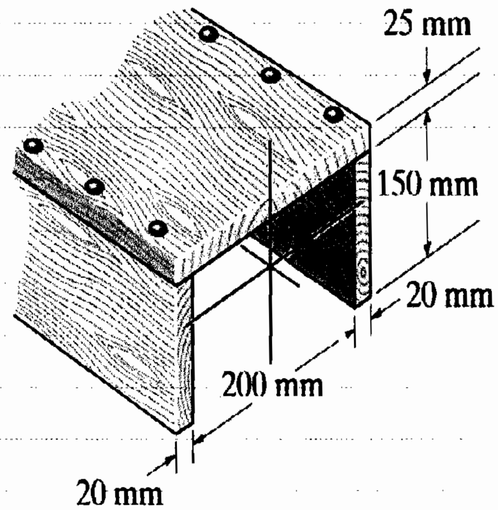
## Problem #1:

**Given:**

The beam crosssection shown

a) $R_n = 6$ KN; $S = 80$ mm

b) $\tau_{allow}^{Glue} = 2$ MPa




**Required:**

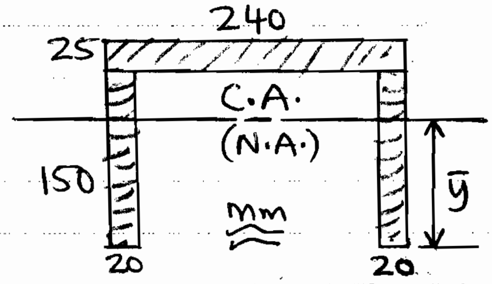
$V_{max}$ for a) and b) above

**Solution:**

a) $q = \dfrac{VQ}{I}$ and $R_n = q\,s$ (How?!)

Thus, we need first to locate the centroid and calculate I.

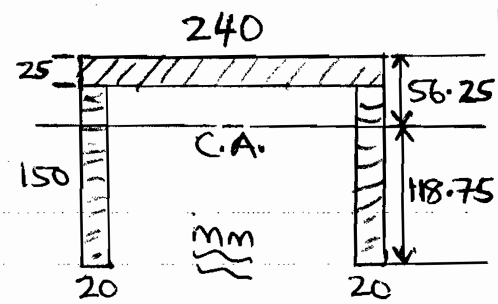
$$\bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i}$$




$$= \frac{25(240)(150+\frac{25}{2}) + 2(20)(150)(\frac{150}{2})}{25(240) + 2(20)(150)}$$

$$= 118.75 \text{ mm} = 0.11875 \text{ m}$$

$$\bar{I} = \sum (\bar{I} + Ad^2)_i$$

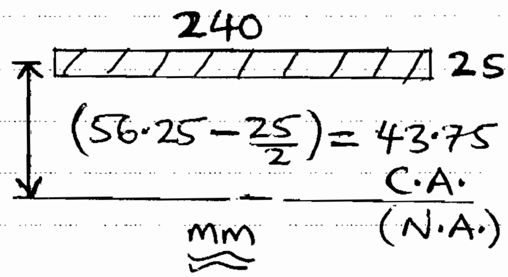



$$= \left[\tfrac{1}{12}(240)(25)^3 + 240(25)(56.25-\tfrac{25}{2})^2\right]$$

$$+ 2\left[\tfrac{1}{12}(20)(150)^3 + 20(150)(\tfrac{150}{2} - 118.75)^2\right]$$

$$= 34{,}531{,}250 \text{ mm}^4 = 3.453125(10)^{-5} \text{ m}^4$$

$$Q = (A\bar{y})_{\text{area of interest}}$$
$$= 240(25)(43.75)$$
$$= 262{,}500 \text{ mm}^3$$
$$= 2.625(10)^{-4} \text{ m}^3$$




$$q = \frac{VQ}{I} = \frac{V(2.625)(10)^{-4}}{3.453125(10)^{-5}} = 7.60181V$$

$$R_n = qs \Rightarrow \underbrace{2}_{\text{two nails}}(6)(10)^3 = (7.60181 V_{max})(80)(10)^{-3}$$

$$\Rightarrow \boxed{V_{max} = 19.73 \text{ kN}}$$

b) $$\tau = \frac{VQ}{It}$$

$$\tau_{max}^{Glue} = 2(10)^6 = \frac{V_{max}(2.625)(10)^{-4}}{3.453125(10)^{-5}(20+20)(10)^{-3}}$$

$$\Rightarrow \boxed{V_{max} = 10.52 \text{ kN}}$$

Note that Q is the same as in part (a). (Why?!)

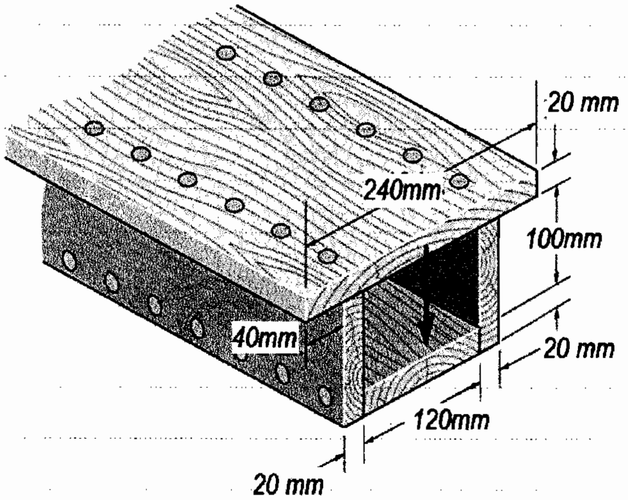
## Problem #2:

**Given:**

The beam cross-section shown; $V = 3\,kN$

a) $S_{\text{vertical nails}} = 50\,mm$

b) $S_{\text{horizontal nails}} = 55\,mm$

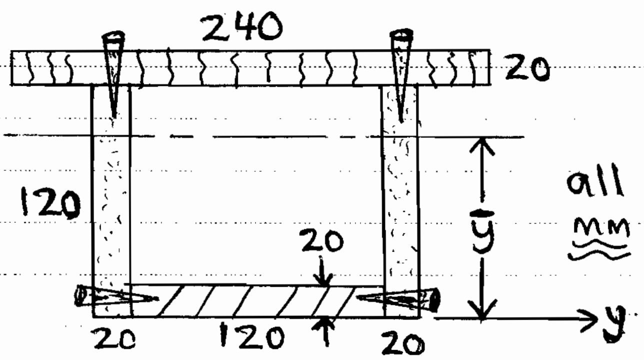



**Required:**

$R_n$ (nail capacity) for a) and b)

**Solution:**

As usual (!), we need first to locate the centroid and calculate the moment of inertia ($I$).




$$\bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = \frac{120(20)(10) + 2(20)(120)\left(\frac{120}{2}\right) + 240(20)(120+10)}{120(20) + 2(20)(120) + 240(20)}$$

$$= 78\,mm \qquad \text{(Reasonable ?!)}$$

$$\bar{I} = \sum (\bar{I} + Ad^2)_i$$

$$= \left[\tfrac{1}{12}(120)(20)^3 + 120(20)(78-10)^2\right]$$

$$+ 2\left[\tfrac{1}{12}(20)(120)^3 + 20(120)(60-78)^2\right]$$

$$+ \left[\tfrac{1}{12}(240)(20)^3 + 240(20)(120+10-78)^2\right]$$

$$= 21{,}312{,}000\,mm^4 = 2.1312(10)^{-5}\,m^4$$

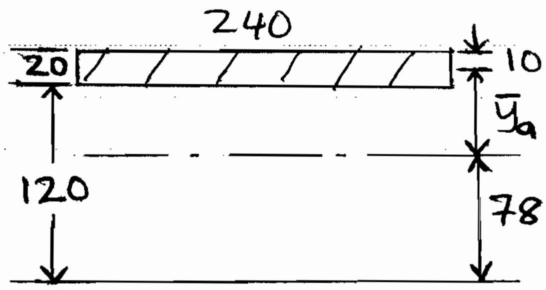
a) $R_n$ = capacity of each nail
$= q s$

$$q = \frac{VQ}{I} \Rightarrow$$

$$2R_n = \frac{VQs}{I} \Rightarrow R_n = \frac{VQs}{2I}$$

two nails (vertical)

$$Q = (A\bar{y})_{\text{area of interest}}$$
$$= 240(20)(120+10-78)$$
$$= 249{,}600 \text{ mm}^3 = 2.496(10)^{-4} \text{ m}^3$$

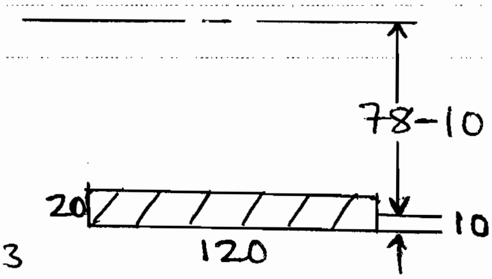



$$R_n = \frac{3(10)^3(2.496)(10)^{-4}(50)(10)^{-3}}{2(2.1312)(10)^{-5}} \Rightarrow \boxed{R_n = 878.4\text{ N}}$$

b) as in (a), $R_n = \dfrac{VQs}{2I}$

Note that there are two horizontal nails also

$$Q = (A\bar{y})_{\text{area of interest}}$$
$$= 120(20)(78-10)$$
$$= 163{,}200 \text{ mm}^3 = 1.632(10)^{-4} \text{ m}^3$$

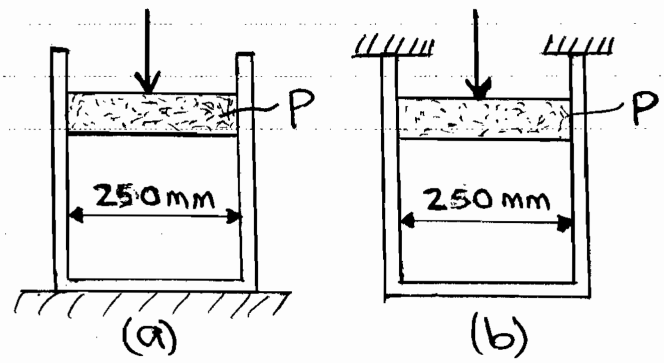



$$R_n = \frac{3(10)^3(1.632)(10)^{-4}(55)(10)^{-3}}{2(2.1312)(10)^{-5}}$$

$$\Rightarrow \boxed{R_n = 631.8\text{ N}}$$

## Problem # 3

Given:
The two figures shown
$P = 0.5\,MPa$; $t = 5\,mm$;
$D = 250\,mm$




Required:
The state of stress in the cylinder wall in cases (a) and (b)

Solution

Case (a)

Note that in case (a), there is a circumferential (hoop) stress, but no longitudinal (axial) stress as it is "free to expand" in the axial (vertical) direction (up).

Thus, $$\sigma_c = \sigma_1 = \frac{Pr}{t} = \frac{0.5(10)^6\left(\frac{250}{2}\right)(10)^{-3}}{5(10)^{-3}}$$

$$\Rightarrow \boxed{\sigma_c = \sigma_1 = 12.5\,MPa}$$

$$\boxed{\sigma_L = \sigma_2 = 0}$$

Case (b)

In case (b) there is an axial stress on the cylinder as we can see it from the boundary conditions. However, $\sigma_c$ is as in (a)

$$\Rightarrow \boxed{\sigma_c = \sigma_1 = 12.5 \text{ MPa}}$$

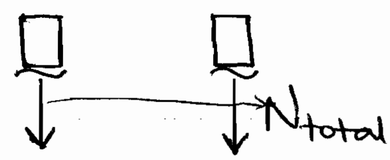
$$\sigma_L = \sigma_2 = \frac{Pr}{2t} = \frac{0.5\left(\frac{250}{2}\right)}{2(5)}$$

$$\Rightarrow \boxed{\sigma_L = \sigma_2 = 6.25 \text{ MPa}}$$

We can also find $\sigma_L$ as we did in the "axially-loaded member" chapter.

For case (a), from the FBD,

$$\boxed{\sigma_L = 0}$$

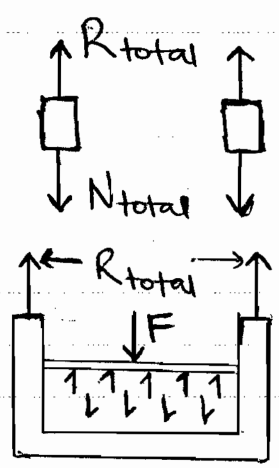



For case (b), from the FBD,

$$N_{total} = R_{total}$$

and $R_{total} = F$, and $F = PA_{pressure}$

Thus, $N = R = F = \text{Pressure (Area)}$



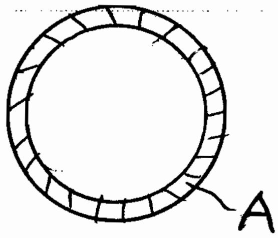
$$\Rightarrow N = 0.5(10)^6 \left[\pi\left(\frac{.250}{2}\right)^2\right](10)^{-6}$$

$$= 7812.5\pi \ (N)$$

$$\sigma_{axial} = \sigma_t = \sigma_2 = \frac{N}{A}$$

$A$ = Area of material

$$\simeq \pi D(t) = 1250\pi \ mm^2$$

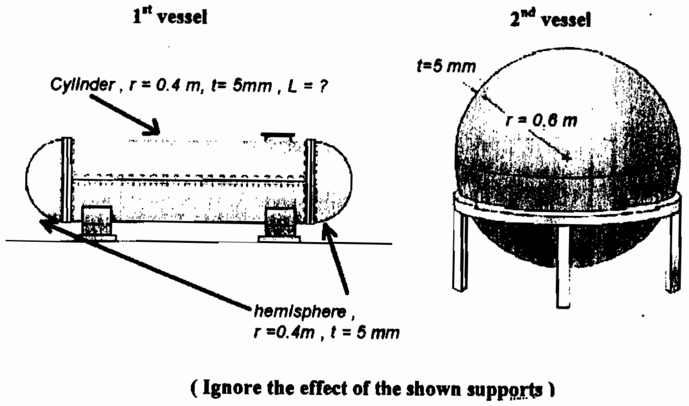



$$\Rightarrow \sigma_t = \sigma_2 = \frac{7812.5\pi}{1250\pi(10)^{-6}} = 6.25 \ MPa$$

(as above)

## Problem #4:

**Given:**
The two vessels shown (cylinder and sphere);
$P = 0.1\,MPa$




( Ignore the effect of the shown supports )

**Required:**
a) $L_{cylinder}$ if $V_{cyl.} = V_{sphere}$
b) $\sigma_L$ and $\sigma_c$ in cylinder
c) $\sigma_{max}$ in sphere

**Solution:**

a) $V_{sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.6)^3 = 0.288\pi\ m^3$

$$V_{cyl.\,total} = \underbrace{\frac{4}{3}\pi(0.4)^3}_{\text{the two "hemispheres" form a sphere!}} + \pi r^2 L = V_{sphere}$$

$$\Rightarrow 0.288\pi \equiv \left(\frac{4}{3}(0.4)^3 + (0.4)^2 L\right)\pi$$

$$\Rightarrow \boxed{L = 1.2667\,m}$$

b) $\sigma_c = \frac{Pr}{t} = \frac{0.1(0.4)}{5(10)^{-3}} \Rightarrow \boxed{\sigma_c = 8 \text{ MPa}}$

$\sigma_L = \frac{1}{2}\sigma_c \Rightarrow \boxed{\sigma_L = 4 \text{ MPa}}$

c) $\sigma_{max} = \sigma_c = \frac{Pr}{2t} = \frac{0.1(0.6)}{2(5)(10)^{-3}}$

$\Rightarrow \boxed{\sigma_c = 6 \text{ MPa}}$

Note that with the same radius for a cylinder and sphere,

$$\sigma_{max}^{sphere} = 50\% \text{ of } \sigma_{max}^{cyl.}$$

For the same Volume of sphere and cylinder ((as in our case)),

$$\sigma_{max}^{sph.} = \frac{6}{8}(100)\,\sigma_{max}^{sph.} = 75\% \text{ of } \sigma_{max}^{cyl.}$$

Thus, the sphere (compared with the cylinder) is more efficient, "looks nicer", has no stress concentration, etc. However, if r gets large, it does not become practical if it is going to be "transported". But, as a stationary storage tank/vessel, it is excellent.