## Problem #1:

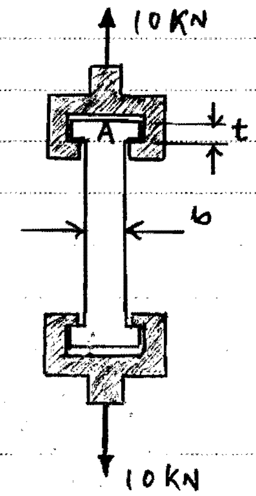
Given:

The figure shown

$(\sigma_t)_{allow} = 12\ MPa$

$\tau_{allow} = 1.2\ MPa$

Width $= 20\ mm$




Required:

The dimensions b and t

So that the two allowable stresses are reached simultaneously

Solution:

$\sigma = \frac{N}{A}$

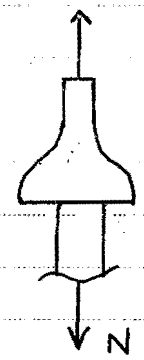
From FBD ①,

$N = 6\ KN\ (T)$

$A = 20\,(b)$

$\Rightarrow \sigma = \frac{6(10)^3}{20(b)}$

Set $\frac{6(10)^3}{20(b)} \equiv 12$




FBD ①

[Note that 1 N/mm² is as 1 MPa]

$\Rightarrow 12(20)b = 6000 \Rightarrow \boxed{b = 25\ mm}$

$\tau = V/A$

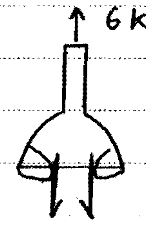
From FBD ②,

$V_{total} = 6\ KN$

$A = 20(t)$

$\Rightarrow \tau = \frac{6000}{2(20)t}$

↳ Double shear

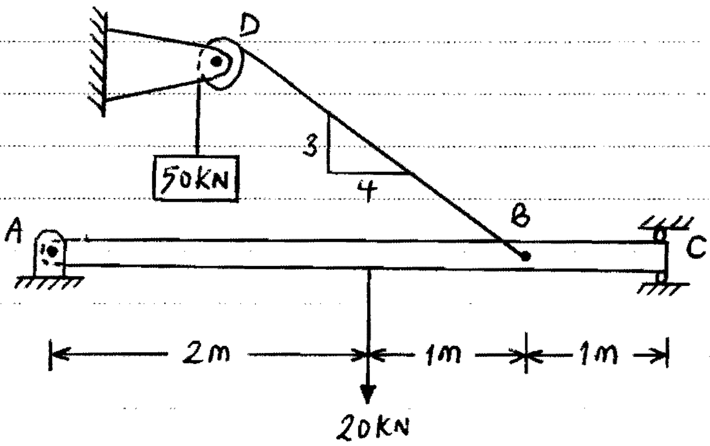



FBD ②

Set $\frac{6000}{2(20)t} \equiv 1.2 \Rightarrow 1.2(2)(20)t = 6000$

$\boxed{t = 125\ mm}$

## Problem #2:




Given;

The figure shown

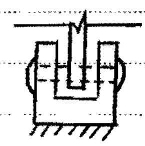
$A_{BD} = 100\ mm^2$

$D_{pins} = 20\ mm$

Required:

$\sigma$ in BD

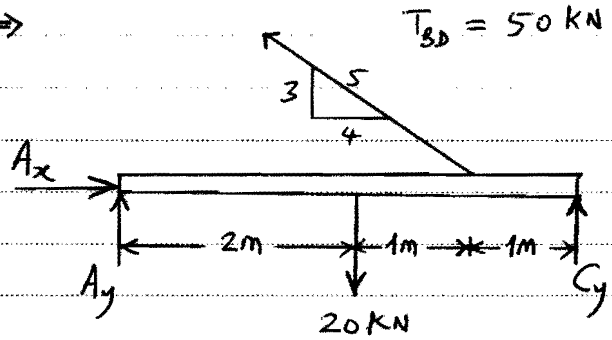
$\tau$ in pins at A and D



Detail of Pin Connector at point A and D

Solution:

First, we need to determine the forces in BD and the pins. ⇒




FBD ①

In FBD ①,

$T_{BD} = W = 50\ KN$ (Why?!)

$$\Rightarrow \sigma_{BD} = \frac{T}{A}$$

$$= \frac{50(10)^3}{100}$$

$$\Rightarrow \boxed{\sigma_{BD} = 500\ MPa}$$

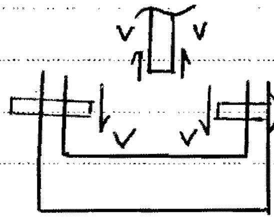
$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow A_x - 50(4/5) = 0 \Rightarrow A_x = 40\ kN$

$\circlearrowleft^{+} \sum M_c = 0 \Rightarrow 20(2) - 50(3/5)(1) - A_y(4) = 0 \Rightarrow A_y = 2.5\ kN$

The shear force in pin A is the resultant of $A_x$ and $A_y$ ⇒

$A = \sqrt{(40)^2 + (2.5)^2} = 40.078\ kN$

Note: the pins at A and D are in <u>double</u> shear

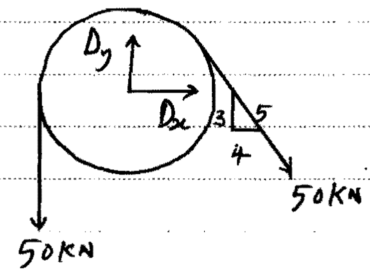



Pins at A $ D

$$\tau_A = \frac{V_A}{2A_{pin}} = \frac{A}{2A_{pin}} = \frac{40.078(10)^3}{2(\pi/4)(20)^2}$$

$$\boxed{\tau_A = 63.79\ MPa}$$

For the forces at D, we need to draw FBD② for the pulley.



FBD②

$\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow D_x + 50(4/5) = 0 \Rightarrow$

$D_x = -40 = 40\text{ KN} \leftarrow$

$+\uparrow \Sigma F_y = 0 \Rightarrow D_y - 50 - 50(3/5) = 0 \Rightarrow$

$D_y = 80\text{ KN}$

The resultant $= D = \sqrt{(40)^2 + (80)^2} = 89.443\text{ KN}$

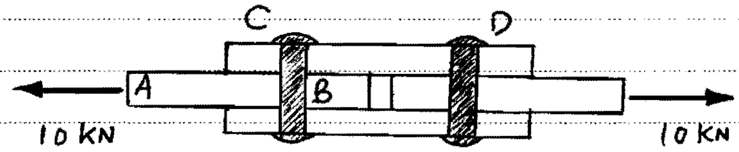
$$\tau_D = \frac{D}{2A_{pin}} = \frac{89.443(10)^3}{2(\pi/4)(20)^2}$$

$$\Rightarrow \boxed{\tau_D = 142.4\text{ MPa}}$$

## Problem #3:

**Given:**

The figure shown




**Required:**

(a) $\tau$ in the 10-mm D rivet

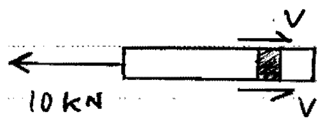
(b) $\sigma_{bearing}$ in the 100-mm × 15-mm plate AB

(c) $\sigma_{bearing}$ in the 100-mm × 5-mm plate CD

**Solution:**

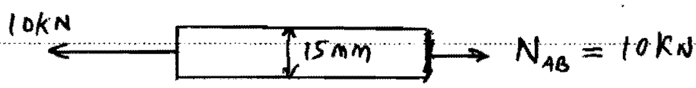
(a) Note that the rivets act in double shear

$$\tau_c = \tau_D = \frac{V_{total}}{2A_{rivet}} \Rightarrow V_{total} = 10 KN$$




$$\tau_{rivet} = \frac{10(10)^3}{2(\pi/4)(10)^2} \Rightarrow \boxed{\tau_{rivet} = 63.66\ MPa}$$

(b) $$(\sigma_{bearing})_{AB} = \frac{N_{AB}}{A_c}$$

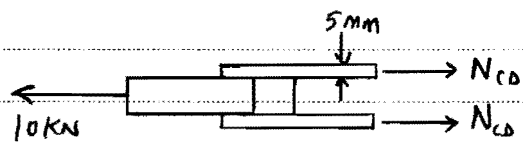



$$A_c \approx Dt = 10(15)$$

$$\Rightarrow (\sigma_{bearing})_{AB} = \frac{10(10)^3}{10(15)}$$

$$\Rightarrow \boxed{(\sigma_{bearing})_{AB} = 66.67\ MPa}$$

$$(\sigma_b)_{CD} = \frac{N_{CD}}{A_c}$$

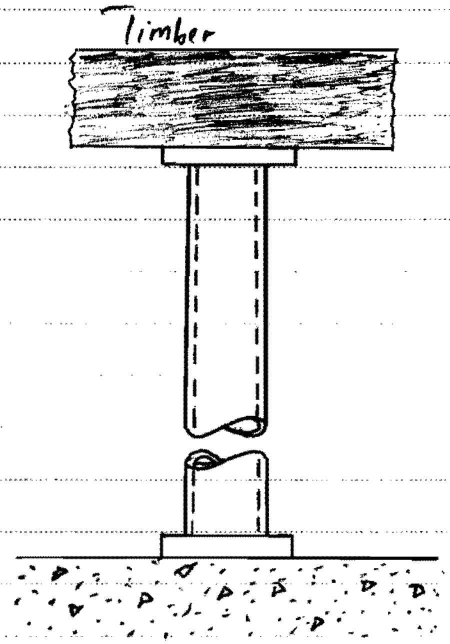



$$2N_{CD} = 10\ KN \Rightarrow N_{CD} = 5KN$$

$$A_c \approx Dt = 10(5)$$

$$\Rightarrow (\sigma_b)_{CD} = \frac{5(10)^3}{10(5)} \Rightarrow \boxed{(\sigma_b)_{CD} = 100 MPa}$$

## Problem # 4:




Given:

The figure shown

$(D_{out})_{column} = 150\ mm, \quad t_{column} = 15\ mm$

$F_{timber} = 150\ kN$

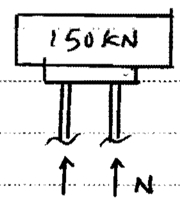
Required:

a) $\sigma_{bearing}$ on steel pipe/bearing plate

b) D of the circular bearing plate if $(\sigma_b)_{max} = 6.5\ MPa$ on wood and Safety factor (SF) = 2

Solution:

a) $\sigma_b = \dfrac{N}{A_c}$




$N = 150\ kN$

$A_c = \dfrac{\pi}{4}(D_{out}^2 - D_{in}^2)$

$= \dfrac{\pi}{4}\left[(150)^2 - \{150 - 2(15)\}^2\right]$

$= 6361.7\ mm^2$

$\sigma_b = \dfrac{150\,(10)^3}{6361.7} \Rightarrow \boxed{\sigma_b = 23.58\ MPa}$

b) $(\sigma_b)_{allow} = \dfrac{(\sigma_b)_{max}}{SF} = \dfrac{6.5}{2} = 3.25\ MPa$

$\sigma_b = \dfrac{N}{A_c} = \dfrac{150\,(10)^3}{\frac{\pi}{4} D_{Plate}^2} = 3.25$

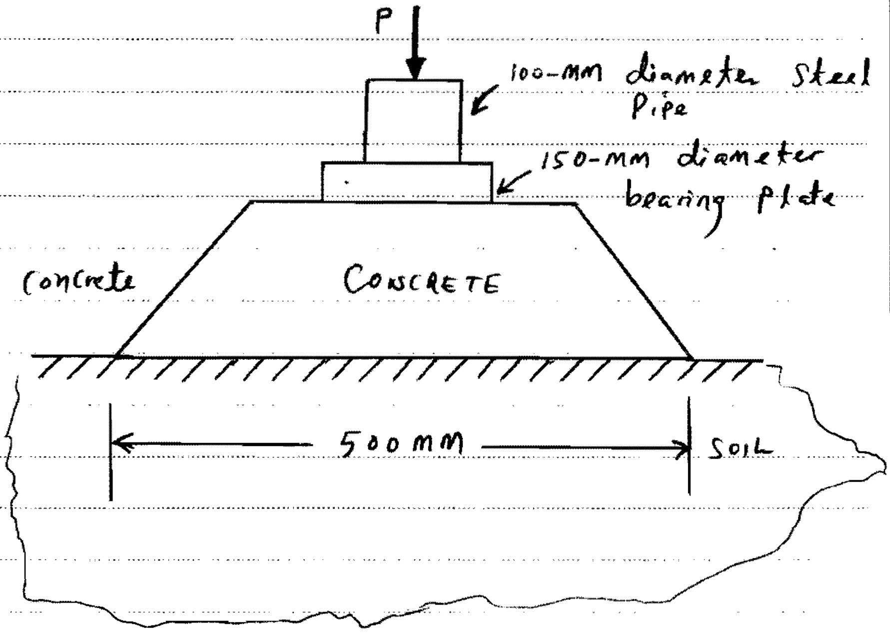
$\Rightarrow \boxed{D_{Plate} = 242.4\ mm}$

Problem #5:

Given;
The figure shown
$(\sigma_{max})_{allow}$ steel $= 100\,MPa$

$(\sigma_{max})_{allow}$ bearing $= 15\,MPa$ in concrete

$(\sigma_{max})_{allow}$ bearing $= 2\,MPa$ in soil

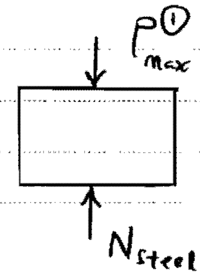



Required:
$P_{max\ allow}$

Solution:
Here, we have 3 criteria to satisfy. There are usually two ways to solve such problems. The first one is to satisfy each criterion, then choose the smallest P for the maximum allowable. (why ?!).
The other method is that we assume one criterion will "control" (i.e. the maximum stress will be reached before the other two), then we check our assumption. If "ok", the problem is solved. If "not", then our assumption is not correct, and we need to assume one of the other criteria will control, and repeat the procedure above.
The first method may be appropriate for hand calculations, while the other one could be suitable for "automation" (e.g. computer programming).
Thus, here, we will use the first method.

First, consider $(\sigma_{normal})_{steel}$

$$\sigma_{steel} = \frac{N_{steel}}{A_{steel}}$$
$$N_{steel} = P^{(1)}_{max}$$

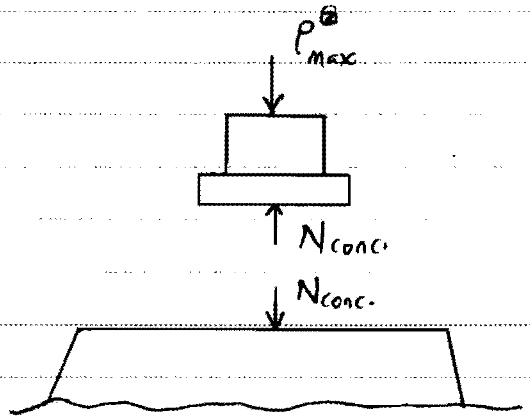


$$A_{steel} = \frac{\pi}{4}(100)^2 = 2500\pi \ mm^2$$

$$\sigma_{steel} = \frac{P^{①}_{max}}{2500\pi} \equiv 100 \quad [\text{set it equal to } 100]$$

$$\Rightarrow \underline{\underline{P^{①}_{max} = 785.4 \ kN}}$$

<u>Second</u>, consider $(\sigma_{bearing})_{concrete}$




$$\sigma_{b_{conc.}} = \frac{N_{conc.}}{A_{c_{conc.}}}$$

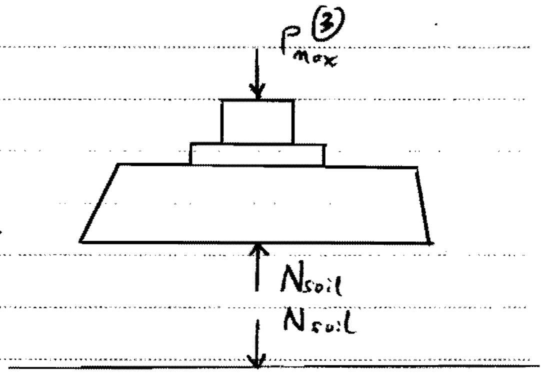
$$N_{conc.} = P^{②}_{max}$$

$$A_c = \frac{\pi}{4}(150)^2 = 5625\pi \ mm^2$$

$$\Rightarrow \frac{P^{②}_{max}}{5625\pi} \equiv 15$$

$$\underline{\underline{P^{②}_{max} = 265.1 \ kN}}$$

<u>Third</u>, consider $(\sigma_{bearing})_{soil}$




$$\sigma_{b_{soil}} = \frac{N_{soil}}{A_{c_{soil}}}$$

$$N_{soil} = P^{③}_{max}$$

$$A_{c_{soil}} = 500 \times 500 = 250{,}000 \ mm^2$$

$$\Rightarrow \frac{P^{③}_{max}}{250{,}000} \equiv 2$$

$$\Rightarrow \underline{\underline{P^{③}_{max} = 500 \ kN}}$$

Thus $P_{\substack{max \\ allow}} = P_{min}\left(P^{①}_{max}, P^{②}_{max}, P^{③}_{max}\right)$

$$\boxed{P_{\substack{max \\ allow}} = 265.1 \ kN}$$

Note that criterion ② (Concrete) "Controls" eventhough $\sigma_{allow}$ is not the smallest !! (Why?!)