\ KN \leftarrow}}$ expected values ?!

Next, apply Temp.:

$\delta^{Free\ expan.} = \alpha \Delta T L$

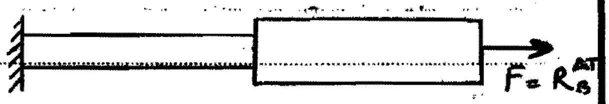
$\delta_{st}^{FE} = 12(10)^{-6}(20-80)(0.5) = -3.6(10)^{-4}\ m$

$\delta_{br}^{FE} = 18(10)^{-6}(20-80)(0.4) = -4.32(10)^{-4}\ m$



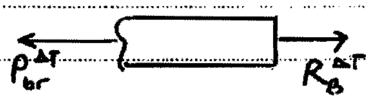
We need to prevent δ_{st}^{FE} and δ_{br}^{FE} by applying $F = R_B^{\Delta T}$.

Note that it is "T" in order to "prevent the contraction". $\Rightarrow$

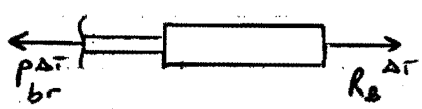


$e^{R_B^{\Delta T}} = \sum\left(\frac{PL}{AE}\right) \Rightarrow$

$P_{br}^{\Delta T} = R_B^{\Delta T}$



$P_{st}^{\Delta T} = R_B^{\Delta T}$



$$= \left[\frac{R_B^{\Delta T}(0.5)}{\pi/4(0.05)^2(200)(10)^9} + \frac{R_B^{\Delta T}\ (0.4)}{\pi/4(0.06)^2(101)(10)^9}\right]$$

$= 1.27324(10)^{-9} R_B^{\Delta T} + 1.40070(10)^{-9} R_B^{\Delta T}$

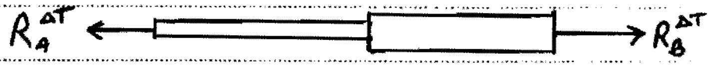
$= 2.67394(10)^{-9} R_B^{\Delta T}$

$e^{R_B^{\Delta T}} + \delta^{FE} = 0 \Rightarrow$

$2.67394(10)^{-9} R_B^{\Delta T} - 3.6(10)^{-4} - 4.32(10)^{-4} = 0 \Rightarrow$

$\underline{\underline{R_B^{\Delta T} = 296.19\ KN}} \rightarrow$

From the FBD,



$R_B^{\Delta T} - R_A^{\Delta T} = 0$ (no load applied) $\Rightarrow$

$\underline{\underline{R_A^{\Delta T} = 296.19\,kN \leftarrow}}$

$(\overset{+}{\rightarrow})\ R_A^{Total} = R_A^{load} + R_A^{\Delta T} = -104.767 - 296.19 \Rightarrow$

$\boxed{R_A^{Total} = 401.0\,kN \leftarrow}$

$(\overset{+}{\rightarrow})\ R_B^{Total} = -95.233 + 296.19 \Rightarrow \boxed{R_B^{Total} = 201.0\,kN \rightarrow}$

b- First apply Temp:

In this particular problem, when the temperature is applied first, there will be no difference in the procedure/solution steps followed in part (a). Thus the answers will be exactly the same!

Thus, $R_A^{\Delta T} = 296.19\,kN \leftarrow$

$R_B^{\Delta T} = 296.19\,kN \rightarrow$

Now, when the load is applied, the exact same procedure followed in part (a) will be followed here; thus, there is no difference when the load is applied after the temperature.

Therefore: $R_A^{load} = 104.767\,kN \longleftarrow$

$R_B^{load} = 95.233\,kN \longleftarrow$

Thus, $R_A^{Total} = -296.19 - 104.767 \Rightarrow \boxed{R_A^{Total} = 401.0\,kN \leftarrow}$

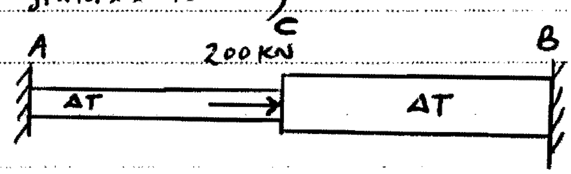
$R_B^{Total} = 296.19 - 95.233 \rightarrow \boxed{R_B^{Total} = 201.0\,kN \rightarrow}$

c- Apply load and temperature simultaneously:

① Equilibrium:

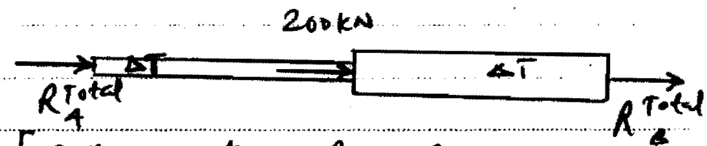
$\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow$

$R_A^{Total} + R_B^{Total} + 200(10)^3 = 0$ ①



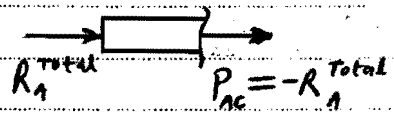
② Geom. Compat:

$\Sigma \delta_{AB} = 0 \Rightarrow$

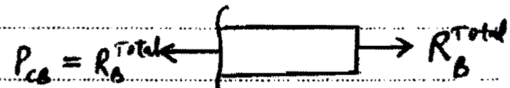


[Both reactions R_A & R_B are assumed in $+x \rightarrow$]

$$\left(\ell_{AC}^{load} + \delta_{AC}^{Free\ expan.}\right) + \left(\ell_{CB}^{load} + \delta_{CB}^{Free\ expan.}\right) = 0$$



$$\left[\frac{-R_A^{Total}(0.5)}{\pi/4\,(0.05)^2(200)(10)^9} + 12(10)^{-6}(20-80)(0.5)\right]$$



$$+\left[\frac{R_B^{Total}(0.4)}{\pi/4\,(0.06)^2(101)(10)^9} + 18(10)^{-6}(20-80)(0.4)\right] = 0$$

[Both internal forces P_{AC} & P_{CB} are assumed + (T)]

$$\Rightarrow -1.27324(10)^{-9} R_A^{Total} - 3.6(10)^{-4} + 1.4007(10)^{-9} R_B^{Total} - 4.32(10)^{-4} = 0$$

$$\Rightarrow \quad 1.4007\, R_B^{Total} - 1.27324\, R_A^{Total} - 7.92(10)^5 = 0 \qquad (2)$$

From eq. (1)

$$R_B^{Total} = -R_A^{Total} - 200(10)^3 \qquad (3)$$

from eq. (3) into eq. (2),

$$R_A^{Total} = -401.1\,kN \Rightarrow \boxed{R_A^{Total} = 401.0\,kN \leftarrow}$$

$$\Rightarrow \text{into (3)}, \quad \boxed{R_B^{Total} = 201.0\,kN \rightarrow}$$

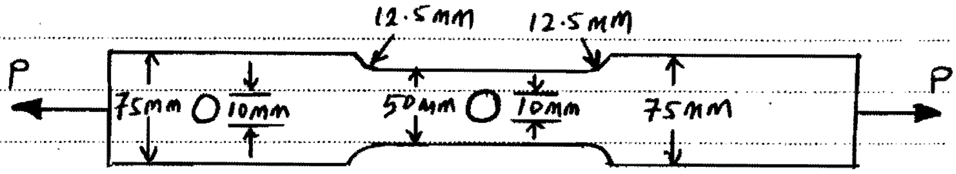
The answers are the same in all methods (as expected!).
In this particular case, Method (c) seems to be better as it is shorter. However, Methods (a) and (b) may be clearer (more clear). This problem could have been interesting (or more interesting) if there had been a gap "somewhere". Due to that gap, we would have noticed real differences among the three methods! Try it yourself!!

Solution of HW #

Problem #5:

Given:

The figure shown



$t = 10mm;$

$\sigma_{fail} = 150MPa;$

factor of safety $= 1.5$

Required:

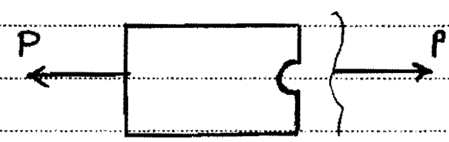
$P_{max\ allow}$

Solution:

There are three locations for possible σ_{max} where there are stress concentrations: the left hole, the right hole, and the fillet.

1) Left hole: (Do you think this will "control"?!)

$\sigma_{max} = k\,\sigma_{average}$



$$\sigma_{average} = \frac{P}{10(75-10)(10)^{-6}} = 1538.46P$$

for K, $\frac{D}{W} = \frac{10}{75} = 0.1333$

Note $\frac{D}{W}$ <u>not</u> $\frac{r}{W}$ as in your textbook that is wrong.

$K \approx 2.57 \Rightarrow$

$\sigma_{max}^{①} = 2.57(1538.46P) = \underline{\underline{3953.8P}}$

2) Right hole

$$\sigma_{avea} = \frac{P}{10(50-10)(10)^{-6}} = 2500P$$

for K, $\frac{D}{W} = \frac{10}{50} = 0.2 \Rightarrow K \approx 2.45$

$\Rightarrow \sigma_{max}^{②} = 2.45(2500P) = \underline{\underline{6125P}}$

3) Fillet

$$\sigma_{allow} = \frac{P}{10(50)(10)^{-6}} = 2000P$$

For K_t $\left\{ \begin{array}{l} r/h = \frac{12.5}{50} = 0.25 \\ w/h = \frac{75}{50} = 1.5 \end{array} \right\} \Rightarrow K \approx 1.62$

$$\Rightarrow \sigma^{③}_{max} = 1.62(2000P) = \underline{\underline{3240P}}$$

From $\sigma^{①}_{max}$, $\sigma^{②}_{max}$, $\sigma^{③}_{max}$, we choose the largest for σ_{max}. (Why not the minimum?!) $\Rightarrow$

$$\sigma_{max} = \sigma^{②}_{max} = 6125P_{max}$$

$$\sigma_{allow} = \frac{\sigma_{fail}}{F.S.} = \frac{150}{1.5} = 100\,MPa$$

$\Rightarrow$ Set $\sigma_{max} = \sigma_{allow} \Rightarrow$

$$6125P_{max} = 100(10)^6 \Rightarrow$$

$$\boxed{P_{max} = 16.33\ kN}$$