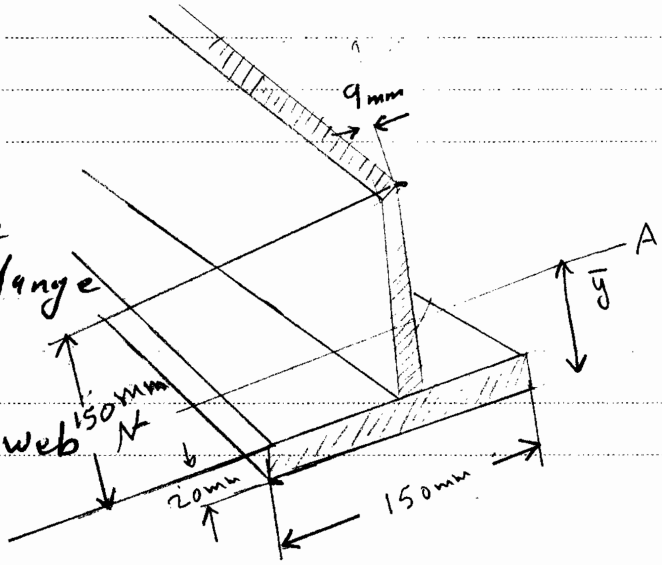
Solution of HW # 11

Problem #1

Given:

The beam cross-section shown

$M = +5\ \text{kN}\cdot\text{m}$



Required:

a) $\sigma_{\text{top and bottom}}$ of flange
Resultant force on the flange

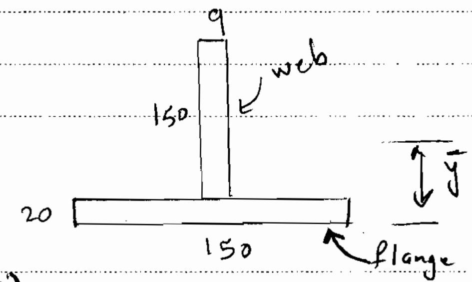
b) $\sigma_{\text{top and bottom}}$ of web
Resultant force on the web

Solution:

$$\sigma = -\frac{My}{I}$$

Thus, first, we need to locate the centroid and calculate I for the cross-section

$$\bar{y} = \frac{\sum \bar{y}_i A_i}{\sum A_i} \Rightarrow$$



$$\bar{y} = \frac{150(20)(10) + 150(9)(20+75)}{150(20) + 150(9)} = 36.3793\ \text{mm}$$

$$\bar{I} = \sum (\bar{I} + Ad^2)_i$$

$$= \left[\frac{1}{12}(150)(20)^3 + 150(20)(36.3793-10)^2\right] + \left[\frac{1}{12}(9)(150)^3 + 9(150)\left(\frac{150}{2} + 20 - 36.3793\right)^2\right]$$

$$= 9.35797\,(10)^6\ \text{mm}^4 = 9.35797\,(10)^{-6}\ \text{m}^4$$

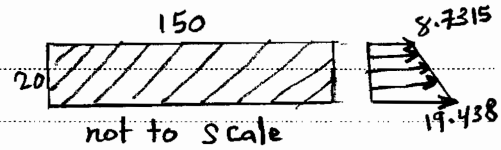
a) $\sigma_{flange-top} = -\dfrac{5(10)^3(-36.3793+20)(10)^{-3}}{9.35797(10)^{-6}} \Rightarrow$

$$\boxed{\sigma_{flange-top} = 8.7515\ MPa \quad \text{"T"}}$$

$\sigma_{flange-bottom} = -\dfrac{5(10)^3(-36.3793)(10)^{-3}}{9.35797(10)^{-6}} \Rightarrow$

$$\boxed{\sigma_{flange-bottom} = 19.438\ MPa \quad \text{"T"}}$$

$F_{flange} = \bar{\sigma} A$ (How ?!)



$= \dfrac{8.7515 + 19.438}{2}(10)^6(150)(20)(10)^{-6} \Rightarrow$

$$\boxed{F_{flange} = 42.28\ kN \quad \text{"T"}}$$

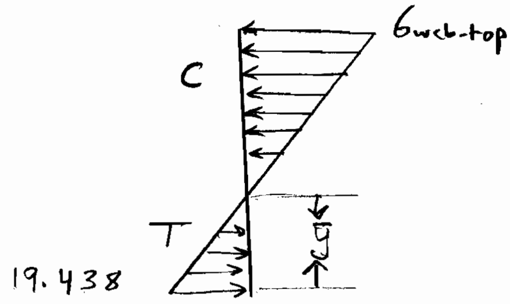
b) $\sigma_{web-top} = -\dfrac{5(10)^3(150+20-36.3793)(10)^{-3}}{9.35797(10)^{-6}} \Rightarrow$

$$\boxed{\sigma_{web-top} = 71.394\ MPa \quad \text{"C"}}$$

OR, similer triangles can be used:

$$\frac{\sigma_{web-top}}{170-\bar{y}} = \frac{19.438}{\bar{y}} \Rightarrow$$

$$\sigma_{web-top} = 71.39 \text{ -- ok}$$



$$\boxed{\sigma_{web-bottom} = \sigma_{flange-top} = 8.7515 \text{ MPa} \quad \text{"T"}}$$

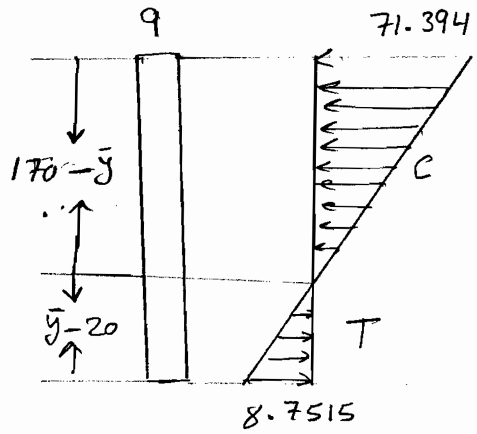
Sure ?!?

$$F_{web} = \bar{\sigma} A$$

$$= \frac{-71.394 + 8.7515}{2} (10)^6 (9)(150)(10)^{-6} \Rightarrow$$

$$\boxed{F_{web} = 42.28 \text{ kN} \quad \text{"C"}}$$

OR, you can separate the "T" from the "C"



$$F_{web} = \frac{1}{2}\,(-71.397)(9)(170-\bar{y}) + \frac{1}{2}\,(+8.7515)(9)(\bar{y}-20) \Rightarrow$$

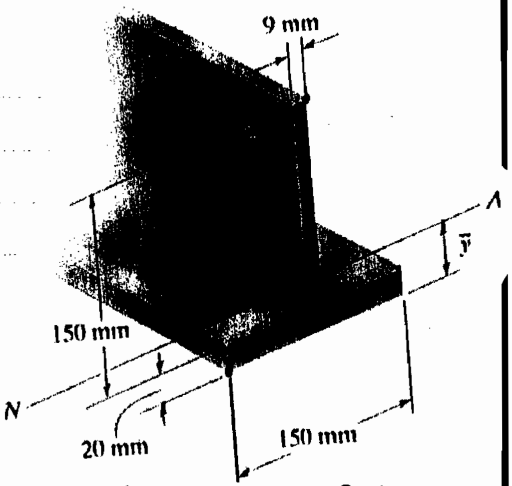
$$F_{web} = -92.28 \quad \text{"as above"}$$

Note that $\Sigma F = F_{flange} + F_{web} = 0$

(Expected ?! Why ?!)

Solution of HW # 11

Problem # 2



Given:

The beam cross-section shown

$V = 80$ kN

Required:

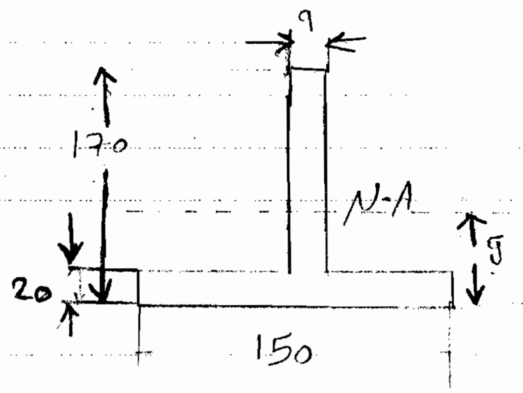
τ @ top, bottom, N.A., bottom of web, top of flange

τ - distribution

Solution:

$$\tau = \frac{VQ}{It} \quad \text{or} \quad \frac{VQ}{Ib}$$

From problem # 1



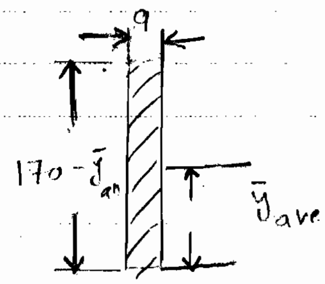
$\bar{y}_{all} = 36.3793$ mm

$\bar{I} = 9.35797 (10)^{-6}$ m^4

$Q = (A\bar{y})_{\text{area of interest}}$

$Q_{top} = 0(170 - \bar{y}_{all}) = 0 \Rightarrow \boxed{\tau_{top} = 0}$

$Q_{bottom} = 0(\bar{y}) = 0 \Rightarrow \boxed{\tau_{bottom} = 0}$



$Q_{N.A.} = [9(170 - \bar{y}_{all})]\left(\frac{170 - \bar{y}_{all}}{2}\right)$

$= 80,345.2$ mm^3

$\bar{y}_{all} = 36.3793$

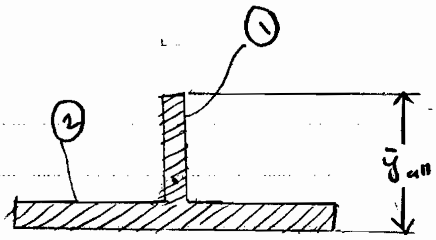
$\bar{y}_{ave} = \frac{170 - \bar{y}_{all}}{2}$

Note that the lower part may be taken as shown below. But it is easier to take the upper part

$$Q_{N.A.} = Q_{1} + Q_{2}$$

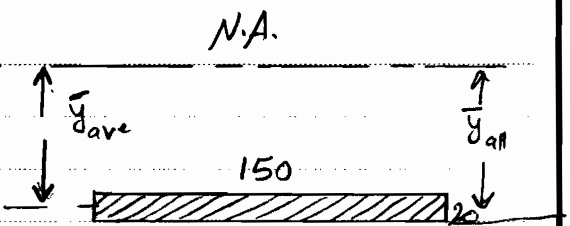
$$= (\bar{y}_{all} - 20)(9)\left(\frac{\bar{y} - 20}{2}\right) + (150 \times 20)(\bar{y}_{all} - 10)$$

$$= 80{,}345.2 \text{ mm}^3 \quad \text{as above}$$



$$\Rightarrow \tau_{N.A.} = \frac{80(10)^3(80{,}345.2)(10)^{-9}}{9.35797(10)^{-6}\underbrace{(9)(10)^{-3}}_{t\,(\text{or } b)}}$$

$$\Rightarrow \boxed{\tau_{N.A.} = 76.32 \ MPa}$$



$$Q_{\text{bottom of web}} = 150(20)(\bar{y}_{all} - 10)$$

$\bar{y}_{ave} = \bar{y}_{all} - 10$

$$= 79{,}137.9 \text{ mm}^3 = Q_{\text{top of flange}} \quad (\text{why ?!})$$

$t_{web} = 9$ mm
$t_{flange} = 150$ mm

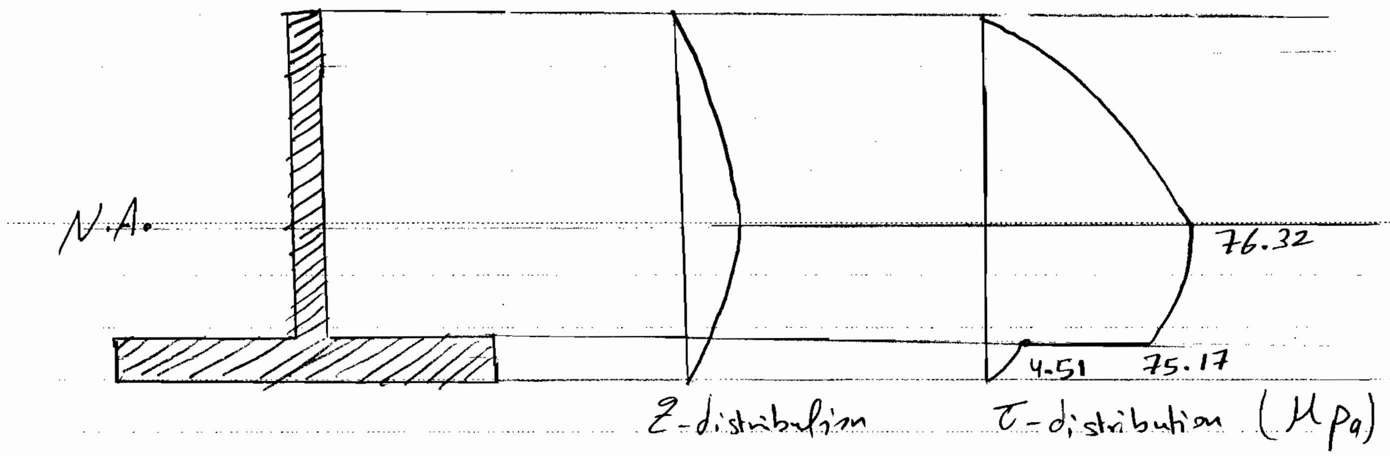
$$\tau_{\text{bottom of web}} = \frac{80(10)^3(79{,}137.9)(10)^{-9}}{9.35797(10)^{-6}(9)(10)^{-3}} \Rightarrow$$

$$\boxed{\tau_{\text{bottom of web}} = 75.17 \ MPa}$$

$$\tau_{top\ of\ flange} = \frac{80(10)^3(79{,}137.9)(10)^{-9}}{9.35797(10)^{-6}(150)(10)^{-3}}$$

$$\boxed{\tau_{top\ of\ flange} = 4.510\ \mu pa} \ll \sigma_{bottom\ of\ web}\ (\text{why?!})$$

τ-distribution is shown below



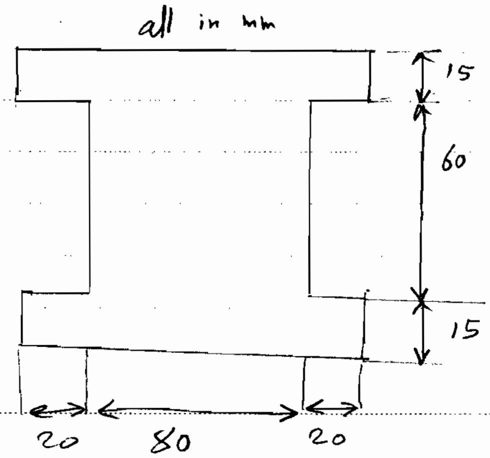
Note: $q = \frac{VQ}{I}$

$\tau = \frac{q}{t}$

Problem #3

Given:
The beam cross-section shown
$\tau_{all} = 40$ Mpa

Required:
The maximum shear force V_{max} that can be supported



Solution:

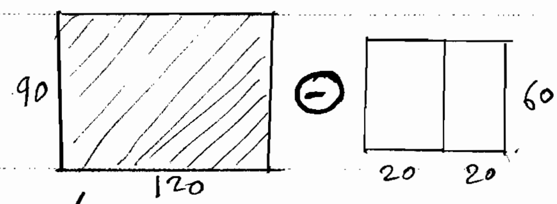
$$\tau = \frac{VQ}{It}$$

τ_{max} @ C.A. [as shown below]

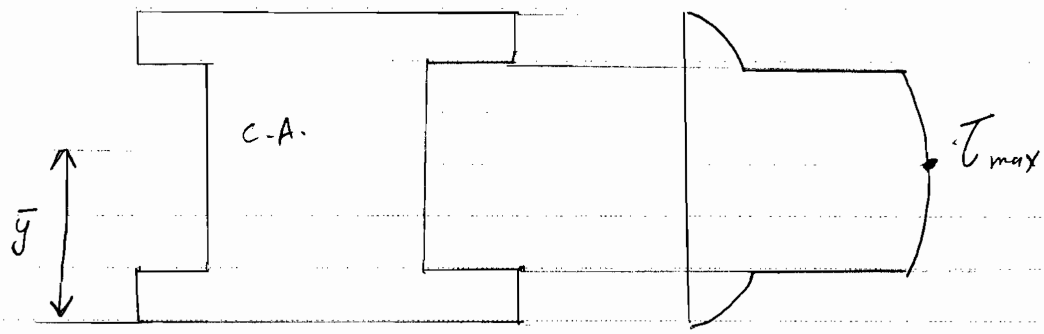
We need $\bar{y}$ and I.

$\bar{y}$ is in the "middle" due to the double symetry.
∴ $\bar{y} = 45$ mm

$$\bar{I} = \frac{1}{12}\left[120\,(90)^3 - 40\,(60)^3\right]$$



$$= 6.57\,(10)^6 \text{ mm}^4 = 6.57\,(10)^{-6} \text{ m}^4$$

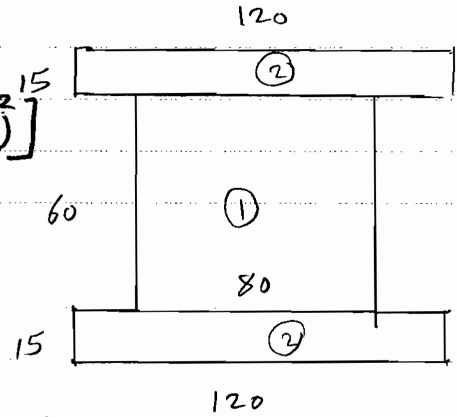


τ- distribution

or (but Longer)

$$\bar{I} = \frac{1}{12}(80)(60)^3 + 2\left[\frac{1}{12}(120)(15)^3 + 120(15)(30+7.5)^2\right]$$

$$= 6.57(10)^6 \text{ mm}^4 \quad \text{as above}$$



we need $Q_{C.A.}$ as τ_{max} @ C.A. (N.A.).

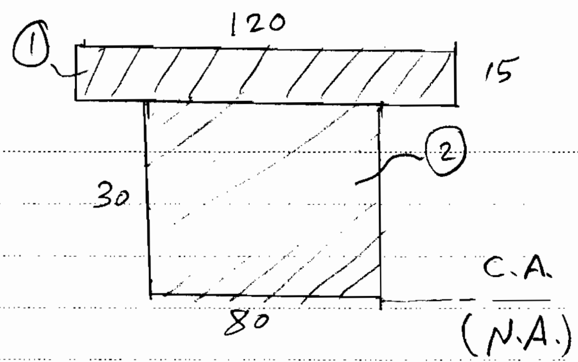
$$Q = A\bar{y}$$

$$Q_{C.A} = Q_{①} + Q_{②}$$

$$= 120(15)(30+7.5)$$

$$+ 80(30)(15)$$

$$= 103{,}500 \text{ mm}^3$$



$t = b = 80$ mm

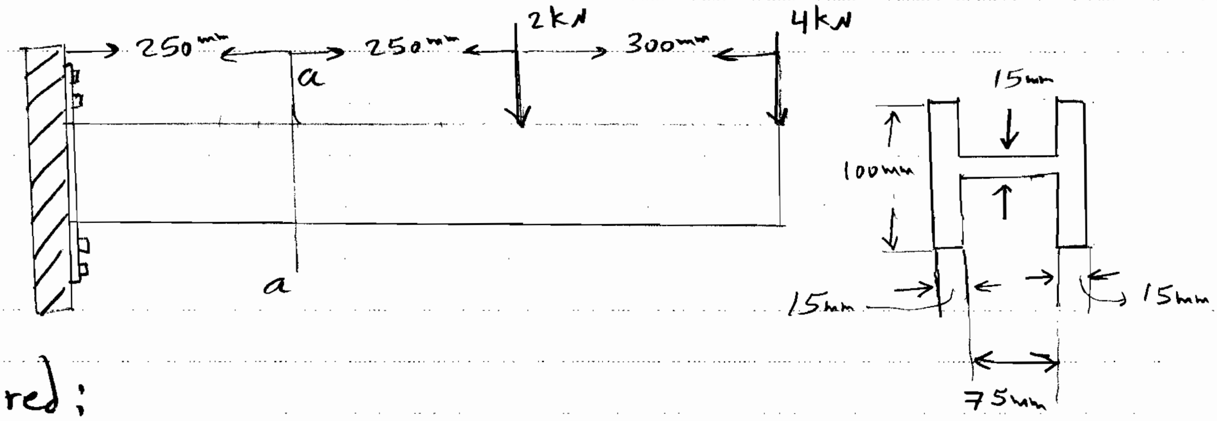
$$\tau_{max} = \frac{V_{max}(103{,}500)\cancel{(10)}^{-9}}{6.57(10)^{-6}(80)\cancel{(10)}^{-3}} \equiv \tau_{allow} = 40(10)^6$$

$$\Rightarrow \boxed{V_{max} = 203.1 \text{ kN}}$$

Problem #4

Given:

The beam with the cross-section shown



Required:

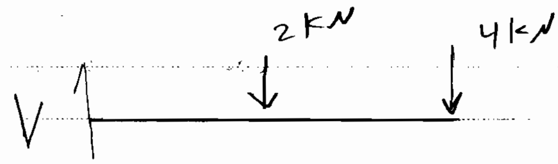
$\tau_{N.A.}$ and τ_{max} @ section a-a

Solution:

$$\tau = \frac{VQ}{Ib}$$

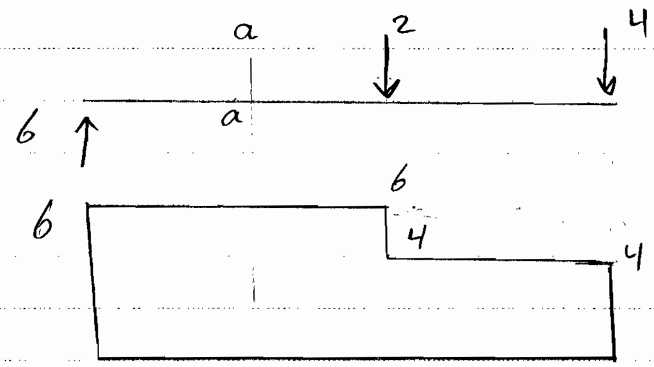
we can get V_{a-a} by either FBD as shown

$+\uparrow \sum F_y = 0 \Rightarrow V = 6\ kN$



or SFD as shown

$V_{a-a} = 6\ kN$

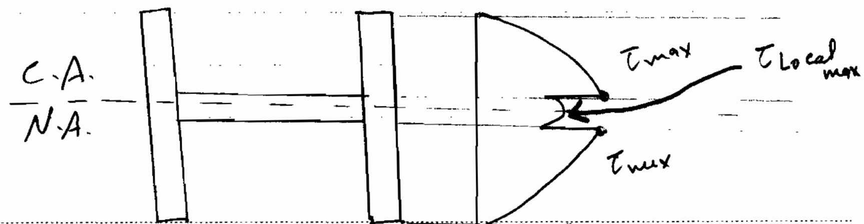


SFD (kN)

We need the centroid and I.
Due to the double symmetry, $\bar{y}$ is
@ the "middle/center".

$$\bar{I} = 2\left[\frac{15(100)^3}{12}\right] + \frac{75(15)^3}{12} = 2.52109(10)^6 \text{ mm}^4 = 2.52109(10)^{-6} \text{ m}^4$$

(No Ad^2. Why ?!)



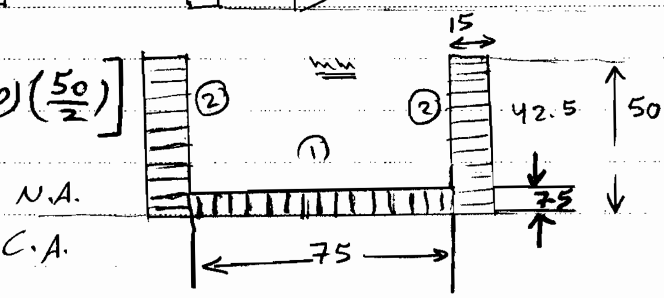
$Q = A\bar{y}$

$$Q_{N.A.} = Q_{1} + 2Q_{2}$$

$$= 75(7.5)\left(\frac{7.5}{2}\right) + 2\left[15(50)\left(\frac{50}{2}\right)\right]$$

$$= 39{,}609.375 \text{ mm}^3$$

$$= 3.96094(10)^{-5} \text{ m}^3$$



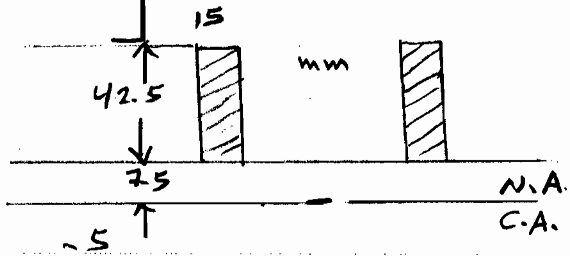
$$\tau_{N.A.} = \frac{VQ_{N.A.}}{I\,b_{N.A.}} = \frac{6(10)^3(3.96094)(10)^{-5}}{2.52109(10)^{-6}\underbrace{(75+2\times15)}_{b=105}(10)^{-3}}$$

$$\Rightarrow \boxed{\tau_{N.A.} = 0.8978 \text{ MPa}}$$

Solution of HW # 11

From the τ-distribution shown, τ_{max} is @ the junction of the horizontal and vertical parts. τ_{max} could have been at the N.A. (?!)

$$Q_{junc} = 2\left[15(42.5)\left(\frac{42.5}{2} + 7.5\right)\right]$$



$$= 36,656.25 \text{ mm}^3$$

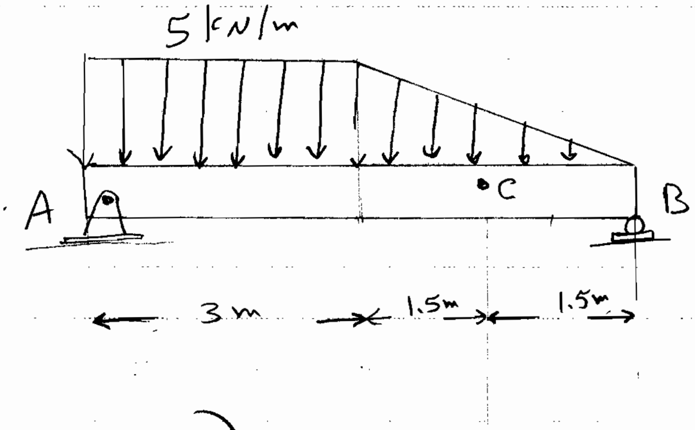
$$= 3.66563(10)^{-5} \text{ m}^3$$

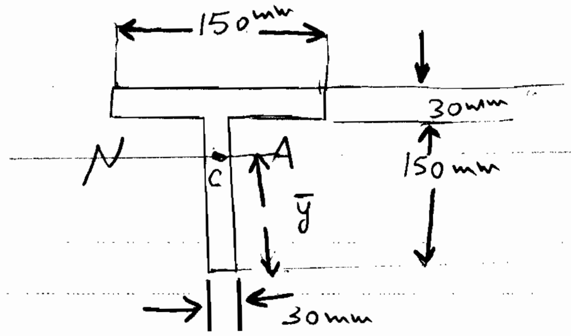
$$\tau_{max} = \frac{6(10)^3(3.66563)(10)^{-5}}{2.52109(10)^{-6}(15+15)(10)^{-3}} \Rightarrow$$

$$\boxed{\tau_{max} = 2.908 \text{ MPa}}$$

Problem # 5:

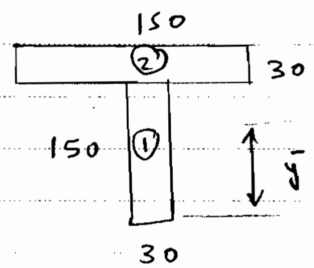
Given:-
The beam with the cross-section shown.





Required:
$\tau_{N.A.}$ @ C

Solution:



$$\tau = \frac{VQ}{It}$$

We need $\bar{y}$ and $\bar{I}$

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i}$$

$$= \frac{30(150)\left(\frac{150}{2}\right) + 150(30)\left(150 + \frac{30}{2}\right)}{30(150) + 150(30)}$$

$$= 120 \text{ mm}$$

$$\bar{I} = \Sigma(\bar{I} + Ad^2)_i$$

$$= \left[\tfrac{1}{12}(30)(150)^3 + 30(150)\left(\tfrac{150}{2} - 120\right)^2\right] + \left[\tfrac{1}{12}(150)(30)^3 + 150(30)\left(150 - 120 + \tfrac{30}{2}\right)^2\right]$$

$\Rightarrow \bar{I} = 2.7\,(10)^{7}\,mm^4 = 2.7\,(10)^{-5}\,m^4$

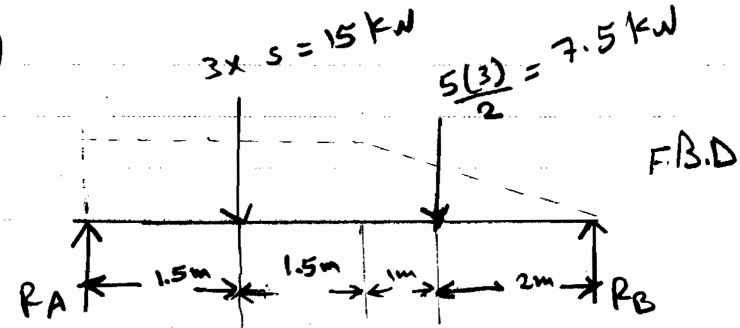
To find V @ section through C, the easiest is to cut through C and take the right part (compared with the left part or drawing the SFD).

However, we need to find R_B first. From the FBD,

$+\circlearrowleft\ \Sigma M_A = 0$ (why A ?!)

$6R_B - 15(1.5) - 7.5(4) = 0$

$\Rightarrow R_B = 8.75\ kN$

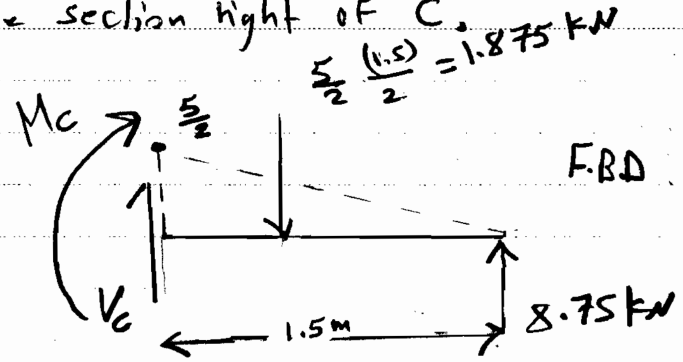


Now, we take the FBD of the section right of C.

$+\uparrow \Sigma F_y = 0 \Rightarrow$

$V_c - 1.875 + 8.75 = 0 \Rightarrow$

$V_c = -6.875\ kN$



The sign is not important (not significant) here.

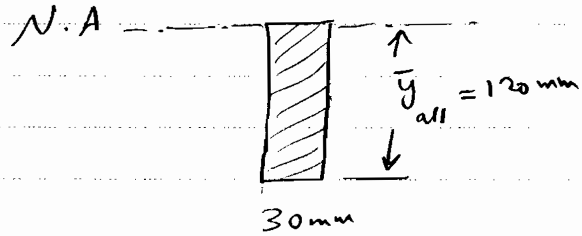
$\Rightarrow |V_c| = 6.875\ kN$

$$Q_{N.A} = A\bar{y}$$

$$= 30(120)\left(\frac{120}{2}\right)$$

$$= 216{,}000 \ mm^3$$

$$= 2.16(10)^{-4} \ m^3$$



Note that it is easier to take the lower part of the area.

Try taking the upper part and compare!

$$\tau_c = \frac{V_c \, Q_c}{I \, t_c}$$

$$= \frac{6.875(10)^3 (2.16)(10)^{-4}}{2.7(10)^{-5}(30)(10)^{-3}} \Rightarrow$$

$$\boxed{\tau_c = 1.833 \ MPa}$$