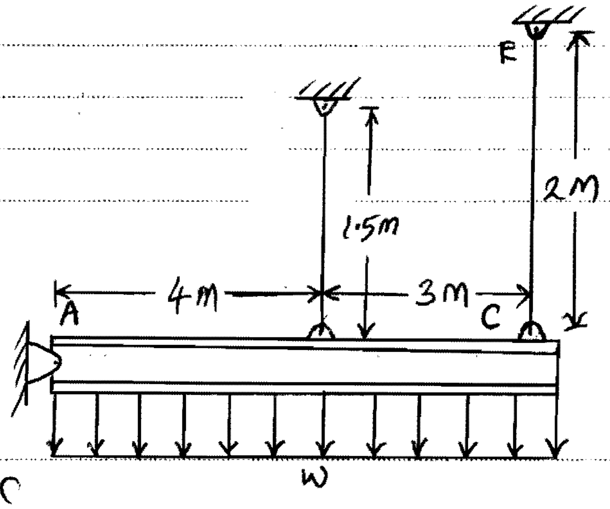
Problem #1

Given:

The figure shown

C displaced by 14 mm

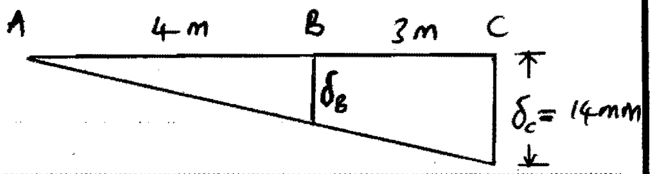


Required:

ε_{CE} and ε_{BD}

Solution:

Note that B and C displaced while A did not as B and C are "free" while A is hinged. Thus, the displacement at ABC is drawn.



From similar triangles,

$$\frac{\delta_B}{\delta_C} = \frac{AB}{AC} \Rightarrow \frac{\delta_B}{14} = \frac{4}{7}$$

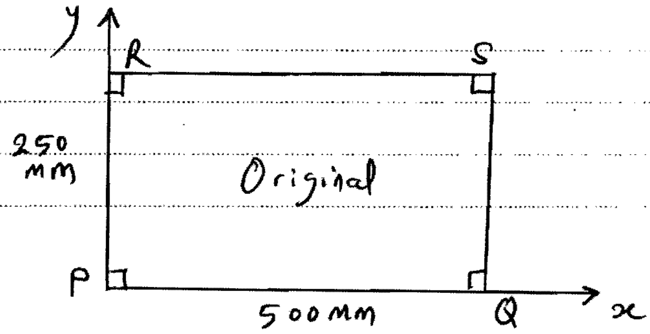
$$\Rightarrow \delta_B = 8 \text{ mm}$$

$$\varepsilon = \frac{\delta L}{L} \Rightarrow \varepsilon_{CE} = \frac{14(10)^{-3}}{2} \Rightarrow \boxed{\varepsilon_{CE} = 0.007 \text{ m/m}}$$

$$\varepsilon_{BD} = \frac{8(10)^{-3}}{1.5} \Rightarrow \boxed{\varepsilon_{BD} = 0.005333 \text{ m/m}}$$

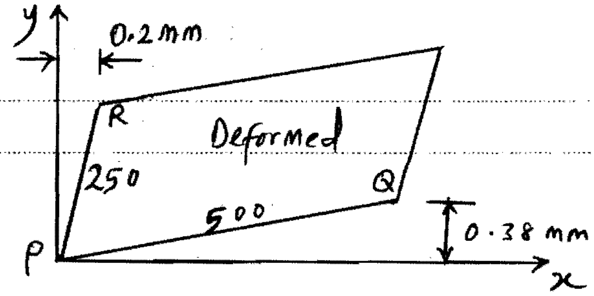
Problem # 2

Given:
The figure shown



Required:
The shear strain at P and Q

Solution:
The shear strain is the change in angle (from the 90° original).
If the angle becomes < 90°, it is a positive shear strain;
If > 90°, it is negative.



(not to scale)

$$\gamma_p = \tan^{-1}\left(\frac{0.38}{500}\right) + \tan^{-1}\left(\frac{0.2}{250}\right)$$

$$\Rightarrow \boxed{\gamma_p = 0.08938^\circ = 0.00156 \text{ rad}}$$

The decrease in the angle at P is the same as the increase in the angle at Q.

Thus, $\boxed{\gamma_q = -0.08938^\circ = -0.00156 \text{ rad}}$

Note that for small angles, $\tan^{-1}\theta \approx \theta$ in radian.
Thus, we could apply it here.

$$\Rightarrow \gamma_p \approx \frac{0.38}{500} + \frac{0.2}{250} = 0.00156 \text{ rad} \quad \text{(as above!)}$$

Problem 3 (I)

Given:

The data from a tensile test.
The diameter of the specimen = 10 mm, and the length = 20 mm.

Required:

Plot the stress-strain diagram.

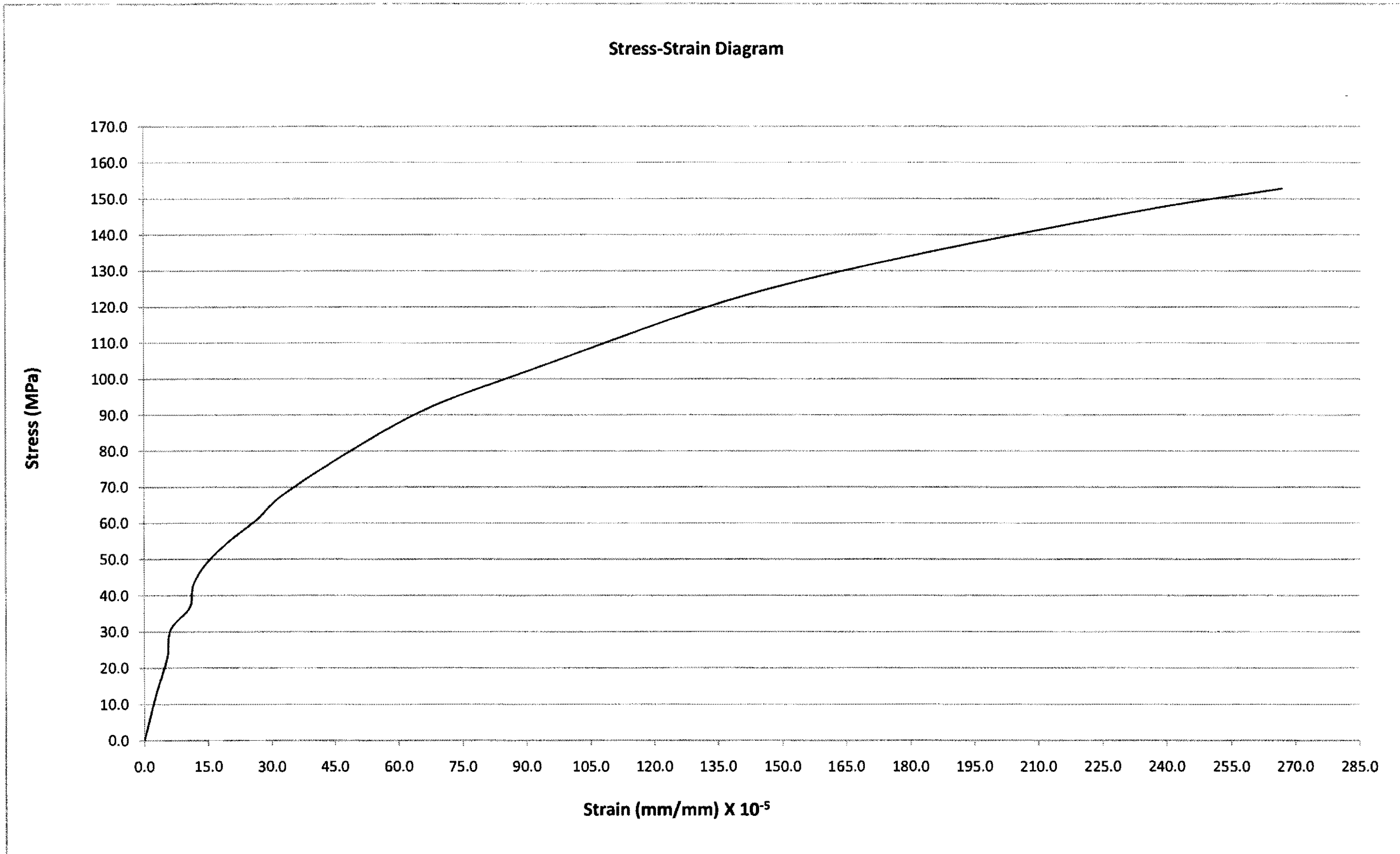
Solution:

We need first to calculate the stress (σ) and the strain (ε)

Area of the specimen = $\pi d^2/4 = \pi(10)^2/4 = 78.54\ mm^2$

Stress = P/A
Strain = Deformation/Length

Load (N)	Deformation (mm)	Stress (MPa)	Strain $X10^{-5}$
0	0	0.0000	0.0000
960	0.000533	12.2231	2.6650
1800	0.001067	22.9183	5.3350
2400	0.001227	30.5577	6.1350
2880	0.002133	36.6692	10.6650
3360	0.002293	42.7807	11.4650
3840	0.002933	48.8923	14.6650
4320	0.004	55.0038	20.0000
4800	0.005333	61.1154	26.6650
5280	0.0064	67.2269	32.0000
6240	0.0096	79.4500	48.0000
7200	0.013333	91.6730	66.6650
8040	0.018133	102.3682	90.6650
9600	0.027733	122.2307	138.6650
10560	0.036267	134.4538	181.3350
11530	0.046933	146.8042	234.6650
12000	0.053333	152.7884	266.6650


Stress-Strain Diagram
Stress (MPa)
170.0
160.0
150.0
140.0
130.0
120.0
110.0
100.0
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
0.0
15.0
30.0
45.0
60.0
75.0
90.0
105.0
120.0
135.0
150.0
165.0
180.0
195.0
210.0
225.0
240.0
255.0
270.0
285.0
Strain (mm/mm) X 10^{-5}

Problem #3 (II)

Given:
The data above

Required:
a) The Modulus of elasticity
b) The proportional limit stress
c) The yield strength at 0.1% offset
d) The ultimate stress
e) The Modulus of resilience
f) The toughness (Use straight lines to approximate the curve).

Solution:

a) $E = \sigma/\varepsilon$ (in elastic range)

$$\Rightarrow E = \frac{12.2231\,(10)^6}{2.665 \times 10^{-5}} = 458.65\,(10)^9 = \boxed{458.65\ GPa}$$

b) $\boxed{\sigma_{PL} \approx 23.75\ MPa}$

c) σ_{ys} from the stress-strain diagram at 0.1% strain
$\boxed{\sigma_{ys} \approx 115\ MPa}$

d) The ultimate stress $\boxed{\sigma_{ult} = 152.79\ MPa}$

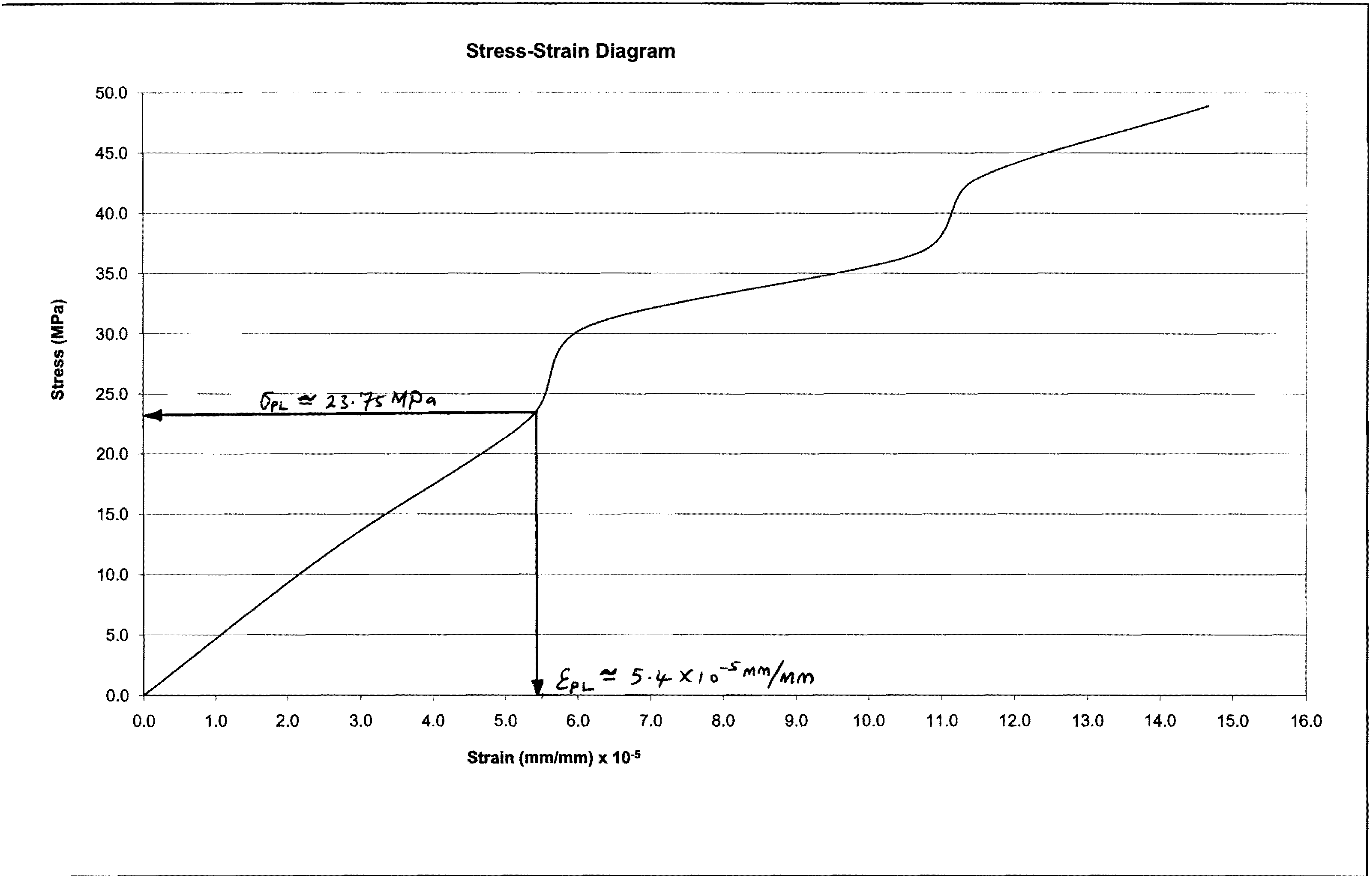
e) The Modulus of resilience:

$$M_R \approx \frac{1}{2}\,\sigma_{PL} * \varepsilon_{PL}$$
$$= \frac{1}{2} * 23.75 \times 10^6 * 5.4 \times 10^{-5}$$

$$\boxed{M_R = 641\ J/m^3}$$

f) The toughness:

The toughness $\approx 10^3 * 105\,[0.0026 - 0.00092] + 10^3\left(\frac{152.79 - 105}{2}\right)[0.0026 - 0.00092]$
$+ 10^3\left(\frac{105 + 75}{2}\right) * [0.00092 - 0.000125] + \left(\frac{75 + 30}{2}\right) \times 10^3 * (0.000125 - 0.00005)$
$+ \frac{1}{2} * 23.75 \times 10^3 * 5.4 \times 10^{-5} = 4{,}613\ J/m^3$ $\therefore$ $\boxed{\text{The toughness} \approx 4{,}613\ J/m^3}$



Stress-Strain Diagram
Stress (MPa)
50.0
45.0
40.0
35.0
30.0
25.0
20.0
15.0
10.0
5.0
0.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
11.0
12.0
13.0
14.0
15.0
16.0
Strain (mm/mm) x 10-5
$\sigma_{PL} \simeq 23.75$ MPa
$\varepsilon_{PL} \simeq 5.4 \times 10^{-5}$ mm/mm

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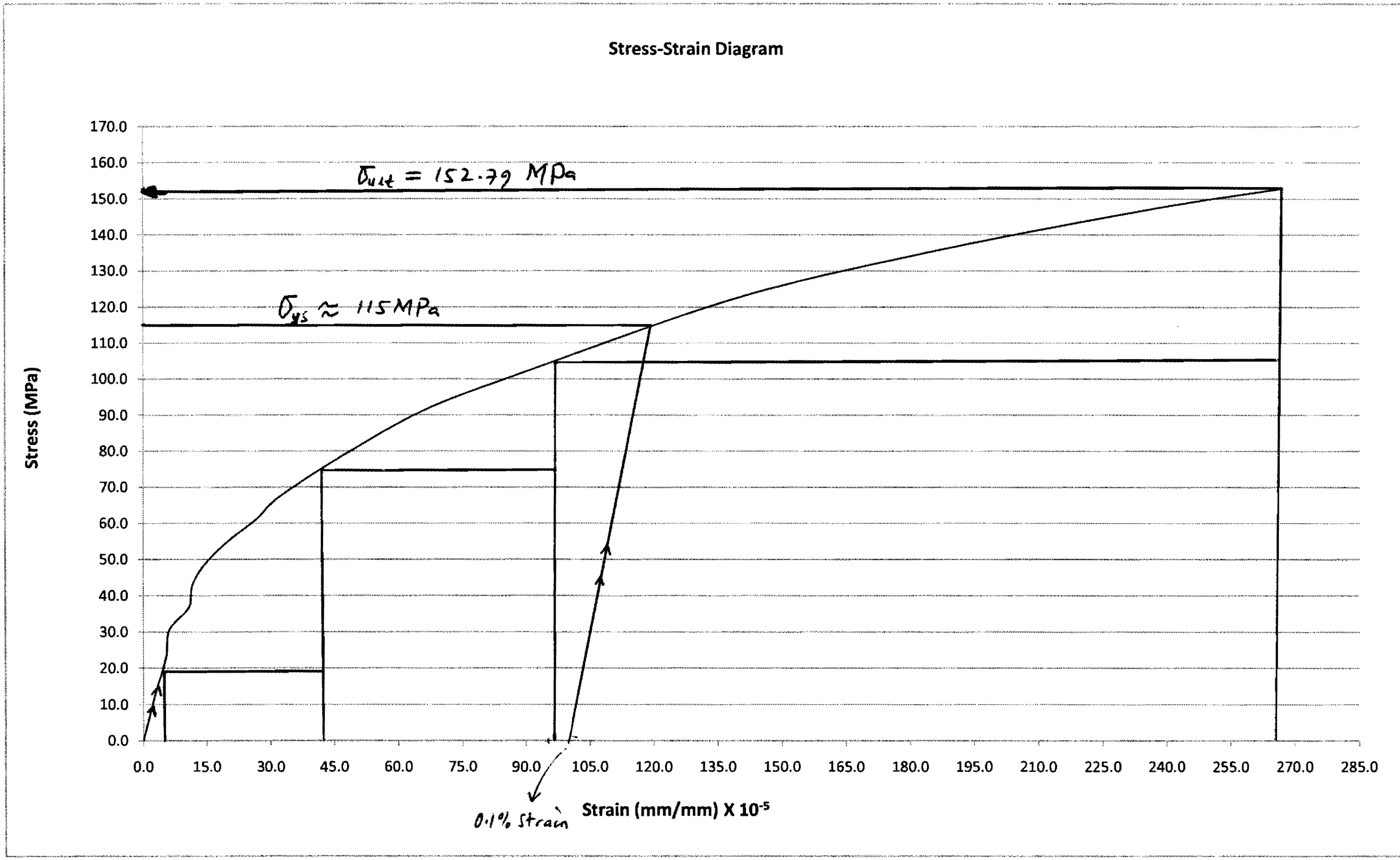
Stress-Strain Diagram
Stress (MPa)
Strain (mm/mm) X 10⁻⁵
170.0
160.0
150.0
140.0
130.0
120.0
110.0
100.0
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
0.0
15.0
30.0
45.0
60.0
75.0
90.0
105.0
120.0
135.0
150.0
165.0
180.0
195.0
210.0
225.0
240.0
255.0
270.0
285.0
σult = 152.79 MPa
σys ≈ 115MPa
0.1% Strain

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Problem # 3 (III)

Given:

The data above

Poisson's ratio = 0.25

Required:

a) The new dimensions of the Specimen
b) The change in the volume
c) The dilatation (unit change in Volume)
d) The percent reduction in area
e) The percent elongation

Solution:

a) $\varepsilon_{axial} = \varepsilon_{final} = \dfrac{l_f - l_o}{l_o} = 0.00267$

$\varepsilon_{lateral} = \varepsilon_l = \dfrac{d_f - d_o}{d_o}$

$\nu = \left|\dfrac{\varepsilon_l}{\varepsilon_a}\right|$

$\therefore\ 0.25 = \left|\dfrac{\varepsilon_l}{\varepsilon_a}\right|$

$\therefore\ \varepsilon_l = \varepsilon_a * 0.25 = -0.00267 * 0.25 = -0.0006675$

$\therefore\ \varepsilon_l = -0.0006675$

ε at failure $= \dfrac{l - 20}{20} = 0.00267 \Rightarrow$

$\therefore\ \boxed{l_f = 20.0534\ mm}$

$\varepsilon_l = -0.0006675 = \dfrac{d - 10}{10}$

$\therefore\ \boxed{d_f = 9.9933\ mm}$

b) The change in volume

$V_o = \dfrac{(\pi)(d_o)^2}{4} * l_o$

$= \dfrac{(\pi)(10)^2}{4} * 20 = 1570.796\ mm^3$

$$V_f = \frac{\pi (d_f)^2}{4} * l_f$$

$$= \frac{(\pi)(9.9933)^2}{4} * 20.0534 = 1572.8805 \text{ mm}^2$$

$$\underline{\underline{\Delta V}} = V_f - V_o = 1572.8805 - 1570.796$$

$$\boxed{\Delta V = 2.084 \text{ mm}^3}$$

c) The dilatation $= e = \dfrac{\Delta V}{V}$

$$\therefore \underline{\underline{e}} = \frac{2.084}{1570.796} \Rightarrow \boxed{e = \underline{\underline{0.001326}}}$$

d) The percent reduction in area $= \dfrac{A_o - A_f}{A_o} \times 100$

$$= \frac{\frac{(\pi)(10)^2}{4} - \frac{(\pi)(9.9933)^2}{4}}{\frac{(\pi)(10)^2}{4}} * 100$$

$$\boxed{\text{percent reduction in area} = \underline{\underline{0.134\%}}}$$

e) The percent elongation $= \dfrac{l_f - l_o}{l_o} * 100$

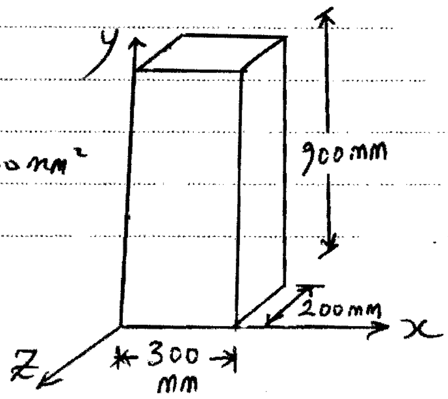
$$= \frac{20.0534 - 20}{20} * 100$$

$$\boxed{\text{Percent elongation} = \underline{\underline{0.267\%}}}$$

Problem #4

Given:

- A tensile test of a bar
- Area of the bar = 200 * 300 = 60000 mm^2
- P at proportional limit = 1200 kN
- The length = 900 mm
- The increase in the length = 0.45 mm
- The 300 mm dimension decreases by = 0.015 mm



Required:

a) The proportional limit
b) The Modulus of elasticity
c) poisson's ratio
d) The new value of the 20 mm dimension

Solution:

a) $\sigma_{PL} = \dfrac{1200 \times 10^3}{60000} = 20\ N/mm^2 \Rightarrow \boxed{\sigma_{PL} = 20 MPa}$

b) $\varepsilon_y = \varepsilon_{PL} = \dfrac{0.45}{900} = 0.0005\ mm/mm$

$\therefore E = \dfrac{\sigma_{PL}}{\varepsilon_{PL}} = \dfrac{20}{0.0005} = 40000\ N/mm^2 \Rightarrow \boxed{E = 40\ GPa}$

c) $\varepsilon_y = 0.0005\ mm/mm$

$\varepsilon_x = \varepsilon_z = \dfrac{t - t_o}{t_o} = \dfrac{-0.015}{300} = -0.00005\ mm/mm$

$\nu = \left| \dfrac{\varepsilon_{lateral}}{\varepsilon_{axial}} \right| = \dfrac{0.00005}{0.0005} \Rightarrow \boxed{\nu = 0.1}$

d) The new value of the 200 mm dimension

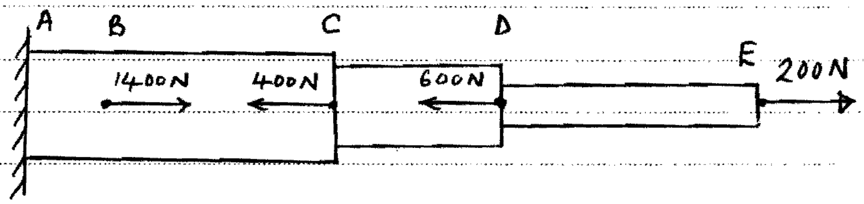
$\varepsilon_y = -0.00005 = \dfrac{w - w_o}{w_o} = \dfrac{w - 200}{200}$

$\therefore \boxed{w = 199.99 mm}$

Problem #5:

Given:

The figure shown



Required:

The displacement of E

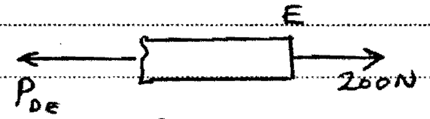
Solution:

The displacement of point E is the total elongation (+ or −) of all Members / segments as point A is fixed. ⇒

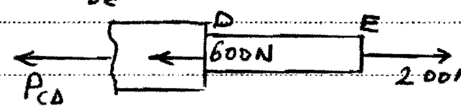
$$\delta_E = \sum e = \sum\left(\frac{PL}{AE}\right) = \left(\frac{PL}{AE}\right)_{AB} + \left(\frac{PL}{AE}\right)_{BC} + \left(\frac{PL}{AE}\right)_{CD} + \left(\frac{PL}{AE}\right)_{DE}$$

L, A, and E are given in the table for all.
We need to determine P for each. We get it from the FBD as it is internal. Thus.

$P_{DE} = +200N$ "T"

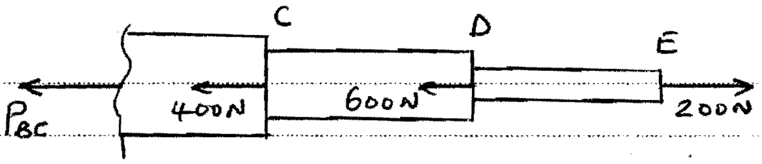


$P_{CD} = -400N$ "C"

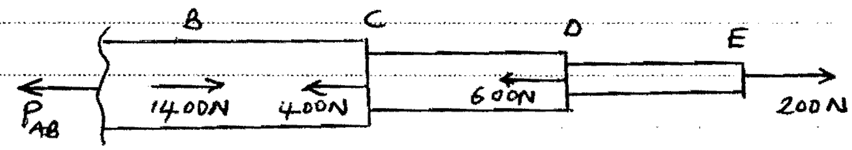


(Be Careful about signs!! Compression is (−).)

$P_{BC} = -800N$ "C"



$P_{AB} = +600N$ "T"



$$\Rightarrow \delta_E = \left[\frac{600(0.5)}{50(10)^{-6}(250)(10)^9} + \frac{-800(1.5)}{50(10)^{-6}(250)(10)^9} + \frac{-400(2)}{32(10)^{-6}(400)(10)^9} + \frac{200(3)}{10(10)^{-6}(600)(10)^9}\right]$$

$$\Rightarrow \boxed{\delta_E = -3.45(10)^{-5}\,m = -0.0345\ mm}$$

(− means moved to the left.)