Problem 2: (20 points)

A bar with the stress-strain diagram shown was originally 1 m long with a square cross-sectional area of 100 mm x 100 mm.
When an axial tension load $F$ is applied, the square cross-section became 99.95 mm x 99.95 mm.

Determine the following:

a) The magnitude of the applied force $F$.

b) The final length of the bar when the load $F$ is applied.

c) The final length of the bar when the load $F$ is released.

d) The final length of the bar when the applied load is 300 kN.

e) The final length of the bar when the 300 kN load is released.

Poisson’s ratio, $v = 0.25$

\[ \text{Stress, MPa} \]

\[ \text{Strain, mm/mm} \]

Permentent \hspace{1cm} Strain

Solution

\[ \nu = \frac{E_{\text{at}}}{E_{\text{long}}} \Rightarrow E_{\text{long}} = \frac{E_{\text{at}}}{\nu} \]

\[ E_{\text{at}} = \frac{99.95 - 100}{100} = -0.0005 \text{ mm/mm} \]

\[ E_{\text{long}} = \frac{-(-0.0005)}{0.25} = 0.002 \text{ mm/mm} \]

From $\sigma - \varepsilon$ diagram when $\varepsilon = 0.002$ \hspace{1cm} $\sigma = 15 \text{ MPa}$

\[ \sigma = \frac{P}{A_0} \Rightarrow P = \sigma A_0 = 15 \times 10000 = 150000 \text{ N} = 150 \text{kN} \]

\[ \varepsilon_{\text{long}} = \frac{L_f - L_0}{L_0} \Rightarrow L_f = (\varepsilon_{\text{long}} + 1) L_0 = (0.002 + 1) \times 1 = 1.002 \text{ m} \]

When the load $F$ is released, it will go back to original length $\sigma = \varepsilon_y$. \hspace{1cm} $L = 1 \text{ m}$.

\[ \sigma = \frac{300000}{10000} = 30 \text{ MPa} \] in the plastic range.

\[ \delta = 30 \text{ MPa} \] \hspace{1cm} $E_{\text{long}} = 0.008 \text{ mm/mm}$

\[ L_f = (0.008)(1) + 1 = 1.008 \text{ m} \]

\[ E = \frac{E_{\text{at}}}{0.002} = 7500 \text{ MPa} \]

\[ \text{Permanent strain} = \frac{30}{7500} = 0.004 \text{ mm/mm} \] or directly from the graph

\[ L_f = (1 \times 0.004) + 1 = 1.004 \text{ m} \]
Problem 5: (20 points)

The steel block shown is subjected to a uniform pressure \( p \) on all the faces. Knowing that the change in length of edge AB is \(-30 \times 10^{-3}\) mm and using \( E = 200\) GPa, and \( G = 75\) GPa, determine the followings:

1. a) The magnitude of the applied pressure, \( p \).
2. b) The strains in the \( x \), \( y \), and \( z \) directions.
3. c) The new length of AB, CB, and BD after the application of the uniform pressure \( p \).

Solution

a) \[ E_{x} = \frac{\Delta L_{AB}}{L_{AB}} = -30 \times 10^{-3} \frac{100}{-3} = -3 \times 10^{-4}\,\text{mm/mm} \]

\[ E_{x} = -3 \times 10^{-4} = \frac{1}{200 \times 10^{9}} [\Delta L_{x} - 0.333 (-p - p)] \]

\[ p = 179.64\,\text{MPa} \]

b) \[ E_{y} = \frac{1}{20 \times 10^{9}} [-179.64 \times 10^{-4} - 0.333 (-2 \times 179.64 \times 10^{-4})] \]

\[ E_{y} = -3 \times 10^{-4}\,\text{mm/mm} \]

Similarly, \[ E_{z} = -3 \times 10^{-4}\,\text{mm/mm} \]

\[ (L_{AB})_{\text{new}} = (-30 \times 10^{-3}) + 100 = 99.7\,\text{mm} \]

\[ (L_{CB})_{\text{new}} = (50 + -3 \times 10^{-4}) + 50 = 99.985\,\text{mm} \]

\[ (L_{BD})_{\text{new}} = (75 + -3 \times 10^{-4}) + 75 = 74.975\,\text{mm} \]

(d) change in volume \( \Delta V = \)

\[ (99.97\,\text{mm})(99.985\,\text{mm})(74.975\,\text{mm}) - (100\,\text{mm})(50\,\text{mm})(75\,\text{mm}) = \]

\[ \Delta V = -337.44\,\text{mm}^3 \]