Problem #1:

Given:
The beam cross-section shown
a) \( R_n = 6 \text{ kN} \); \( S = 80 \text{ mm} \)
b) \( \gamma_{\text{allow}} = 2 \text{ MPa} \)

Required:
\( V_{\text{max}} \) for a) and b) above

Solution:
a) \( q = \frac{VQ}{I} \) and \( R_n = q \cdot S \) (How?!!)

Thus, we need first to locate the centroid and calculate \( I \).

\[
\bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = \frac{25(240)(150 + \frac{25}{2}) + 2(20)(150)(\frac{150}{2})}{25(240) + 2(20)(150)}
\]

\[
= 118.75 \text{ mm} = 0.11875 \text{ m}
\]

\[
\bar{I} = \frac{\sum (I_i + A_i d^2)}{\sum A_i} = \left[ \frac{1}{12} (240) (25)^3 + 240(25)(56.25 - \frac{25}{2}) \right]
\]

\[
+ 2 \left[ \frac{1}{12} (20) (150)^3 + 20(150)(\frac{150}{2} - 118.75)^2 \right]
\]

\[
= 34531 \text{ mm}^4 = 3.4531 \times 10^{-5} \text{ m}^4
\]
Q = \text{(Area of interest)} = 240 \times 25 \times (43.75) \\
= 262500 \text{ mm}^3 \\
= 2.625 \times 10^{-4} \text{ m}^3 \\

q = \frac{VQ}{I} = \frac{V(2.625)(10)^{-4}}{3.453125(10)^{-5}} = 7.60181V \\

R_n = q \delta 
\Rightarrow 2(6)(10)^3 = (7.60181\max)(80)(10)^{-3} \\
\Rightarrow \max = 19.73 \text{ KN} \\

b) \quad \gamma = \frac{VQ}{IE} \\
\gamma_{\max} = \frac{2(10)^6}{3.453125(10)^{-5}(20+20)(10)^{-3}} \\
\Rightarrow \max = 10.52 \text{ KN} \\

Note that Q is the same as in part (a). (Why ?!!)
Problem #2:

Given:
The beam cross-section shown: $V = 3\text{kN}$

a) $S_{\text{vertical nails}} = 50\text{mm}$

b) $S_{\text{horizontal nails}} = 55\text{mm}$

Required:
$R_n$ (nail capacity) for a) and b)

Solution:
As usual (!), we need first to locate the centroid and calculate the moment of inertia ($I$).

$$
\bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = \frac{120(20)(10) + 2(20)(120)(\frac{120}{2}) + 240(20)(120 + 10)}{120(20) + 2(20)(120) + 240(20)}
$$

$$
= \frac{78}{120} \text{mm} \quad \text{(Reasonable?!)}
$$

$$
\bar{I} = \sum (I + Ad^2)
$$

$$
= \left[\frac{1}{12}(120)(20)^3 + 120(20)(78 - 10)^2\right] + 2\left[\frac{1}{12}(20)(120)^3 + 20(120)(60 - 78)^2\right] + \left[\frac{1}{12}(240)(20)^3 + 240(20)(120 + 10 - 78)^2\right]
$$

$$
= 21,312,000 \text{ mm}^4 = 2.1312 \times 10^{-5} \text{ m}^4
$$
a) \( R_n = \text{capacity of each nail} = q s \)

\[ q = \frac{VQ}{I} \Rightarrow \]

\[ 2R_n = \frac{VQs}{I} \Rightarrow R_n = \frac{VQs}{2I} \]

\[ \text{two nails (vertical)} \]

\[ Q = (A \bar{y})_{\text{area of interest}} = 240 \times (20)(120+10-78) \]

\[ = 249,600 \text{ mm}^3 = 2.496 \times 10^7 \text{ m}^3 \]

\[ R_n = \frac{3 \times (10)^3 \times (2.496) \times (10)^4 \times (50)(10)^3}{2 \times (2.1312)(10)^5} \Rightarrow R_n = 878.4 \text{ N} \]

b) as in (a), \( R_n = \frac{VQs}{2I} \)

Note that there are two horizontal nails also

\[ Q = (A \bar{y})_{\text{area of interest}} = 120 \times (20)(78-10) \]

\[ = 163,200 \text{ mm}^3 = 1.632 \times 10^7 \text{ m}^3 \]

\[ R_n = \frac{3 \times (10)^3 \times (1.632) \times (10)^4 \times (55)(10)^3}{2 \times (2.1312)(10)^5} \Rightarrow R_n = 631.8 \text{ N} \]
**Problem #3**

**Given:**
The two figures shown.
- \( P = 0.5 \text{ MPa} \); \( t = 5 \text{ mm} \);
- \( D = 250 \text{ mm} \)

**Required:**
The state of stress in the cylinder wall in cases (a) and (b)

**Solution**

**Case (a)**

Note that in case (a), there is a circumferential (hoop) stress, but no longitudinal (axial) stress as it is "free to expand" in the axial (vertical) direction (up).

Thus,

\[
\sigma_c = \sigma_1 = \frac{P r}{t} = \frac{0.5 \times 10^6 \times \left( \frac{250}{2} \right) \times 10^{-3}}{5 \times 10^{-3}}
\]

\[
\Rightarrow \sigma_c = \sigma_1 = 12.5 \text{ MPa}
\]

\[
\sigma_2 = 0
\]
Case (b)

In case (b) there is an axial stress on the cylinder as we can see it from the boundary conditions. However, $\sigma_c$ is as in (a)

\[
\Rightarrow \sigma_c = \sigma_1 = 12.5 \text{ MPa}
\]

\[
\sigma_l = \sigma_2 = \frac{P_c}{2t} \leq \frac{0.5 \left(\frac{250}{2}\right)}{2(5)}
\]

\[
\Rightarrow \sigma_l = \sigma_2 = 6.25 \text{ MPa}
\]

We can also find $\sigma_l$ as we did in the "axially-loaded member" chapter.

For case (a), from the FBD,

\[
\sigma_l = 0
\]

For case (b), from the FBD,

\[
N_{total} = R_{total}
\]

and $R_{total} = F$, and $F = PA_{pressure}$

Thus, $N = R = F = \text{Pressure} \times \text{Area}$
\[ N = 0.5 (10)^6 \left[ \pi \left( \frac{250}{2} \right)^2 \right] (10)^{-6} \]

\[ N = 7812.5 \pi (N) \]

\[ \sigma_{axial} = \sigma_l = \sigma_z = \frac{N}{A} \]

\[ A = \text{Area of material} \]

\[ A = \pi D (t) = 1250 \pi \text{ mm}^2 \]

\[ \Rightarrow \sigma_l = \sigma_z = \frac{7812.5 \pi}{1250 \pi (10)^{-6}} = 625 \text{ MPa} \]

(as above)
Problem #4:

Given:
The two vessels shown (cylinder and sphere);
P = 0.1 MPa

Required:
a) \(V_{cyl} = V_{sphere}\)
b) \(\sigma_i\) and \(\sigma_c\) in cylinder
c) \(\sigma_{max}\) in sphere

Solution:
a) \(V_{sphere} = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (0.6)^3 = 0.288 \pi \ m^3\)

\(V_{cyl\ total} = \frac{4}{3} \pi (0.4)^3 + \pi r^2 L = V_{sphere}\)

the two "hemispheres" form a sphere!

\(\Rightarrow 0.288 \pi = \left(\frac{4}{3} (0.4)^3 + (0.4)^2 L\right) \pi\)

\(\Rightarrow L = 1.2667 \ m\)
b) $\sigma_c = \frac{P_r}{t} = \frac{0.1 \times 0.4 \times 5}{(10)^{-3}} \Rightarrow \sigma_c = 8 \text{ MPa}$

$\sigma_t = \frac{1}{2} \sigma_c \Rightarrow \sigma_t = 4 \text{ MPa}$

$c) \sigma_{\text{max}} = \sigma_c = \frac{P_r}{2t} = \frac{0.1 \times 0.6}{2 \times 5 \times (10)^{-3}} \Rightarrow \sigma_c = 6 \text{ MPa}$

Note that with the same radius for a cylinder and sphere,

$\sigma_{\text{max}}^{\text{sphere}} = 50\% \text{ of } \sigma_{\text{max}}^{\text{cyl.}}$

For the same volume of sphere and cylinder (as in our case),

$\sigma_{\text{max}}^{\text{sph.}} = \frac{6}{8} \times (100) \sigma_{\text{max}}^{\text{sph.}} = 75\% \text{ of } \sigma_{\text{max}}^{\text{cyl.}}$

Thus, the sphere (compared with the cylinder) is more efficient, "looks nicer," has no stress concentration, etc. However, if $r$ gets large, it does not become practical if it is going to be "transported." But, as a stationary storage tank/vessel, it is excellent.