Problem #1:

Given:
The figure shown steel tank filled
with water.
\( \gamma_{\text{water}} = 10 \text{ kN/m}^3 \); \( \gamma_{\text{steel}} = 78 \text{ kN/m}^3 \)

Required:
State of stress at A.

Solution:
The FBD after making a section
through A and taking the "upper"
part (why?!) is shown.
The stress at A is caused by the weight
of the steel above A. "Steel carries itself" and "water carries
itself" vertically. In addition, the water causes "hydrostatic
pressure" on the steel wall according to "Pascal's law"
that is \( P = (\gamma_h) \text{water} \). (Review it if you took it: read
about it or ask your instructor if you did not)
This \( P \) causes \( \sigma_{\text{H}} \) (or \( \sigma_0 \)) as studied in the pressure
vessels section.

\[ W_{\text{steel}} = \gamma_{\text{steel}} V_{\text{steel}} \]

\[ = 78 \left( \frac{(612)^2 - (600)^2}{2} \right) \pi (10)^6 (1.5) = 5.34588 \text{ kN} \]

\[ \sigma_{\text{vertical}} = \sigma_{\text{long}} = \sigma_{\text{axial}} = \frac{W_{\text{steel}}}{A_{\text{steel}}} \]

\[ = \frac{5.34588 \times (10)^3}{(612)^2 - (600)^2} \pi (10)^6 \text{ at that section} \]

\[ \sigma_{\text{vertical}} = -117 \text{ kPa} = 117 \text{ kPa} "C" \]

\[ \sigma_{\text{horizontal}} = \sigma_{\text{hoop}} = \sigma_{\text{circumferential}} = \frac{P_r}{t} \]
CE 203 – 112
Solution of HW #13

\[ P = \gamma_{\text{water}} \cdot h_{\text{water}} = 10(10)^3(1.5) = 15 \text{ kPa} \]

\[ \sigma_h = \frac{15 (600)}{12} = \frac{9000}{12} = 750 \text{ kPa} \]

"T" = 0 \( h \)

The state of stress at A is shown.

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Note that the internal pressure due to water does not cause longitudinal stress in the open tank (why? !)
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Problem #: 2:

Given:
The beam with the cross-section shown.

Required:
$\sigma$ and $\gamma$ at A and B

Solution:

\[
\begin{align*}
\bar{y} &= 118.75 \text{ mm} = 0.11875 \text{m} \quad \text{See HW#12} \\
I_z &= 3.453125 \times 10^5 \text{ m}^4
\end{align*}
\]

To calculate $\sigma$ and $\gamma$, we need to determine the internal forces at section 2-m from left that passes through A and B, as shown in the FBD below.

$\Sigma F_x = 0 \Rightarrow F_x = 0$

$\Sigma M_A = 0 \Rightarrow -40 + 40(1) + M_2 = 0 \Rightarrow M_2 = 0$

$\Sigma F_y = 0 \Rightarrow V = -40 \text{ kN} = 40 \text{ kN}$

$\Sigma M_A = 0 \Rightarrow -40 + 40(1) + M_z = 0 \Rightarrow M_z = 0$

$\sigma_x = \frac{F_x}{A} + \frac{M_2}{I_z} + \frac{M_y}{I_y}$

$\sigma$ is zero throughout the section \( \Rightarrow \sigma_A = \sigma_B = 0 \)

$\gamma = \frac{V_y}{A} + \frac{Q_z}{I_z} \quad I_z$ is not taken by student.

Only $V_y$ is present. $\sigma$ and $\gamma$ distribution throughout the section are as shown.

$Q_z = A \bar{y} = 0 \Rightarrow \gamma_a = 0$
\( Q_z^B = (A\gamma)_B = 20(50)(118.75 - \frac{50}{2}) = 93750 \text{ mm}^3 \)

\[ \gamma_B = \frac{V_y Q_z}{I_z E_z} = \frac{40(10^3)(9.375)(10)^5}{3453125(10)^5(20)(10)^3} \]

\[ \gamma_B = 5.43 \text{ MPa} \]

The states of stress at A and B are shown.

\[ \text{A} \quad \text{"free"} \quad \text{B} \quad 5.43 \text{ MPa} \]
Problem #3:

Question:
The solid block with the loading shown.
Required:
0 @ A and B; show them on differential elements.
Solution:

\[
\sigma_x = \pm \frac{F_z}{A_z} \pm \frac{M_{x,y}}{I_x} \pm \frac{M_{y,x}}{I_y}
\]

First, we need to determine the internal forces at the bottom of the block (through A and B), as shown (F=B).

\[
\begin{align*}
\sum F_x &= 0 \Rightarrow F_x = 0 \\
\sum F_y &= 0 \Rightarrow F_y = -100 + 90 + 50 = 340 \text{ N upwards} \\
\sum F_z &= 0 \Rightarrow F_z = 200 + 40 + 20 = 260 \text{ N inward}
\end{align*}
\]

\[
\mathbf{M} = \begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} = \begin{bmatrix}
r_x \\
r_y \\
r_z
\end{bmatrix}
\]

Since only normal stresses are required, no need to calculate \( M_z \); which causes shear stress only. \( \Rightarrow \sum M_x = 0 \Rightarrow M_x = \sum M \text{ all forces} = 0 \)

\[
M_x = -\sum [(r_y F_z) - (r_z F_y)]
\]

\[
= \left[ (20 + 40) (-200) + \left\{ -10 + \frac{40}{2} \right\} (-90) \right] \\
+ 40 (100) - (110) (100)
\]

\[
= 164 \text{ N m}
\]
My = \sum \left[ (A_x F_x - (A_z F_z)^T) \right] = \left[ \left( -\frac{30}{2} \right)(-200) + \left( -\frac{30}{2} \right)(-90) + \left( 50 + \frac{30}{2} \right)(-50) \right] = 1.1 \text{ Nm.}

The stress distributions due to $F_2$, $M_x$ and $M_y$ are shown below.

\( A_z = 40 \left( 50 + 30 + 50 \right) + 2(20)(30) = 6,400 \text{ mm}^2 \)

The stress due to $F_2$ is calculated as:

\( \sigma_{F_2} = \frac{F_2}{A_z} = \frac{340}{6400 \times 10^{-6}} = 531.25 \text{ kPa} \)

\( I_x = \frac{1}{12} \left[ 30(80)^3 + 2(50)(40)^3 \right] = 1.8133 \times 10^{-6} \text{ m}^4 \)

\( I_y = \frac{1}{12} \left[ 40(130)^3 + 2(20)(30)^3 \right] = 7.4133 \times 10^{-6} \text{ m}^4 \)

The stress due to $M_x$ is calculated as:

\( \sigma_{M_x} = \frac{M_x y}{I_x} = \frac{16.4(40)(10)^3}{1.8133 \times 10^{-6}} = 361.77 \text{ kPa} \)

The stress due to $M_y$ is calculated as:

\( \sigma_{M_y} = \frac{M_y x}{I_y} = \frac{11.1(15)(10)^3}{7.4133 \times 10^{-6}} = 22.257 \text{ kPa} \)

The total stress $\sigma_A$ is given by:

\( \sigma_A = -581.25 - 361.77 + 22.257 = -890.8 \text{ kPa} \)
Solution of HW #13

\[ \sigma_{x}^{B} \text{ due to } M_{x} = \frac{M_{x} y}{I_{x}} = \frac{16.4 (20) (10)^{-3}}{1.8133 (10)^{-6}} = 180.88 \text{ kPa } "C" \]

\[ \sigma_{y}^{B} \text{ due to } M_{y} = \frac{M_{y} x}{I_{y}} = \frac{1.1 (65) (10)^{-3}}{7.4133 (10)^{-6}} = 9.64 \text{ kPa } "T" \]

\[ \sigma_{z}^{B} = -531.25 - 180.88 + 9.64 = -702.5 \text{ kPa } "C" \]

\[ \sigma_{z}^{B} = -702.5 = 702.5 \text{ kPa } "C" \]

"Shear stress not shown."
Problem 4:

Given: 
The figure shown. \( W_{AB} = 10 \text{ kN} \).
\( Y_{in} = 68 \text{ mm} \); \( Y_{out} = 75 \text{ mm} \).

Required: 
The state of stress at C and D
Show them on differential elements.

Solution:

\[ \sigma_X = \sigma_{due to} F_X + \sigma_{due to} M_y + \sigma_{due to} M_x \]
\[ \tau = \tau_{due to} F_y + \tau_{due to} M_x \]

We need to find the internal forces first.

\[ \sum F = 0 \Rightarrow F_O + 10 \hat{i} - 6(3.6)(1.8) \hat{j} = 0 \]
\[ \Rightarrow F_O = -10 \hat{i} + 38.88 \hat{j} \text{ kN} \]

\[ \sum M = 0 \Rightarrow M_O + \begin{bmatrix} -1.8 & 0 & -1.8 \\ 10 & -38.88 & 0 \end{bmatrix} = 0 \]
\[ M_O = 69.984 \hat{i} + 18 \hat{j} - 69.984 \hat{k} (\text{kN m}) \]

Normal Stress:

Thus, the stress distributions are as shown below at the section of interest.
\[ A_x = \pi \left[ (75)^2 - (68)^2 \right] = 1001\pi \approx 3144.73 \text{ mm}^2 \]

\[ I_y = I_1 = \frac{1}{4} \left[ (75)^4 - (68)^4 \right] = 8.0576 \times 10^6 \text{ mm}^4 \]

\[ \sigma_C = \frac{-10(10)^3}{3144.73(10)^6} + 0 + \frac{18(10)^3(68)(10)^{-3}}{8.0576 (10)^{-6}} \]

\[ \sigma_C = -3.1799 + 151.91 \]

\[ \sigma_C = 148.7 \text{ MPa} \]

\[ \sigma_D = \frac{-10(10)^3}{3144.73(10)^6} + \frac{69.984 (10)^3 (75)(10)^{-3}}{8.0576 (10)^{-6}} \]

\[ \sigma_D = -3.1799 + 651.22 \]

\[ \sigma_D = 648.2 \text{ MPa} \]

Note that \( \sigma \) due to bending >> \( \sigma \) due to axial.

Similar procedures can be followed for any other points (e.g., E and F). Once you have drawn the correct stress distribution, the problem becomes easy.
Shear Stress:

\[ V_y = 38.88 \text{ kN} \; ; \; V_z = 0 \; ; \; T = M_x = 69.984 \text{ kNm} \]

\[ \gamma_{xy}^C = \gamma_{V_y} - \gamma_T = \frac{V_y Q z}{I_z t_y} = \frac{M_x Y}{J_0} \]

\[ J_0 = 2 I_x; I_z = 2(t) = 2(75.68) = 14.14 \text{ mm} \]

\[ Q \frac{z}{2} = (A y) \text{ area of interest} = Q \text{ total} - Q \text{ hole} \]

\[ = \frac{\pi}{2} (75)^2 \left( \frac{4 \times 75}{8\pi} \right) - \frac{\pi}{2} (68)^2 \left( \frac{4 \times 68}{8\pi} \right) \]

\[ = 71.628.7 \text{ mm}^3 \]

\[ \gamma_{xy}^C = \frac{38.88 \left( \frac{71.628.7}{10^3} \right) (10)^{-9} - 69.984 \left( \frac{68}{10^3} \right) (10)^{-9}}{8.0576 (10)^{-6} (14) (10)^{-3}} \]

\[ \Rightarrow \gamma_{xy}^C = -270.6 \text{ MPa} \]

\[ \gamma_{yz}^C = 0 \quad \text{(Horrible!)} \]

\[ \beta_z = 0 \quad \text{(why?!) \Rightarrow} \]

\[ \gamma_{xz}^D = \frac{69.984 \left( \frac{75}{10^3} \right)}{2 \left( \frac{8.0576}{10} \right) (10)^{-6}} \]

\[ \gamma_{xz}^D = 325.7 \text{ MPa} \]

\[ \gamma_{xy}^D = \frac{V_y Q z}{I_z t_y} \Rightarrow \gamma_{xy}^D = 0 \]
The states of stress at C and D are shown below. All MPa.

2-D

3-D
**Problem #5**

**Given:**
The figure shown

\[ D = 36 \text{ mm} \Rightarrow r = 40 \text{ mm} \]

**Required:**
States of stress at A and C
Show them on differential elements

**Solution:**
First, the FBD is drawn and the internal forces are calculated

\[ \sum \mathbf{F} = 0 \]

\[ \mathbf{F}_0 = 300 \mathbf{i} + 500 \mathbf{j} + 400 \mathbf{k} = 0 \]

\[ \mathbf{F}_0 = 800 \mathbf{i} + 500 \mathbf{j} - 400 \mathbf{k} (N) \]

\[ \sum \mathbf{M}_0 = 0 \Rightarrow \mathbf{M}_0 + Y \times \mathbf{F}_{\text{applied}} = 0 \]

\[ \mathbf{F}_{\text{applied}} = -300 \mathbf{i} + 500 \mathbf{j} + 400 \mathbf{k} (N) \]

\[ \mathbf{r}_2 = (1.5 \mathbf{i} + 1 \mathbf{j} + 0 \mathbf{k}) \]

\[ \mathbf{M}_0 = -\mathbf{r}_2 \times \mathbf{F}_{\text{applied}} = -\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 1 & 0 \\ -300 & 500 & 400 \end{vmatrix} = -400 \mathbf{i} + 600 \mathbf{j} - 1050 \mathbf{k} (N\text{m}) \]

\[ \delta y_1 = \pm \frac{F_y}{A_y} \text{ due to axial load } (F_y) \]

\[ \delta y_2 = \pm \frac{M_x Z}{I_y} \text{ due to bending moment } (M_x) \]
\[ \sigma_y = \pm \frac{M_z x}{I_z} \text{ due to bending moment (M_z)} \]

\[ \gamma_{yx} = \frac{I_x}{J_y} \text{ due to twisting moment (T = My)} \]

\[ \gamma_{yx} = \frac{V_x g z}{I_x k_x} \text{ due to shear force (V_x)} \]

\[ \gamma_{yz} = \frac{V_z g x}{I_x k_x} \text{ due to shear force (V_z)} \]

**Normal Stress:**

\[ F_y = 500 \text{ N} \quad \alpha T \]

\[ A_y = \pi R^2 = \pi (40)^2 = 5026.55 \text{ mm}^2 = 5.02655 \times 10^{-3} \text{ m}^2. \]

\[ M_x = 400 \text{ Nm} \]

\[ Z_A = Z_C = 0 \]

\[ I_x = I_z = \frac{\pi R^4}{4} = \frac{\pi}{4} (40)^4 = 2010699 \text{ mm}^4 = 2.010699 \times 10^6 \text{ m}^4 \]

\[ M_z = 1050 \text{ Nm} \]

\[ X_A = X = 40 \text{ mm} = 0.04 \text{ m} \]

\[ X_C = 0 \]

Thus, the stress distribution are drawn.
$\sigma_A = \frac{500}{5.026 \times 10^{-3}} + 0 + \frac{1050 \times (0.04)}{2.01062 \times 10^{-6}} = 0.09947 \times 20.89 \Rightarrow$

$\sigma_A = 20.99 \text{ MPa}$

Note that $\sigma$ due to axial load is usually "noticeably smaller" than $\sigma$ due to bending moment in typical application.

$\sigma_C = \frac{500}{5.026 \times 10^{-3}} + 0 + 0 \Rightarrow \sigma_C = 99.47 \text{ kPa}$

Note that it is now easy to find the compound normal stress at any point in the section.

Shear stress:

$T = My = 600 \text{ Nm}$

$r_A = 0.04 \text{ m}$

$r_C = 0$

$J_0 = \frac{1}{2} r_A^4 = 2I = 4.02124 \times 10^{-6} \text{ m}^4$

$V_n = 800 \text{ N}$ ; $V_z = -400 \text{ N}$

$\tau_A^x = A \gamma$ = 0 (x) = 0

$\tau_C^z = A \gamma = \frac{1}{2} (0.04)^2 \left( \frac{u(0.04)}{3 \pi} \right)$

$= 4.26667 \times 10^{-5} \text{ m}^3$

$\tau_A^x = \tau_C^z = \frac{1}{2} \times 4.26667 \times 10^{-5} \text{ m}^3 \text{(How?!) }$

$t_A^x = 2x = 0.08 \text{ m} = t_C^z$

$t_A^z = 0$

$d = 2x = 0.08 \text{ m}$. 
Thus, from above and the drawing shown.

**Point A:**

\[ \gamma_{\text{due to } T} = \frac{f_2}{J} = \gamma_{yz} \]

\[ = -\frac{600(0.04)}{4.02(10)^6} \Rightarrow -0.0157 \text{ MPa} \]

\[ \gamma_{\text{due to } V_y} = \frac{V_x}{I_x} = 0 \]

\[ \Rightarrow \gamma_{\text{due to } V_x} = \frac{V_2}{I_x} = \frac{-400(4.2667)(10)^5}{2.01062(10)^6 (0.08)} \Rightarrow \]

\[ \gamma_{yz} = -0.1061 \text{ MPa} \]

\[ \Rightarrow \gamma_A = \gamma_{yz} = 5.9683 + 0.1061(\downarrow) \]

\[ \gamma_A = \gamma_{yz} = 6.074 \text{ MPa (\downarrow)} \]

**Point C:**

\[ \gamma_{\text{due to } T} = \frac{f_2}{J} = 0 \]

\[ \Rightarrow \gamma_{\text{due to } V_x} = \frac{V_x}{I_x} = \frac{300(4.2667)(10)^5}{2.01062(10)^6 (0.08)} \]

\[ = 0.079578 \text{ MPa} = \gamma_{yz} \Rightarrow \]

\[ \gamma_{\text{due to } V_x} = \frac{V_2}{I_x} = \gamma_{\text{due to } V_x} \text{ (How??!!)} \Rightarrow \]

\[ \gamma_{yz} = -0.1061 \text{ MPa} \downarrow \]
Solution of HW #13

\[ \vec{\tau}_c = 0.07958 \hat{\mathbf{r}} - 0.1061 \hat{\mathbf{z}} \text{ (MPa)} \]

We can take the resultant (from the forces first or directly from the stress):

\[ \gamma = \sqrt{\gamma_{y_1}^2 + \gamma_{y_2}^2} \Rightarrow \gamma_c = 0.1326 \text{ MPa} \]

\[ \theta = \tan^{-1} \left( \frac{-0.1061}{0.07958} \right) \Rightarrow \theta = 53.13^\circ \]

Note that shear stresses are usually lower than normal stresses.

The states of stress are shown below.