Problem #1:

Given:
The figure shown
AB: A = 80 mm²
E = 70 GPa
CD: A = 30 mm²
E = 210 GPa

Required:
The stresses in AB and CD

Solution:
To find \( \sigma_B \) and \( \sigma_{CD} \), we need to find the forces in these members.

In the FBD shown, there are 4 unknowns and only 3 equilibrium equations. Thus the problem is statically indeterminate.

Note that AB and CD are two-force members (why?!) and that both EA and ED are assumed "T" (why?!) and

\[ \sum F_x = 0 \Rightarrow E_x = 0 \] "not needed!"

1. Equilibrium

\[ \sum M_E = 0 \quad \text{(Why point E?)} \] \[ 150 (F_{EA}) - 450 (F_{CD}) - 600 (3) = 0 \] \[ F_{EA} - 3 F_{CD} - 12 = 0 \] 0

2. Geometric Compatibility:

From similar triangles,

\[ \frac{\Delta C}{450} = \frac{\Delta E}{150} \Rightarrow \] \[ \Delta C = 3 \Delta E \] 2
Solution of HW # 5

\[ \delta_b = \frac{F_{ba}}{4E_{ba}} = \frac{F_{ba} (0.2)}{80(10)^{-6}(70)(10)^2} = 5.35714 (10)^{-1} F_{ba} \]  \hspace{1cm} (3)

\[ \delta_c = -\frac{F_{cd}}{4E_{cd}} = -\frac{F_{cd} (0.2)}{30(10)^{-6}(210)(10)^9} = -3.7529 (10)^{-8} F_{cd} \]  \hspace{1cm} (4)

Note that \((-\)\) sign is used as \(CD\) is shortened.

From (3) and (4) into (2)

\[ -3.7529 F_{cd} = 3(5.35714) F_{ba} \]  \hspace{1cm} (5)

From (5) into (0), \[ F_{ba} = 0.74131 \text{ KN} \] "T"

\[ F_{cd} = -3.7529 \text{ KN} \] "C"

Thus, \[ \sigma_{ba} = \frac{F_{ba}}{A_{ba}} = \frac{0.74131 (10)^3}{80 (10)^{-6}} \] "T"

\[ \sigma_{cd} = -3.7529 (10)^3 \]

\[ \sigma_{cd} = 125.1 \text{ MPa} \] "C"
Solution of HW #5

Problem #2:

Given:
The figure shown

\[ F = 600 \text{ kN} \]
\[ A = 0.05 \text{ m}^2 \]
\[ E = 200 \text{ GPa} \]
\[ L = 2 \text{ m} \]

Required:

\[ \sigma_{AF}, \sigma_{BG}, \sigma_{spring} \]

\[ a) \delta = 0 \]
\[ b) \delta = 0.5 \text{ mm} \]
\[ c) \delta = 1.5 \text{ mm} \]

Solution:

\[ a) \delta = 0 \]

Note that all are assumed to be "T" from the FBD shown, there are 3 unknowns and 2 "useable" equilibrium equations. Thus the problem is statically indeterminate (SIE). Therefore, we need to use the geometric compatibility equation.

1) Equilibrium

\[ + \sum F_y = 0 \Rightarrow F_{AF} - F_{SP} - F_{BG} = 600 \text{ kN} \]
\[ - \sum M_A = 0 \Rightarrow 6F_{SP} - 15F_{BG} - 6(10) = 0 \]

2) Geom. Compat.

Depending on the values of the elongations in the different members, the deformation of the rigid bar may be as shown in 1 or 2.
The values we get for d's will "tell" us which one is the "actual".

Let's assume $d_4 = d_0$

$$\frac{d_b - d_4}{d_0 - d_4} \Rightarrow 2.5 \frac{d_b - d_0}{d_0 - d_4} = 1.5$$

In case deformation $d_4$ is assumed, we get

$$\frac{d_b - d_0}{d_0 - d_4} \Rightarrow 2.5 \frac{d_b - d_0}{d_0 - d_4} = 1.5$$

So, $d_0 = 2d_4 - 1.5d_b$

Dividing by 2

$$2.5 \frac{d_b - d_0}{d_0 - d_4} = 1.5$$

Which is the same as (3)!

2. Material behaviour

$$\delta_4 = -\varepsilon_{sp} = -\frac{F_{sp}}{R}$$

$$\delta_8 = -\varepsilon_{ef} = \frac{FL}{AE}$$

$$\delta_0 = -\varepsilon_{o6} = -\frac{FL}{4e}$$

Be careful about sign of the member is shortened !!!

From equations (4) - (6) into equation (3),

$$2.5 \left[ \frac{2.5 F_{BF}}{0.005(200)(10)^2} \right] - \left[ -3 F_{o9} \right] - 1.5 \left[ \frac{F_{sp}}{900(10)^2} \right] = 0$$
Solution of HW #5

1. \(6.25 \times 10^{-9} F_{BA} + 6.25 \times 10^{-6} F_{BA} + \varepsilon_A(10)^{-6} F_{SP} = 0 \)  

Solving Eqs. 1, 2, and 4 yields:

- \( F_{BF} = 331.43 \text{ KN} \) \( T \)
- \( F_{DG} = -267.43 \text{ KN} = 267.43 \text{ KN} \) \( C \)
- \( F_{SP} = -1.142 \text{ KN} \Rightarrow F_{SP} = 1.142 \text{ KN} \) \( C \)

\[ \sigma = \frac{F}{A} \Rightarrow \sigma_{BF} = \frac{331.43 (10)^3}{0.005} \Rightarrow \sigma_{BF} = 66.29 \text{ MPa} \) \( T \)

\[ \sigma_{DG} = \frac{-267.43(10)^3}{0.004} \Rightarrow \sigma_{DG} = 6.686 \text{ MPa} \) \( C \)

2. \( b = 0.5 \text{ mm} \)

First, we need to check whether the gap (d) closes or not. Thus, we ignore the spring first (i.e., the gap does not close) and then calculate \( F_{BF} \) and \( F_{DG} \) (Why?). After that, we calculate \( \sigma_d \) and compare it with \( \sigma_{gap} \).

In the f.B.D.,

- \( \sum M_A = 0 \)
  \[ F_{BF} \times 4 - F_{DG} \times 9 = 0 \]
  \[ F_{DG} = -266.667 \text{ KN} \) \( C \)
  \[ F_{SP} = 0 \) (assumed \( T \);

- \( \sum F_y = 0 \)
  \[ F_{BF} - 600 - (-266.667) = 0 \Rightarrow F_{BF} = 333.333 \text{ KN} \) \( T \)

[Note: the problem is statically determinate]

Now, we need to calculate the elongation:

\[ \varepsilon_{BF} = \frac{333.333(10)^2}{0.005 \times 200 \times (10)^2} = 8.3333 \times (10)^{-6} \text{ m} \]

\[ \Rightarrow \delta_B = 8.3333(10)^{-6} \text{ m (down)} \]

\[ \varepsilon_{DG} = -266.667(10)^2{(3)} = -1.66667(10)^{-6} \text{ m} \]

\[ \Rightarrow \delta_D = 1.66667(10)^{-6} \text{ m (down)} \]
The geometry is drawn next.

\[ \delta A - \delta D = \frac{9}{15} (8.3333 - 1.6667)(10)^{-4} = \frac{\delta A - 1.6667(10)^{-4}}{15} \]

\[ \Rightarrow \delta A = 1.2778(10)^{-2} \text{m} = 1.278 \text{mm} > 0.5 \text{mm} \]

The gap closes, and the problem becomes 5a. Thus, procedures similar to that of Part (a) will be followed.

Eqs. 0 to 6 are still valid except Eq. 4 which becomes

\[ \delta A = -R_{sp} + \delta_{gap} = \frac{-F_{sp} + 0.5(10)^{-5}}{R} \]

Thus Eq. 7 becomes

\[ 6.25(10)^{-3}F_{40} + 6.25(10)^{-3}F_{eq} + \frac{5}{3}(10)^{-5}F_{sp} - 0.75(10)^{-2} = 0 \]

Then solving Eqs. 0, 02, and 07 yields

\[ F_{BF} = 332.13 \text{ kN} \text{ "T"} \]

\[ F_{eq} = -267.13 \text{ kN} = 267.13 \text{ kN } \text{ "C"} \]

\[ F_{sp} = -0.6955 \text{ kN} \Rightarrow F_{sp} = 0.6955 \text{ kN } \text{ "E"} \]

As expected, \( |F_{sp}| \) decreased and the other two increased.

The values of the forces are between \( F_{40} \), \( \delta_{gap} \), and \( F_{sp} \) not closed.

(Reasonable?!) \[ \Rightarrow \delta_{BF} = \frac{332.13(10)^2}{0.005} \Rightarrow \delta_{BF} = 66.43 \text{ MPa } \text{ "T"} \]
\[
\sigma_{Dq} = \frac{-269.13 \times (10)^2}{0.04} = 6.67 \text{ MPa} \quad \text{"C"}
\]

\[
\sigma_{Dq} = 6.67 \text{ MPa} \quad \text{"C"}
\]

\[
\sigma_{Dq} = 6.67 \text{ MPa} \quad \text{"C"}
\]

\[
F_{sp} = 0
\]

\[
F_{af} = 333.33 \text{ kN} \quad \Rightarrow \quad \sigma_{af} = 66.67 \text{ MPa} \quad \text{"T"}
\]

\[
F_{df} = -266.67 \text{ kN} \quad \Rightarrow \quad \sigma_{df} = 66.67 \text{ MPa} \quad \text{"C"}
\]

Reasonable answers?!
Problem #3:
Given:
The figure shown
$D_{st} = 12 \text{mm}$
$D_{out \ br} = 30 \text{mm}$
$D_{in \ br} = 20 \text{mm}$
$\Delta T_{st} = +50^\circ \text{C}$
$E_{st} = 210 \text{ GPa}$
$\alpha_{st} = 12(10)^{-6}/\text{C}$
$\Delta T_{br} = -40^\circ \text{C}$
$E_{br} = 105 \text{ GPa}$
$\alpha_{br} = 18(10)^{-6}/\text{C}$

Required:
\begin{enumerate}
\item The displacement of point A
\item The stresses in the steel and brass
\end{enumerate}

Solution:
\begin{enumerate}
\item $\delta_T = \alpha \Delta T L$

Steel expanded by $\delta_{st} = (12)(10)^{-6}(50)(700)$

\begin{align*}
\delta_{st} &= 0.42 \text{ mm} \\
\text{Brass contracted by } & \delta_{br} = (18)(10)^{-6}(40)(400)
\end{align*}

\begin{align*}
\delta_{br} &= 0.288 \text{ mm}
\end{align*}

The steel expansion moved point A down, and the brass contraction moved it down also! Thus,
\begin{align*}
\delta_A &= \delta_{st} + \delta_{br} \Rightarrow \\
\delta_A &= 0.708 \text{ mm down}
\end{align*}

\item Since the problem is statically determinate, the materials can expand freely, and thus are stress-free. (Why?!) \Rightarrow
\begin{align*}
\sigma_{st} &= 0 \text{ br} = 0
\end{align*}
\end{enumerate}
**Problem #4:**

**Given:**
- Figure shown as follows:
  - Section: 1-36 steel
  - Section: brass

- $T_1 = 80^\circ C$
- $T_2 = 20^\circ C$
- $P = 200 \text{ kN}$

**Required:**
- To determine the reactions at A and B by
  - Applying load, then temperature
  - Applying temperature, then load
  - Applying load and temperature simultaneously

**Solution:**

**a.** First apply $P$: 

The problem is statically indeterminate as there are two reactions and only one equilibrium equation ($\Sigma F_x = 0$).

In the FBD,

$$+ \Sigma F_x = 0 \Rightarrow R_{load} + R_{load} + 200 (10) = 0 \quad (1)$$

*Note that both reactions are assumed positive (i.e., $\rightarrow$) for why?*  

You may assume "something else" (e.g., both to the left as they are expected to be).

**Geometric Compatibility:**

$$\Sigma \text{ elongations} = 0 \Rightarrow$$

$$\delta_{BA} = 0 = R_{AC} + \varepsilon_{AB} \Rightarrow$$

$$P_{AC} = -R_{load}$$

**$R_{load}$**  

**"Assumed T"**
\[
\begin{align*}
\text{P}_{\text{cb}} &= R^\text{load}_A \\
\Rightarrow \left( \frac{PL}{4E} \right) + \left( \frac{PL}{4E} \right)_{bc} &= 0 \\
\left[ -\frac{R_A^{\text{load}} (0.5)}{N_4 (0.05)^2 (200) (10)} \right] + \left[ \frac{R_A^{\text{load}} (0.4)}{N_4 (0.06)^2 (101) (10)^2} \right] &= 0 \\
- \frac{4 \pi}{(10)^9} R_A^{\text{load}} + \frac{4 \cdot 400 \cdot 4 (10)^{-9} R_A^{\text{load}}}{\pi} &= 0 \\
\Rightarrow R_A^{\text{load}} &= 104.767 \text{ KN} \quad \text{expected values? !}
\end{align*}
\]

Next, apply Temp.
\[
\delta_{\text{free exp}} = \alpha \Delta T L
\]
\[
\delta_{\text{free}} = 12 (10)^{-4} (200 - 80)(0.5) = -5.6 (10)^{-4} \text{ M}
\]
\[
\delta_{\text{br}} = 18 (10)^{-4} (200 - 80)(0.4) = -4.32 (10)^{-4} \text{ M}
\]

We need to prevent \( \delta_{\text{st}} \) and \( \delta_{\text{br}} \) by applying \( F = R^\Delta T \).

Note that if \( T \) "in" order to "prevent" the contraction, \( \delta^\Delta T = \sum \left( \frac{PL}{4E} \right) \Rightarrow \)
\[
\begin{align*}
\delta^\Delta T &= \left[ \frac{R_A^{\text{load}} (0.5)}{N_4 (0.05)^2 (101) (10)^2} \right] + \left[ \frac{R_A^{\text{load}} (0.4)}{N_4 (0.06)^2 (101) (10)^2} \right] \\
&= 1.27324 (10)^{-9} R_A^{\Delta T} + 1.4007 (10)^{-9} R_A^{\Delta T} \\
&= 2.67394 (10)^{-9} R_A^{\Delta T} \\
\Rightarrow \delta^\Delta T + \delta_{\text{free}} &= 0 \Rightarrow \]
\[
2.67394 (10)^{-9} R_A^{\Delta T} + 2.6 (10)^{-4} - 4.32 (10)^{-4} = 0 \Rightarrow R_A^{\Delta T} = 278.19 \text{ KN}
\]
Solution of HW #5

From the FBD,
\[ R^A_{ac} = R^A_{p1} = 0 \quad \text{(No load applied)} \Rightarrow \]
\[ R^A_{ac} = 296.19 \text{ kN} \]

\[ (\Rightarrow) R^A_{total} = R^A_{load} + R^A_{ac} = -104.367 - 296.19 \Rightarrow \]
\[ R^A_{total} = 401.0 \text{ kN} \]

\[ (\Rightarrow) R^B_{total} = -95.232 + 296.19 \Rightarrow R^B_{total} = 201.0 \text{ kN} \]

b- First apply Temp:
In this particular problem, when the temperature is applied first, there will be no difference in the procedure/solution steps followed in part (a). Thus, the answers will be exactly the same!
Thus, \[ R^A_{ac} = 296.19 \text{ kN} \]
\[ R^B_{ac} = 296.19 \text{ kN} \]

Now, when the load is applied, the exact same procedure followed in part (a) will be followed here; thus, there is no difference when the load is applied after the temperature. Therefore:
\[ R^A_{load} = 104.367 \text{ kN} \]
\[ R^B_{load} = 95.232 \text{ kN} \]

Thus, \[ R^A_{total} = -296.19 - 104.367 \Rightarrow R^A_{total} = 401.0 \text{ kN} \]
\[ R^B_{total} = 296.19 - 95.232 \Rightarrow R^B_{total} = 201.0 \text{ kN} \]

C- Apply load and temperature Simultaneously:

1) Equilibrium:
\[ \Rightarrow \sum F_x = 0 \Rightarrow R^A_{total} + R^B_{total} + 200(10)^3 = 0 \]

2) Geom. Compat.: 
\[ \sum \delta_{ab} = 0 \Rightarrow \]
[Both reactions \( R_A \) & \( R_B \) are assumed \( y + x \rightarrow \)]
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Solution of HW # 5

\[(\delta \text{ Load} + \delta \text{ Free expa.}) + (\delta \text{ Load} + \delta \text{ Free expa.}) = 0 \]

\[ R_{\text{A total}} = -R_{\text{A}} \]

\[ P_{\text{C}} = R_{\text{B total}} \]

\[ R_{\text{B total}} = [R_{\text{A total}} (0.4)] + 18(10^{-6})(20-80)(0.4) = 0 \]  
[Both internal forces \(P_{\text{A}} \times P_{\text{C}}\) are assumed \(+ (\tau)\)]

\[ \Rightarrow -1.27324 (10)^{-6} R_{\text{A total}} = 3.6(10)^{-4} + 1.4007 (10)^{-9} R_{\text{B total}} - 4.22 (10)^{-5} = 0 \]

\[ \Rightarrow 1.4007 R_{\text{B total}} - 1.27324 R_{\text{A total}} - 7.92 (10)^{-5} = 0 \]  \[\square\]

From eq. 1,

\[ R_{\text{B total}} = -R_{\text{A total}} - 200 (10)^{2} \]  \[\square\]

From eq. 3 into eq. 2,

\[ R_{\text{A total}} = -401.1 \text{ KN} \Rightarrow R_{\text{A}} = 401.0 \text{ KN} \]

\[ \Rightarrow \text{ into } 3 \]

\[ R_{\text{B total}} = 201.0 \text{ KN} \]

The answers are the same in all methods (as expected!).

In this particular case, Method (c) seems to be better as it is shorter. However, Methods (a) and (b) may be clearer (more clear).

This problem could have been interesting (or more interesting) if there had been a gap "somewhere". Due to that gap, we would have noticed real differences among the three methods.

Try it yourself!!
Problem #5:

Given:

Re-figured

Shown

\[ t = 10 \text{ mm}; \]
\[ \sigma_{\text{fail}} = 150 \text{ MPa}; \]
\[ \text{factor of safety} = 1.5 \]

Required:

\( P_{\text{max allowable}} \)

Solution:

There are three locations for possible \( \sigma_{\text{max}} \) where there are stress concentrations: the left hole, the right hole, and the fillet.

1) Left hole: (Do you think this will "control"?)

\[
\sigma_{\text{max}} = k \sigma_{\text{average}}
\]

\[
\sigma_{\text{average}} = \frac{P}{10(75 - 10)(10)^{-6}}
\]

\[ = 1538.46 \text{ MPa} \]

For \( K \),

\[ \frac{D}{W} = \frac{10}{75} = 0.1333 \]

Note \( \frac{P}{W} \) not \( \frac{P}{W} \) as in your textbook that is wrong.

\[ K \approx 1.57 \Rightarrow \]

\[ \sigma_{\max} = 2.57 (1538.46 P) = 3953.8 P \]

2) Right hole

\[
\sigma_{\text{average}} = \frac{P}{10(50 - 10)(10)^{-6}} = 2500 P
\]

For \( K \),

\[ \frac{D}{W} = \frac{10}{50} = 0.2 \Rightarrow k \approx 2.45 \]

\[ \Rightarrow \sigma_{\max} = 2.45 (2500 P) = 6125 P \]
3) **Fillet**

\[
\sigma_{\text{allow}} = \frac{P}{10(50)(10)^{-6}} = 2000\, \text{N/mm}^2
\]

For \( K \)

\[
\begin{align*}
\% = \frac{12.5}{50} = 0.25 \\
\% = \frac{75}{50} = 1.5
\end{align*}
\]

\[ K \approx 1.62 \]

\[ \sigma_{\text{max}}^0 = 1.62 (2000\, \text{N/mm}^2) = 3240\, \text{N/mm}^2 \]

From \( \sigma_{\text{max}}^0 \), \( \sigma_{\text{max}}^0 \), \( \sigma_{\text{max}}^0 \), we choose the largest for \( \sigma_{\text{max}} \) (why not the minimum?!) \(
\sigma_{\text{max}} = \sigma_{\text{max}}^0 = 6125\, \text{N/mm}^2
\]

\[ \sigma_{\text{allow}} = \frac{150}{1.5} = 100\, \text{MPa} \]

\[ \Rightarrow \sigma_{\text{max}} = \sigma_{\text{allow}} \Rightarrow \]

\[ 6125\, \text{N/mm}^2 = 100(10)^{b} \Rightarrow \]

\[ P_{\text{max}} = 16.32\, \text{kN} \]