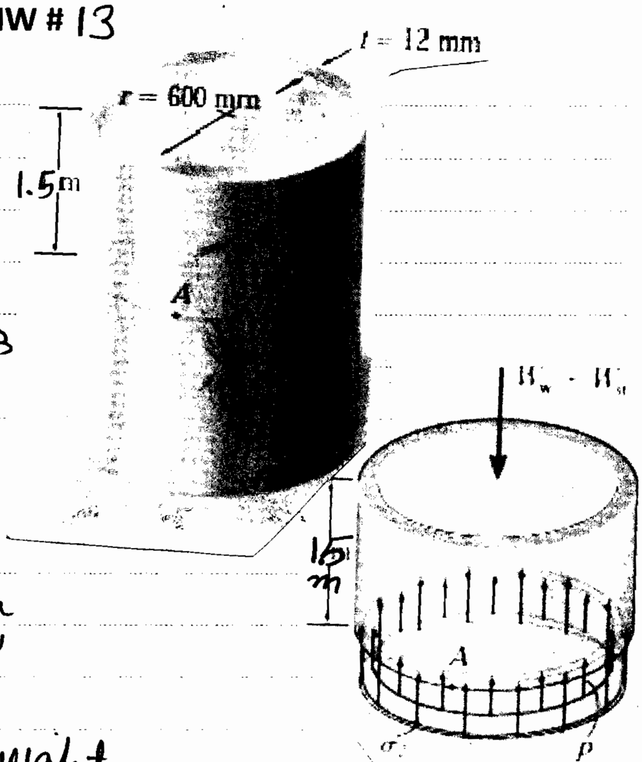
CE 203 – 112

Solution of HW # 13



Problem #1:-

Given:-

The figure shown steel tank filled with water.

$\gamma_{water} = 10\,kN/m^3$; $\gamma_{steel} = 78\,kN/m^3$

Required:

State of stress at A.

Solution:-

The FBD after making a section through A and taking the "upper" part (why?!) is shown.

The stress at A is caused by the weight of the steel above A. "Steel carries itself" and "water carries itself" vertically. In addition, the water causes "hydrostatic pressure" on the steel wall according to "Pascal's law" that is $P = (\gamma h)_{water}$. (Review it if you took it; read about it or ask your instructor if you did not)

This P causes σ_h (or σ_c) as studied in the pressure vessels section.

$W_{steel} = \gamma_{steel} V_{steel}$

$$= 78\,[(612)^2 - (600)^2]\,\pi\,(10)^{-6}(1.5) = 5.34588\,kN$$

$$\sigma_{vertical} = \sigma_{long} = \sigma_{axial} = \frac{W_{steel}}{A_{steel}} \leftarrow \text{at that section}$$

$$= \frac{-5.34588 \times (10)^3}{[(612)^2 - (600)^2]\,\pi\,(10)^{-6}}$$

$$\boxed{\sigma_{vert} = -117\,kPa = 117\,kPa\ \text{"C"}}$$

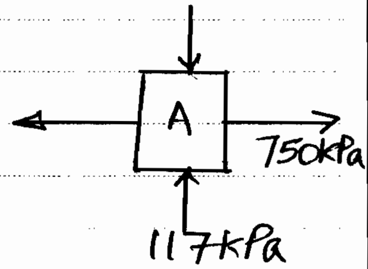
$$\sigma_{horizontal} = \sigma_{hoop} = \sigma_{circumferential} = \frac{Pr}{t}$$

Solution of HW # 13

$P = \gamma_{water} \cdot h_{water} = 10(10)^3(1.5) = 15\ kPa.$

$\Rightarrow \sigma_h = \frac{15\ (600)}{12} = \boxed{750\ kPa\ \text{"T"} = \sigma_h}$

The state of stress at A is shown.

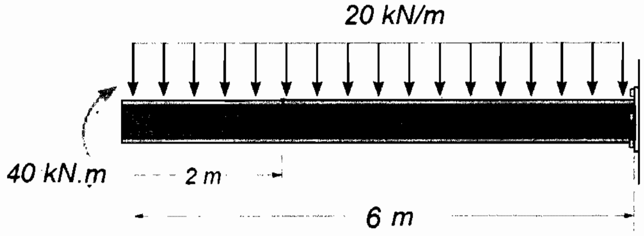


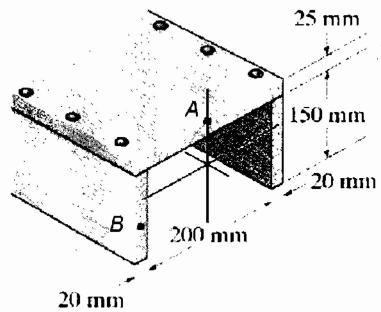
Note that the internal pressure due to water does not cause longitudinal stress in the open tank. (why?!).

Problem #2 :-

Given:-

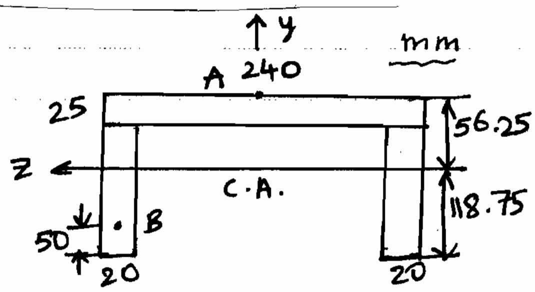
The beam with the cross-section shown.





Required:

σ and τ @ A and B



Solution :-

$\bar{y} = 118.75$ mm $= 0.11875$ m } See
$I_z = 3.453125(10)^{-5}$ m^4. } HW#12

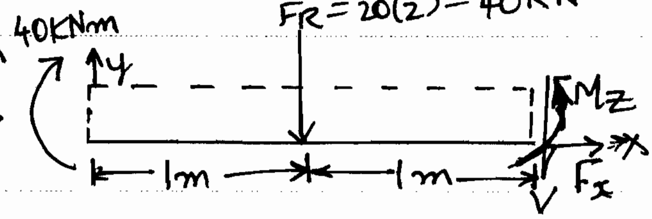
To calculate σ and τ, we need to determine the internal forces at a section 2-m from left that passes through A and B, as shown in the FBD below.

$\Sigma F_x = 0 \Rightarrow F_x = 0$

$+\uparrow \Sigma F_y = 0 \Rightarrow V = -40\text{KN} = 40\text{KN}\uparrow$

$+\circlearrowleft \Sigma M_A = 0 \Rightarrow -40 + 40(1) + M_z = 0 \Rightarrow$

$M_z = 0$



$M_y = 0$ ((2-D))

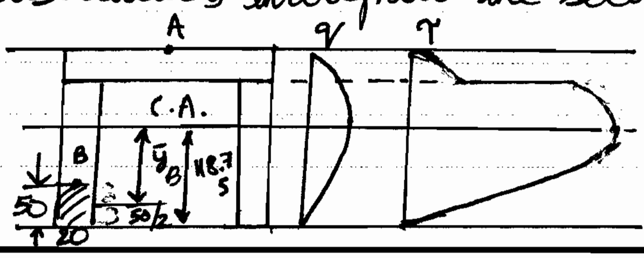
$$\sigma_x = \pm \frac{\cancelto{0}{F_x}}{A_x} \pm \frac{\cancelto{0}{M_z} y}{I_z} \pm \frac{\cancelto{0}{M_y} z}{I_y} \Rightarrow$$

σ is zero throughout the section $\Rightarrow$ $\boxed{\sigma_A = \sigma_B = 0}$

$$\tau = \pm \frac{V_y Q_z}{I_z t_z} \pm \frac{\cancelto{0}{V_z} Q_y}{I_y t_y} \pm \cancelto{0}{T}(*)$$

↑ not taken by students.

Only V_y is present. q and τ-distribution throughout the section are as shown.



$Q_z^A = A\bar{y} = 0 \Rightarrow$

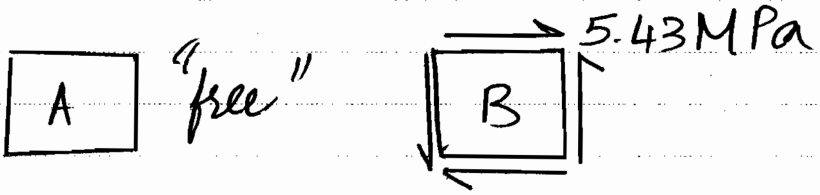
$\boxed{\tau_A = 0}$

$$Q_z^B = (A\bar{y})_B = 20(50)\left(118.75 - \frac{50}{2}\right) = 93750\ mm^3$$

$$\tau_B = \left|\frac{V_y Q_z}{I_z t_z}\right| = \frac{40(10)^{-3}(9.375)(10)^{-5}}{3.453125(10)^{-5}(20)(10)^{-3}}$$

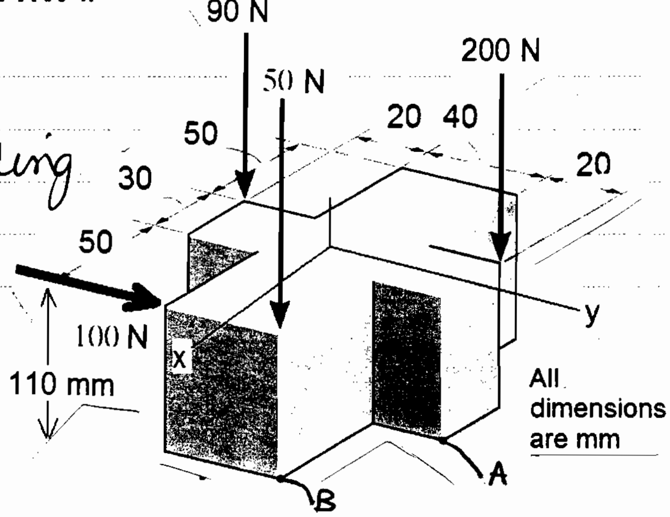
$$\boxed{\tau_B = 5.43\ MPa}$$

The states of stress at A and B are shown.



Problem #3:-

Given:-
The solid block with the loading shown.

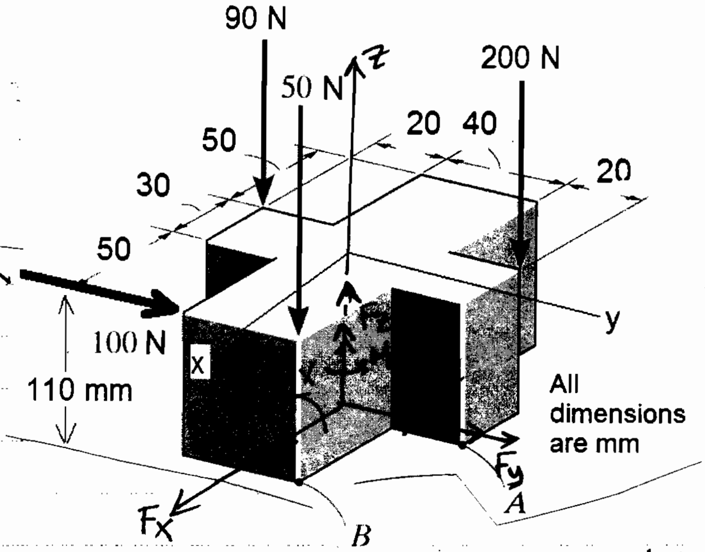


Required:
σ @ A and B; show them on differential elements.

Solution:-

$$\sigma_z = \pm \frac{F_z}{A_z} \pm \frac{M_x y}{I_x} \pm \frac{M_y X}{I_y}$$

First, we need to determine the internal forces at the bottom of the block (through A and B), as shown (FBD).



$\overset{+}{\swarrow} \Sigma F_x = 0 \Rightarrow F_x = 0$

$\searrow^{+} \Sigma F_y = 0 \Rightarrow F_y = -100\ N = 100 N \nwarrow$

$+\uparrow \Sigma F_z = 0 \Rightarrow F_z = 200 + 90 + 50$
$= 340\ N \uparrow$ "C"

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Since, only normal stresses are required, no need to calculate M_z as it is "T" which causes shear stress only. $\Rightarrow \Sigma M_x = 0 \Rightarrow M_x + \Sigma M_{\text{all forces}} = 0$

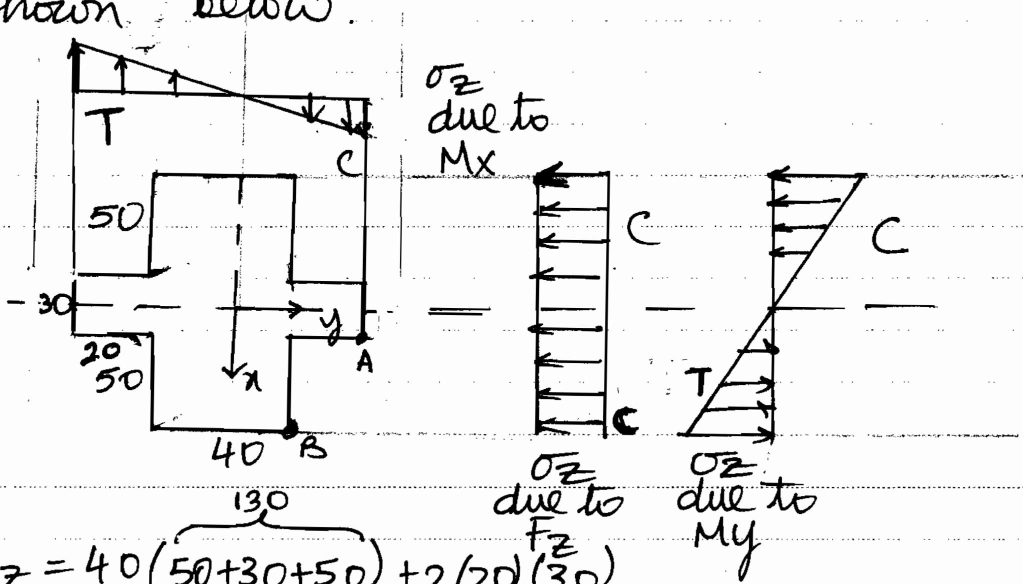
$$\Rightarrow M_x = -\Sigma\left[(r_y F_z) - (r_z F_y)\right]$$

$$= -\left[(20 + \frac{40}{2})(-200) + \left\{(-(20 + \frac{40}{2})(-90)\right\} + \frac{40}{2}(-50) - (110)(100)\right]$$

$$= 16.4\ N.m$$

$$M_y = -\Sigma - \left[(r_x F_z - (r_z \cancel{F_x})^0 \right]$$
$$= \left[\left(-\frac{30}{2}\right)(-200) + \left(\frac{-30}{2}\right)(-90) + \left(50+\frac{30}{2}\right)(-50) \right]$$
$$= 1.1 \text{ Nm}.$$

The stress distributions due to F_z, M_x and M_y are shown below.



$$A_z = 40\overbrace{(50+30+50)}^{130} + 2(20)(30)$$
$$= 6400 \text{ mm}^2$$

$$|\sigma_z|_{\text{due to } F_z}^{\text{all section}} = \frac{F_z}{A_z} = \frac{340}{6400(10)^{-6}} = 531.25 \text{ kPa "C"}.$$

$$I_x = \frac{1}{12}\left[30(80)^3 + 2(50)(40)^3\right] = 1.81333(10)^{-6} \text{ m}^4.$$

$$I_y = \frac{1}{12}\left[40(130)^3 + 2(20)(30)^3\right] = 7.41333(10)^{-6} \text{ m}^4.$$

$$|\sigma_z|_{\text{due to } M_x}^{A} = \frac{M_x y}{I_x} = \frac{16.4(40)(10)^{-3}}{1.8133\times(10)^{-6}} = 361.77 \text{ kPa "C"}.$$

$$|\sigma_z|_{\text{due to } M_y}^{A} = \frac{M_y X}{I_y} = \frac{1.1(15)(10)^{-3}}{7.4133(10)^{-6}} = 2.2257 \text{ kPa "T"}.$$

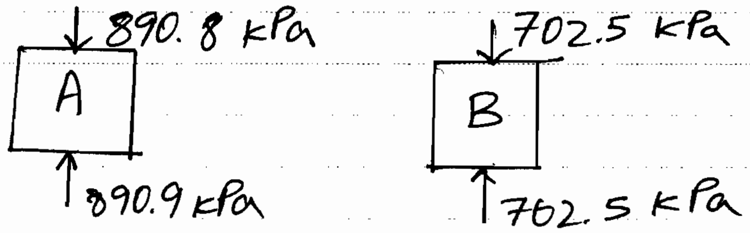
$$\Rightarrow \sigma_z^A = -531.25 - 361.77 + 2.2257 \Rightarrow \boxed{\sigma_z^A = -890.8 = 890.8 \text{ kPa "C"}}$$

$$|\sigma_z|^B_{\text{due to } M_x} = \frac{M_x\, y}{I_x} = \frac{16.4\,(20)\,(10)^{-3}}{1.8133\,(10)^{-6}} = 180.88 \text{ kPa ``C''}$$

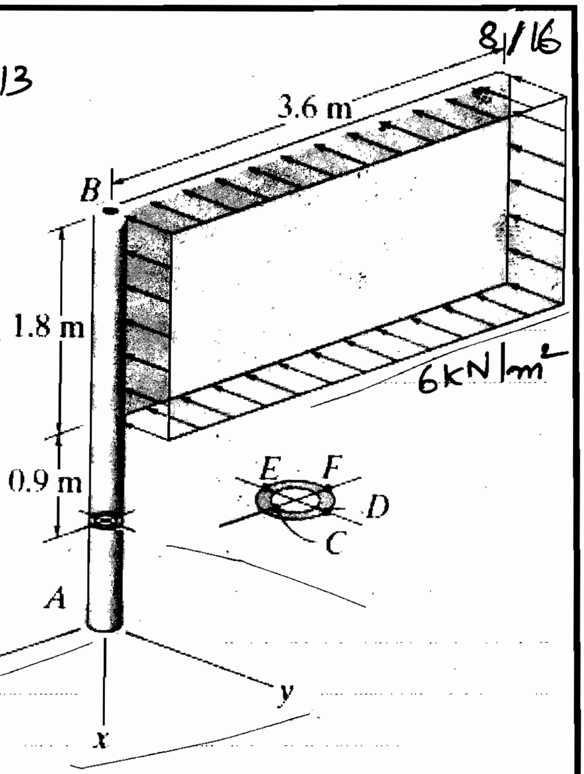
$$|\sigma_z|^B_{\text{due to } M_y} = \frac{M_y\, X}{I_y} = \frac{1.1\,(65)(10)^{-3}}{7.4133\,(10)^{-6}} = 9.6448 \text{ kPa ``T''}$$

$$\sigma_z^B = -531.25 - 180.88 + 9.6448$$

$$\boxed{\sigma_z^B = -702.5 = 702.5 \text{ kPa ``C''}}$$



"shear stress not shown".



Problem # 4 :-

Given :

The figure shown; $W_{sign} = 10\,kN$.

$r_{AB}^{in} = 68\,mm$; $r_{AB}^{out} = 75\,mm$.

Required :

The state of stress at C and D

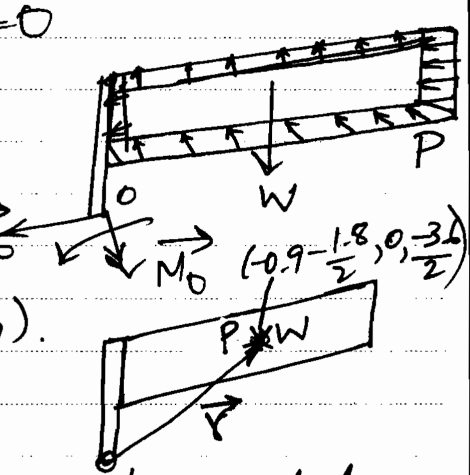
Show them on differential elements.

Solution :

$$\sigma_x = \sigma_{\text{due to } F_x} + \sigma_{\text{due to } M_y} + \sigma_{\text{due to } M_z}$$

$$\vec{\tau} = \vec{\tau}_{\text{due to } F_y} + \vec{\tau}_{\text{due to } F_z} + \vec{\tau}_{\text{due to } M_x (T)}.$$

We need to find the internal forces first.



$$\Sigma \vec{F} = 0 \Rightarrow \vec{F_O} + 10\vec{i} - 6(3.6)(1.8)\vec{j} = 0$$

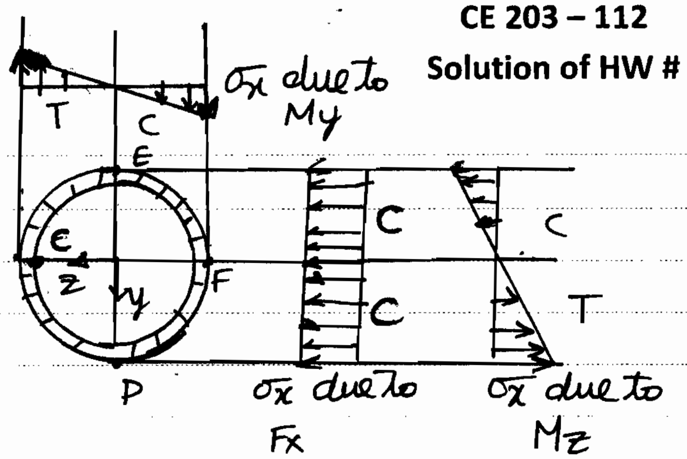
$$\Rightarrow \vec{F_O} = -10\vec{i} + 38.88\vec{j}\ kN$$

$$\Sigma \vec{M} = \vec{0} \Rightarrow \vec{M_O} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1.8 & 0 & -1.8 \\ 10 & -38.88 & 0 \end{vmatrix} = 0$$

$$\vec{M}_O = 69.984\vec{i} + 18\vec{j} - 69.984\vec{k}\ (kN.m).$$

Normal Stress :-

Thus, the stress distributions are as shown below at the section of interest.



$$A_x = \pi\left[(75)^2-(68)^2\right] = 1001\pi \approx 3144.73\ mm^2$$

$$I_y = I_z = \frac{\pi}{4}\left[(75)^4-(68)^4\right] = 8.0576(10)^6\ mm^4$$

$$\sigma_C = \frac{-10(10)^3}{3144.73(10)^{-6}} + 0 + \frac{18(10)^3(68)(10)^{-3}}{8.0576(10)^{-6}}$$

$$= -3.1799 + 151.91$$

$$\boxed{\sigma_C = 148.7\ MPa\ \text{"T"}}$$

$$\sigma_D = \frac{-10(10)^3}{3144.73(10)^{-6}} + \frac{69.984(10)^3(75)(10)^{-3}}{8.0576(10)^{-6}}$$

$$= -3.1799 + 651.22$$

$$\boxed{\sigma_D = 648.2\ MPa\ \text{"T"}}$$

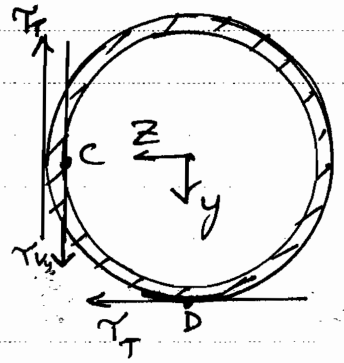
Note that $\sigma_{\text{due to bending}} >> \sigma_{\text{due to axial}}$.

Similar procedures can be followed for any other points (eg. E and F). Once you have drawn the correct stress distribution, the problem becomes easy.

Solution of HW # 13

<u>Shear stress :-</u>

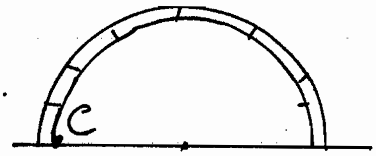
$V_y = 38.88\ kN$; $V_z = \underline{0}$; $T = M_x = 69.984\ kNm$.



$$\tau_{xy}^{C} = \tau_{V_y} - \tau_T = \frac{V_y Q_z}{I_z t_z} - \frac{M_x r}{J_0}$$

$J_0 = 2 I_x$; $t_z = 2(t) = 2(75-68) = 14\ mm$

$Q_z^C = (A\bar{y})_{\text{area of interest}} = Q_{total} - Q_{hole}$



$$= \frac{\pi}{2}(75)^2\left(\frac{4 \times 75}{3\pi}\right) - \frac{\pi}{2}(68)^2\left(\frac{4 \times 68}{3\pi}\right)$$

$$= 71,628.7\ mm^3$$

$$\tau_{xy}^{C} = \frac{38.88(10)^3(71,628.7)(10)^{-9}}{8.0576(10)^{-6}(14)(10)^{-3}} - \frac{69.984(10)^3(68)(10)^{-3}}{2(8.0576)(10)^{-6}}$$

$\Rightarrow \boxed{\tau_{xy}^{C} = -270.6\ MPa}$; $\boxed{\tau_{xz}^{C} = 0}$ (How?!)

$Q_z^D = 0$ (why?!) $\Rightarrow$

$$\tau_{xz}^{D} = \frac{69.984(10)^3(75)(10)^{-3}}{2(8.0576)(10)^{-6}} \Rightarrow$$

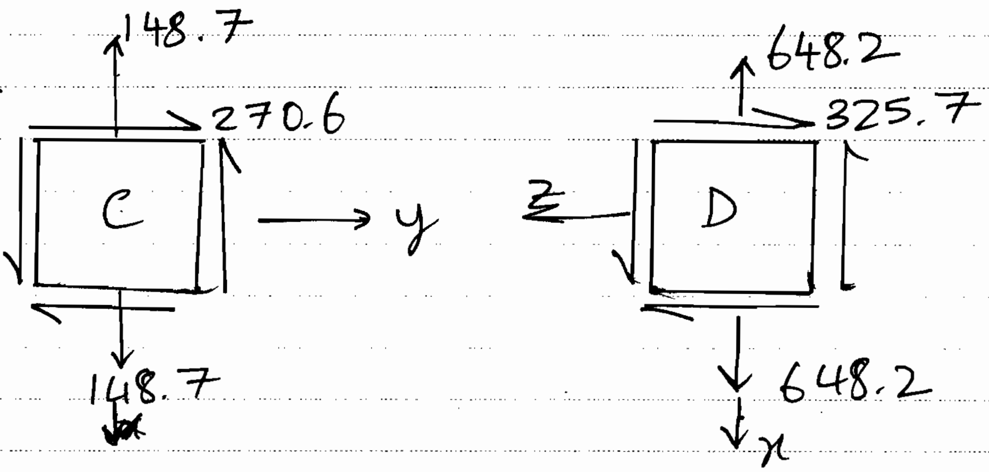
$$\boxed{\tau_{xz}^{D} = 325.7\ MPa}$$

$$\tau_{xy}^{D} = \frac{V_y \cancel{Q_z}^{\,0}}{I_z t_z} \Rightarrow \boxed{\tau_{xy}^{D} = 0}$$

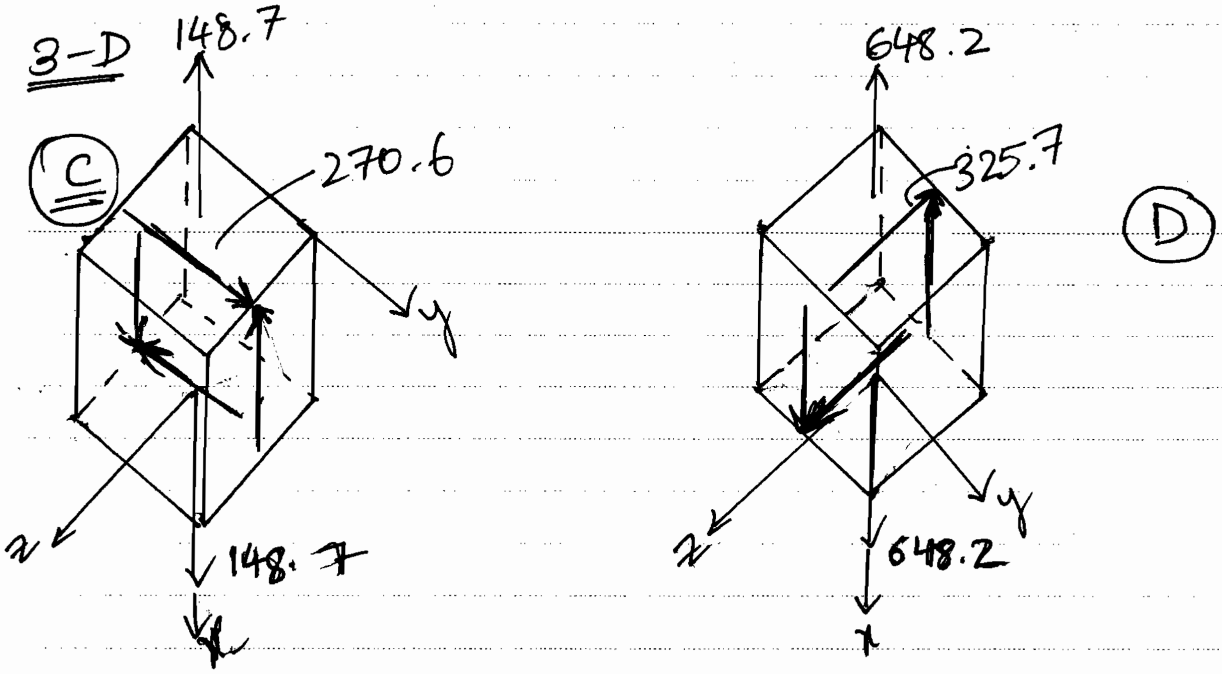
Solution of HW # 13

The states of stress at C and D are shown below. All MPa.

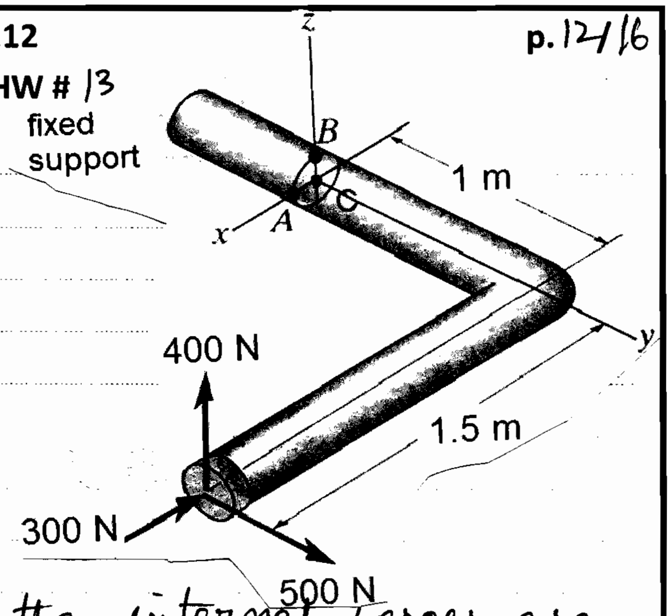
2-D



3-D



Solution of HW # 13



Problem # 5

Given:-

The figure shown

$D = 80\,mm \Rightarrow r = 40\,mm.$

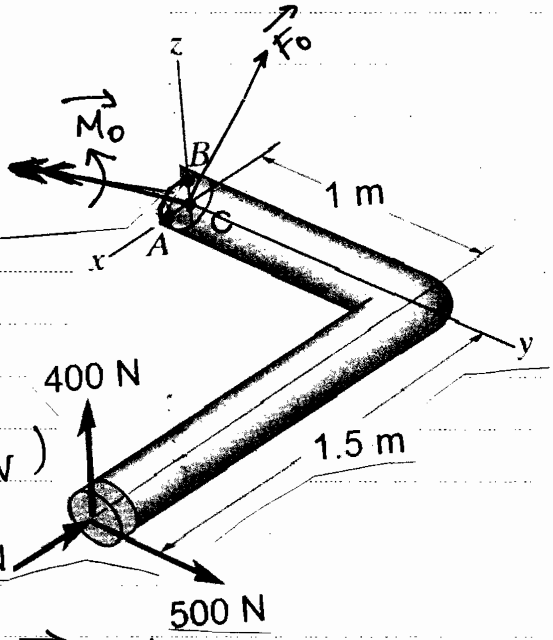
Required:-

States of stress at A and C. Show them on differential elements

Solution:-

First, the FBD is drawn and the internal forces are calculated

$\sum \vec{F} = \vec{0}$



$\vec{F_o} - 300\vec{i} + 500\vec{j} + 400\vec{k} = 0.$

$\vec{F_o} = 300 i - 500\vec{j} - 400\vec{k}\ (N).$

$\sum \vec{M_o} = 0 \Rightarrow \vec{M_o} + \vec{r} \times \vec{F}_{applied} = 0.$

$\vec{F}_{applied} = -300\vec{i} + 500\vec{j} + 400\vec{k}\ (N)$

$\vec{r} = (1.5\vec{i} + 1\vec{j} + 0\vec{k})$

$$\vec{M_o} = -\vec{r} \times \vec{F}_{applied} = -\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.5 & 1 & 0 \\ -300 & 500 & 400 \end{vmatrix}$$

$$= -400\vec{i} + 600\vec{j} - 1050\vec{k}\ (Nm)$$

$\sigma_y = \pm \dfrac{F_y}{A_y}$ due to axial load (F_y).

$\sigma_y = \pm \dfrac{M_x z}{I_x}$ due to bending moment (M_x)

Solution of HW # 13

$\sigma_y = \pm \dfrac{M_z x}{I_z}$ due to bending moment (M_z)

$\tau_{ys} = \dfrac{T r}{J_o}$ due to twisting moment ($T = M_y$)

$\tau_{yx} = \dfrac{V_x Q_z}{I_z t_z}$ due to shear force (V_x).

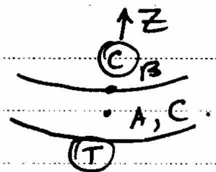
$\tau_{yz} = \dfrac{V_z Q_x}{I_x t_x}$ due to shear force (V_z).

Normal Stress:—

$F_y = 500\ N$ "T"

$A_y = \pi r^2 = \pi (40)^2 = 5026.55\ mm^2 = 5.02655(10)^{-3}\ m^2$.

$M_x = 400\ Nm$



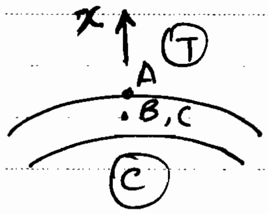
$z_A = z_C = 0$

$I_x = I_z = \dfrac{\pi r^4}{4} = \dfrac{\pi}{4}(40)^4 = 2010619\ mm^4 = 2.01062(10)^{-6}\ m^4$

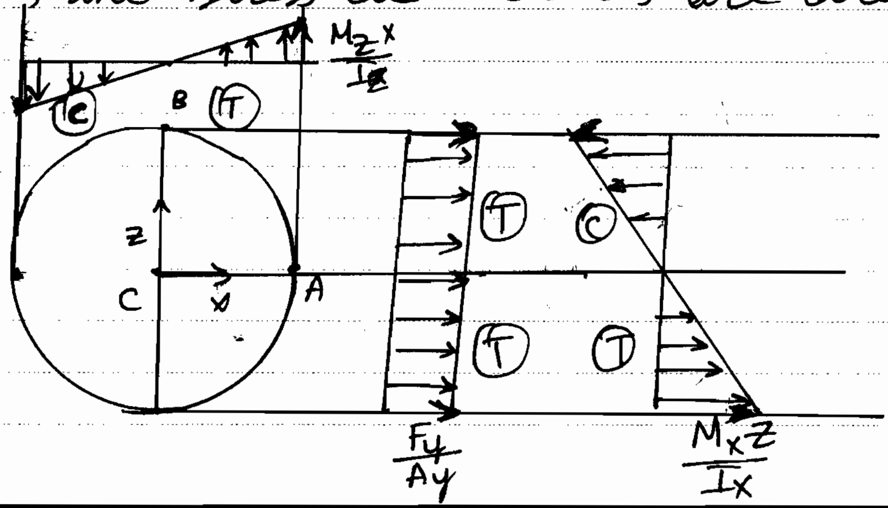
$M_z = 1050\ Nm$

$x_A = r = 40\ mm = 0.04\ m$

$x_C = 0$



Thus, the stress distributions are drawn.



$$\sigma_A = \frac{500}{5.02655(10)^{-3}} + 0 + \frac{1050(0.04)}{2.01062(10)^{-6}} = 0.09947 + 20.89 \Rightarrow$$

$$\boxed{\sigma_A = 20.99\ MPa\ \text{"T"}}$$

Note that σ due to axial load is usually "noticeably smaller" than σ due to bending moment in "typical" applications.

$$\sigma_C = \frac{500}{5.026(10)^{-3}} + 0 + 0 \Rightarrow \boxed{\sigma_C = 99.47\ kPa\ \text{"T"}}$$

Note that it is now easy to find the compound normal stress at any point in the section

Shear stress:-

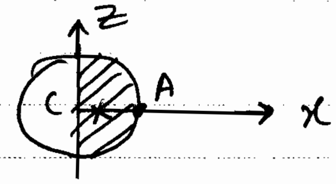
$T = M_y = 600\ Nm.$

$r_A = 0.04\ m$

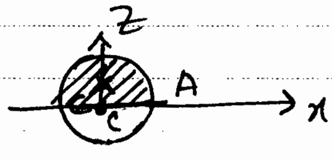
$r_C = 0$

$J_O = \frac{\pi}{2} r^4 = 2I = 4.02124(10)^{-6} m^4.$

$V_x = 300\ N\ ;\ V_z = -400\ N$



$Q_z^A = A\bar{y} = 0(r) = 0$



$Q_z^C = A\bar{y} = \frac{\pi}{2}(0.04)^2 \left(\frac{4(0.04)}{3\ \pi}\right)$

$= 4.26667(10)^{-5} m^3$

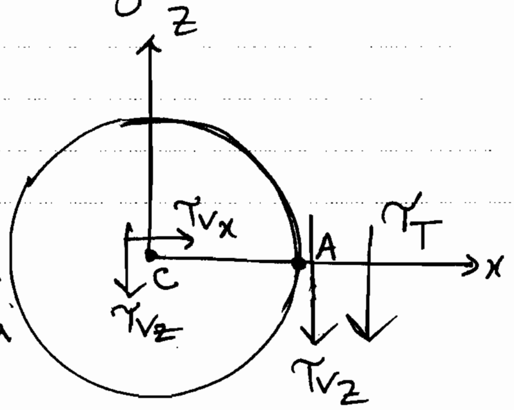
$Q_x^A = Q_x^C = Q_z^C = 4.26667(10)^{-5} m^3$ (How?!)

$t_x^{@A} = 2r = 0.08 m = t_x^C$

$t_z^A = 0$

$t_z^C = 2r = 0.08 m.$

Thus, from above and the drawing shown.



Point A:–

$$\tau_{\text{due to } T} = \frac{Tr}{J} = \tau_{yz}$$

$$= \frac{-600(0.04)}{4.02124(10)^{-6}} \Rightarrow -5.9683\text{MPa}$$

$$\tau_{\text{due to } V_x} = \frac{V_z \cancel{Q_z}^{\,0}}{I_z t_z} = 0$$

$$\tau_{\text{due to } V_z} = \frac{V_z Q_x}{I_x t_x} = \frac{-400(4.26667)(10)^{-5}}{2.01062(10)^{-6}(0.08)} \Rightarrow$$

$$\tau_{yz} = -0.106103 \text{ MPa.}$$

$$\Rightarrow \tau_A = \tau_{yz}^A = 5.9683 + 0.106103 (\downarrow)$$

$$\boxed{\tau^A = \tau_{yz}^A = 6.074 \text{ MPa} (\downarrow).}$$

Point C:–

$$\tau_{\text{due to } T} = \frac{T\cancel{r}^{\,0}}{J} = 0$$

$$\tau_{\text{due to } V_x} = \frac{V_x Q_z}{I_z t_z} = \frac{300(4.26667)(10)^{-5}}{2.01062(10)^{-6}(0.08)}$$

$$= 0.079578 \text{ MPa} = \tau_{yx} (\longrightarrow).$$

$$\tau_{\text{due to } V_z} = \frac{V_z Q_x}{I_x t_x} = \tau^A_{\text{due to } V_z} \text{ (How?!!!)} \Rightarrow$$

$$\tau_{yz} = -0.106103\text{MPa} \downarrow$$

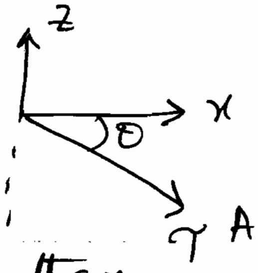
Solution of HW # 13

$$\boxed{\vec{T_c} = 0.07958\,\vec{i} - 0.1061\,\vec{k}\ (\text{MPa})}$$

We can take the resultant (from the forces first or directly from the stress).

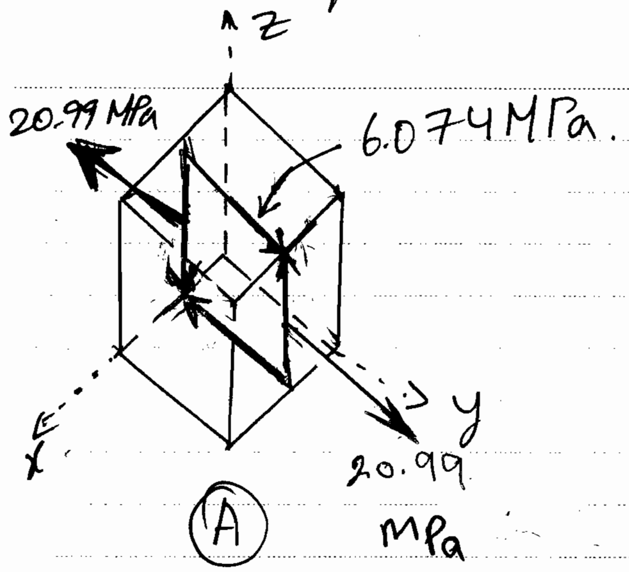
$$\tau = \sqrt{\tau_{yx}^2 + \tau_{yz}^2} \Rightarrow \boxed{\tau^C = 0.1326\ \text{MPa}}$$

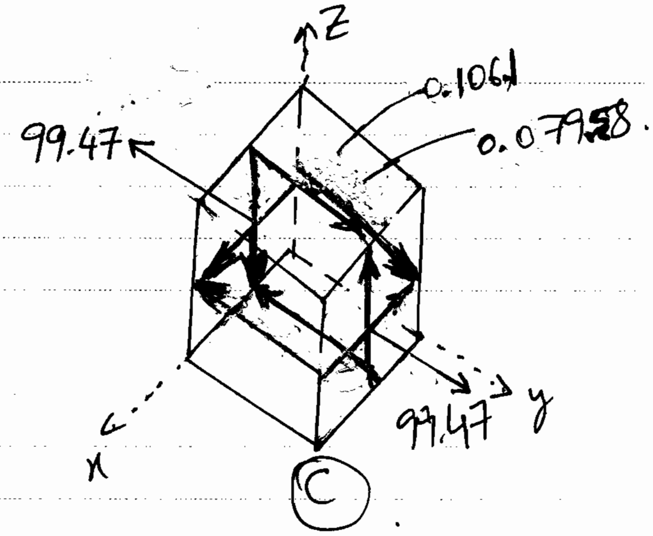
$$\theta = \tan^{-1}\frac{-0.1061}{0.07958} \Rightarrow \boxed{\theta = -53.13^\circ}$$



Note that shear stresses are usually lower than normal stresses.

The states of stress are shown below.





(kPa)