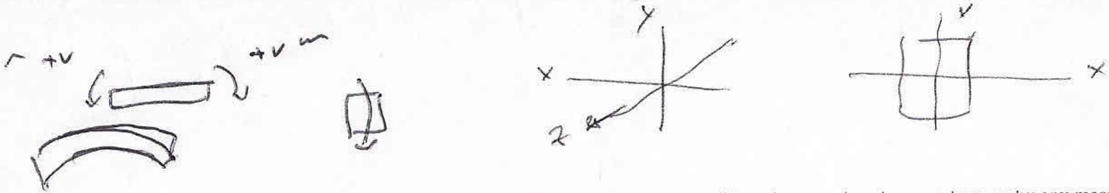
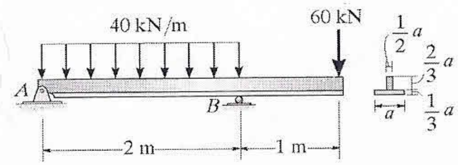
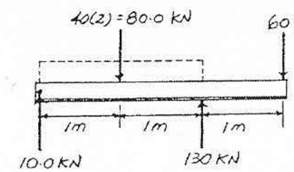


6–93. The beam is subjected to the loading shown. Determine its required cross-sectional dimension a, if the allowable bending stress for the material is $\sigma_{allow} = 150$ MPa.



Support Reactions: As shown on FBD.



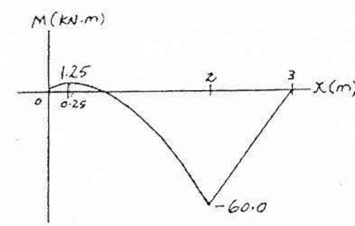
Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{\frac{1}{6}a\left(\frac{1}{3}a\right)a + \frac{2}{3}a\left(\frac{2}{3}a\right)\left(\frac{1}{2}a\right)}{\left(\frac{1}{3}a\right)a + \left(\frac{2}{3}a\right)\left(\frac{1}{2}a\right)} = \frac{5}{12}a \quad \text{5} \quad \frac{5}{12}(150)$$

$$I = \frac{1}{12}(a)\left(\frac{1}{3}a\right)^3 + a\left(\frac{1}{3}a\right)\left(\frac{5}{12}a - \frac{1}{6}a\right)^2 + \frac{1}{12}\left(\frac{1}{2}a\right)\left(\frac{2}{3}a\right)^3 + \frac{1}{2}a\left(\frac{2}{3}a\right)\left(\frac{2}{3}a - \frac{5}{12}a\right)^2$$

$$= \frac{37}{648}a^4 \qquad \frac{37}{648}(150)^4$$

Internal Moment: As shown on the moment diagram.

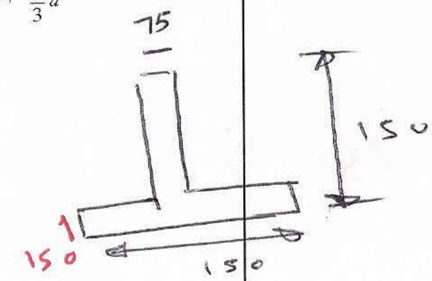


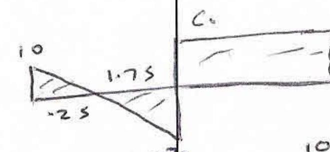
Allowable Bending Stress: The maximum moment is $M_{max} = 60.0$ kN · m as indicated on the moment diagram. Applying the flexure formula

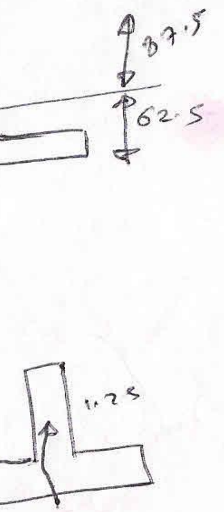
$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}c}{I}$$

$$150(10^6) = \frac{60.0(10^3)\left(a - \frac{5}{12}a\right)}{\frac{37}{648}a^4}$$

$$a = 0.1599 \text{ m} = 160 \text{ mm} \qquad \text{Ans}$$







$\frac{10}{x} = \frac{70}{2-x}$

$20 - 10x = 70x$

$20 = 80x$

(−ve) mod

+m

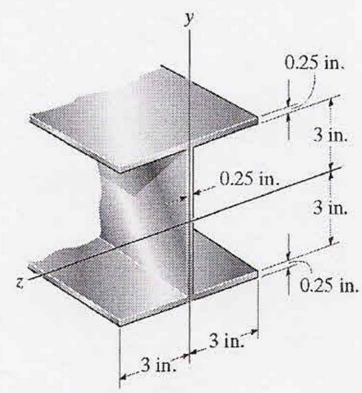
comp $= \frac{(1.25\times10^3)(.0875)}{.00002890625} = 3.874$ MPa (→) moment

ten $= \frac{(1.25\times10^3)(.0625)}{.000002890625} = 2.7$ MPa

max comp $= \frac{(60\times10^3)(.0625)}{.00002890625} = 129.73$ MPa

max tensile $= \frac{60\times10^3(.0875)}{0.00002890625} = 181.62$ MPa

6–49. A beam has the cross section shown. If it is made of steel that has an allowable stress of $\sigma_{\text{allow}} = 24$ ksi, determine the largest internal moment the beam can resist if the moment is applied (a) about the z axis, (b) about the y axis.



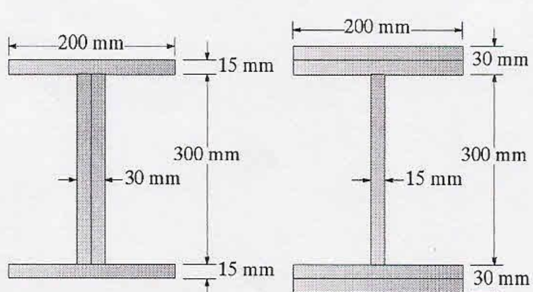
$$I_z = \frac{1}{12}(6)(6.5^3) - \frac{1}{12}(5.75)(6^3) = 33.8125 \text{ in}^4$$

$$I_y = 2[\frac{1}{12}(0.25)(6^3)] + \frac{1}{12}(6)(0.25^3) = 9.0078 \text{ in}^4$$

a) $$(M_{\text{allow}})_z = \frac{\sigma_{\text{allow}} I_z}{c} = \frac{24(33.8125)}{3.25}$$
$$= 249.7 \text{ kip} \cdot \text{in.} = 20.8 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$

b) $$(M_{\text{allow}})_y = \frac{\sigma_{\text{allow}} I_y}{c} = \frac{24(9.0078)}{3}$$
$$= 72.0625 \text{ kip} \cdot \text{in.} = 6.00 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$

6–50. Two considerations have been proposed for the design of a beam. Determine which one will support a moment of $M = 150 \text{ kN} \cdot \text{m}$ with the least amount of bending stress. What is that stress? By what percentage is it more effective?



(a) (b)

Section Property:

For section (a)

$$I = \frac{1}{12}(0.2)\left(0.33^3\right) - \frac{1}{12}(0.17)(0.3) = 0.21645(10^{-3}) \text{ m}^4$$

For section (b)

$$I = \frac{1}{12}(0.2)\left(0.36^3\right) - \frac{1}{12}(0.185)\left(0.3^3\right) = 0.36135(10^{-3}) \text{ m}^4$$

Maximum Bending Stress: Applying the flexure formula $\sigma_{\max} = \frac{Mc}{I}$

For section (a)

$$\sigma_{\max} = \frac{150(10^3)(0.165)}{0.21645(10^{-3})} = 114.3 \text{ MPa}$$

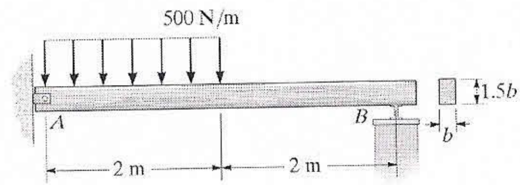
For section (b)

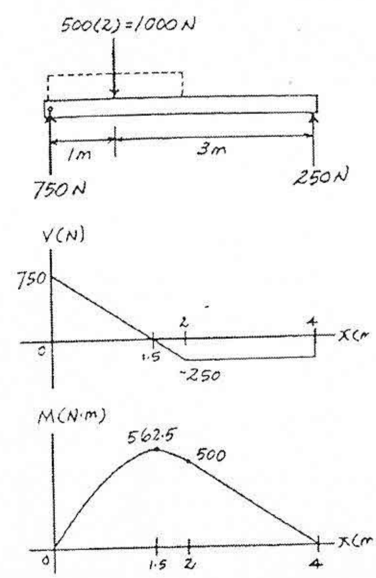
$$\sigma_{\max} = \frac{150(10^3)(0.18)}{0.36135(10^{-3})} = 74.72 \text{ MPa} = 74.7 \text{ MPa} \qquad \textbf{Ans}$$

By comparison, section (b) will have the least amount of bending stress.

$$\% \text{ of effectiveness} = \frac{114.3 - 74.72}{74.72} \times 100\% = 53.0\ \% \qquad \textbf{Ans}$$

6–99. The wood beam has a rectangular cross section in the proportion shown. Determine its required dimension b if the allowable bending stress is $\sigma_{allow} = 10$ MPa.





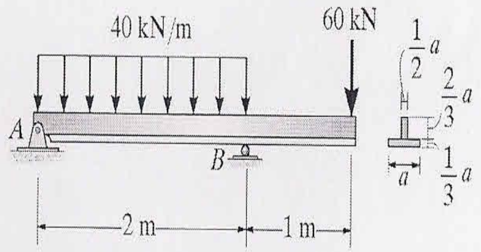
Allowable Bending Stress: The maximum moment is $M_{max} = 562.5$ N · m as indicated on the moment diagram. Applying the flexure formula

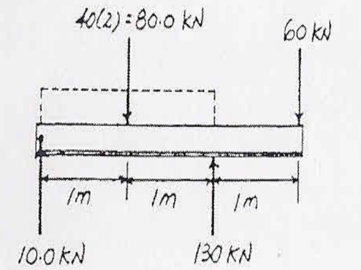
$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}\, c}{I}$$

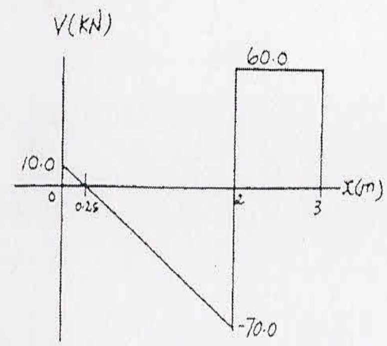
$$10(10^6) = \frac{562.5(0.75b)}{\frac{1}{12}(b)(1.5b)^3}$$

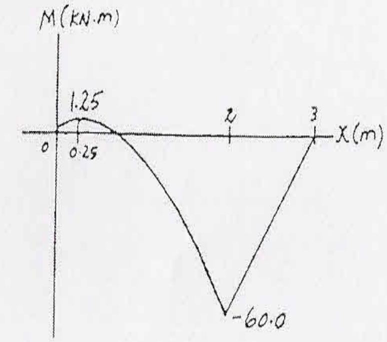
$$b = 0.05313 \text{ m} = 53.1 \text{ mm} \qquad \text{Ans}$$

*6–92. The beam is subjected to the loading shown. If its cross-sectional dimension $a = 180$ mm, determine the absolute maximum bending stress in the beam.









Section Properties:

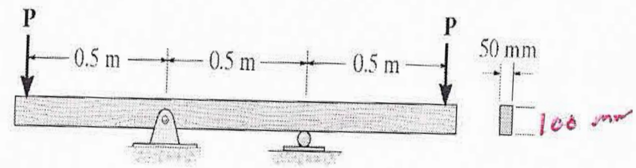
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.03(0.18)(0.06) + 0.12(0.12)(0.09)}{(0.18)(0.06) + (0.12)(0.09)} = 0.075 \text{ m}$$

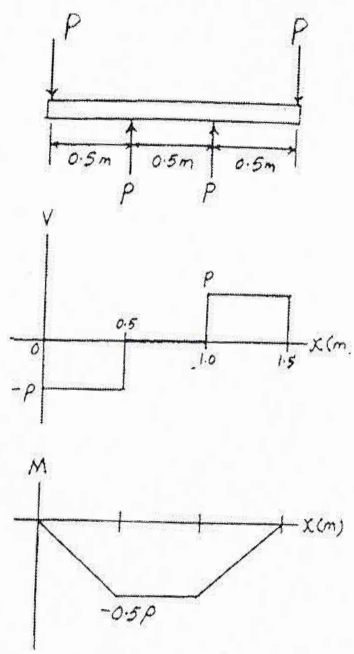
$$I = \frac{1}{12}(0.18)\left(0.06^3\right) + 0.18(0.06)(0.075 - 0.03)^2$$
$$+ \frac{1}{12}(0.09)\left(0.12^3\right) + 0.09(0.12)(0.12 - 0.075)^2$$
$$= 59.94\left(10^{-6}\right) \text{ m}^4$$

Allowable Bending Stress: The maximum moment is $M_{max} = 60.0$ kN · m as indicated on the moment diagram. Applying the flexure formula

$$\sigma_{max} = \frac{M_{max}\, c}{I}$$
$$= \frac{60.0(10^3)(0.18 - 0.075)}{59.94(10^{-6})}$$
$$= 105 \text{ MPa} \qquad \text{Ans}$$

6–90. The beam has a rectangular cross section as shown. Determine the largest load P that can be supported on its overhanging ends so that the bending stress in the beam does not exceed $\sigma_{max} = 10$ MPa.





Absolute Maximum Bending Stress: The maximum moment is $M_{max} = 0.5P$ as indicated on the moment diagram. Applying the flexure formula

$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

$$10(10^6) = \frac{0.5P(0.05)}{\frac{1}{12}(0.05)(0.1^3)}$$

$$P = 1666.7 \text{ N} = 1.67 \text{ kN} \qquad \text{Ans}$$

Problem 4: (20 points)

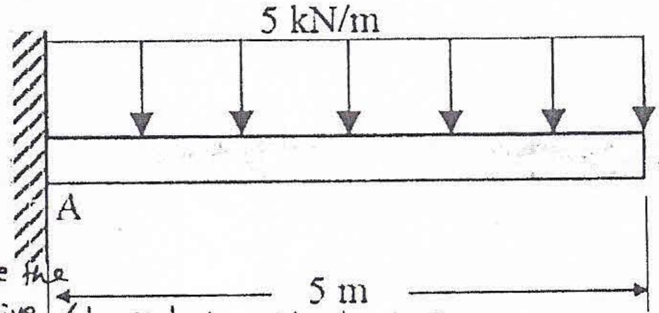
A cantilever beam along with its cross-section is shown in the figure below. Determine the following at point *A* of the beam:

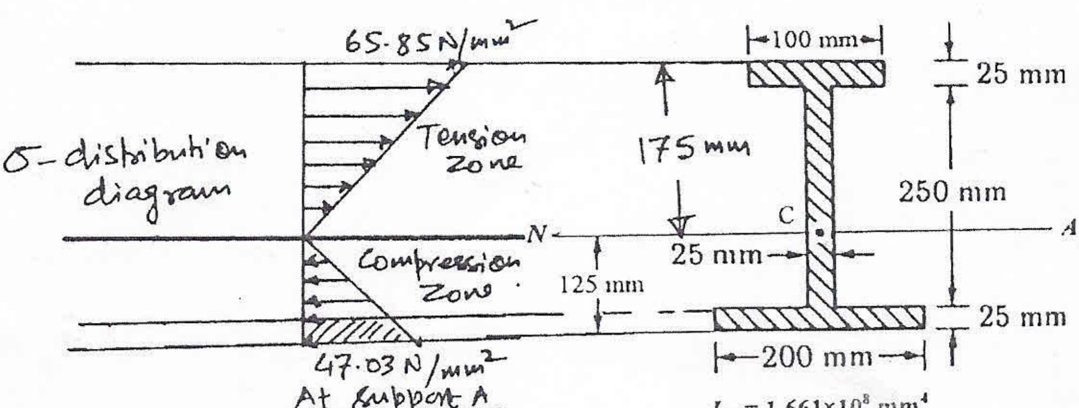
a) Maximum tensile stress and its location
b) Maximum compressive stress and its location
c) Normal stress distribution along the depth of the cross-section
d) Magnitude of the normal force on the lower flange.

M at support A:

$M_A = 5 \times 5 \times 2.5$
$= 62.5$ kN-m

The moment will Cause tensile Stress above the neutral axis and Compressive Stress below the neutral axis





$I_{NA} = 1.661 \times 10^8$ mm⁴

(a) $\left[\sigma_{max,t}\right]_A = \dfrac{M_A C_{top}}{I_{NA}} = \dfrac{62.5 \times 10^6 \times 175}{1.661 \times 10^8} = 65.85\ N/mm^2\ (T)$

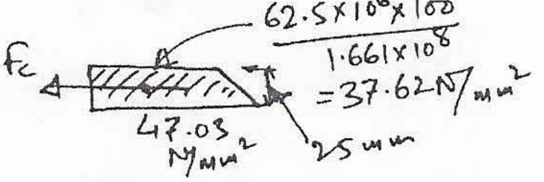
(Location: top most point of the section) Ans

(b) $\left[\sigma_{max,c}\right]_A = \dfrac{M_A C_{bottom}}{I_{NA}}$

$= \dfrac{62.5 \times 10^6 \times 125}{1.661 \times 10^8} = 47.03\ N/mm^2$ (C) Ans (Location: bottom most point of the section).

(C) Normal Stress distribution diagram Shown above.

(d) From σ- distribution diagram, the Stress over lower flange Can be Shown below:



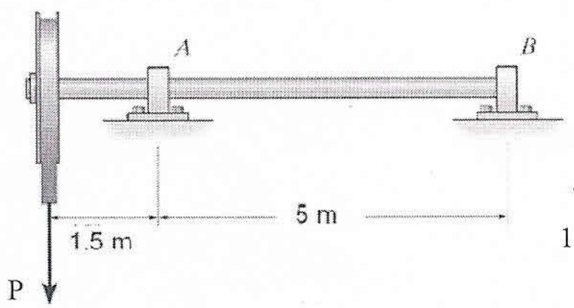
$F_c = \dfrac{[47.03 + 37.62]}{2} \times 25 \times 200$

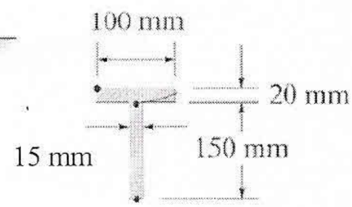
$= 211625$ N

$= 211.62$ kN Ans

Problem 5

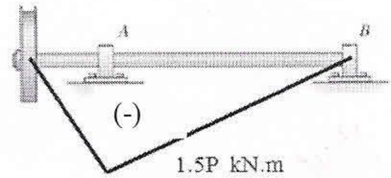
The given beam has a T- cross section as shown below. Determine the largest magnitude of P that can be safely applied. The allowable tensile and compressive stresses are 30 MPa and 40MPa respectively, and the allowable shear stress is 20 MPa. (The 2 supports exert vertical reactions only).



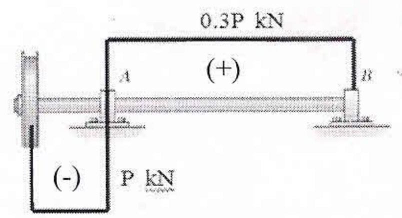


Solution:

Bending Moment diagram (kN.m)

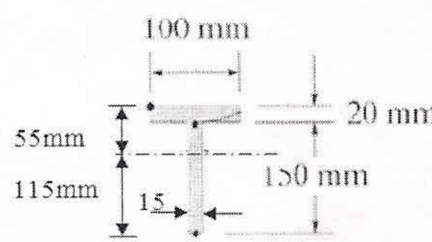


Shear Force diagram (kN)



$$\bar{y} = \frac{\sum A\,y}{\sum A}$$

$$\bar{y} = \frac{(100*20*10)+(150*15)*95}{(100*20+150*15)} = 55\ mm$$



$$I_{NA} = \frac{100*20^3}{12} + 100*20*(55-10)^2 + \frac{150^3*15}{12} + 150*15*(95-55)^2$$

$$= 11.935*10^6 \quad mm^4$$

$$(\sigma_t)_{\max} = \frac{M}{I} y_t = 30$$

$$\Rightarrow M_{\max} = \frac{(\sigma_t)_{\max} I}{y_t} = \frac{30*11.935*10^6}{55} = 6.51 kN.m$$

$$M_{\max} = 1.5P = 6.51$$

$$p = 4.34 \qquad \ldots\ldots\ldots\ldots\ldots \qquad (1)$$

$$Q_A = \bar{y}_2' \; A' = 0.036*0.012*0.12 = 51.84*10^{-6} \; m^3$$

$$(\sigma_c)_{\max} = \frac{M}{I} y_b = 30$$

$$\Rightarrow M_{\max} = \frac{(\sigma_c)_{\max} I}{y_t} = \frac{40*11.935*10^6}{115} = 4.51 \; kN.m$$

$$M_{\max} = 1.5P = 4.51$$

$$p = 2.77 kN \qquad \ldots\ldots\ldots\ldots\ldots \qquad (2)$$

$$Q_{\max} = \sum \bar{y}' A' = 100*20*(10+35) + 35*15*\frac{35}{2} = 99.188*10^3 \; mm^3$$

$$\tau_{\max} = \frac{V Q_{\max}}{It} = \frac{V*99.188*10^3}{11.935*10^6*15} = 20 MPa$$

$$\Rightarrow V = \frac{\tau_{\max} I t}{Q_{\max}} = \frac{20*11.935*10^6*15}{99.188*10^3} = 36.1 \; kN$$

$$V_{\max} = P = 36.1 \; kN \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (3)$$

The largest magnitude of *P that can be be safely applied* is the minimum of the resulting ***P*** values given by (1) , (2) & (3):

$$P = 2.77 \; kN$$

Problem # 4

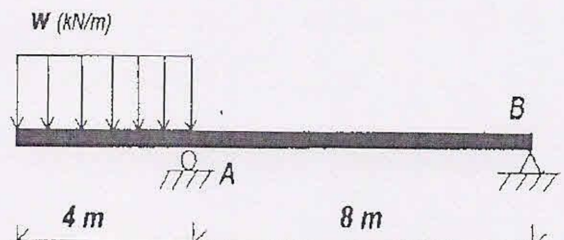
The given beam is subjected to a downward uniformly distributed load **W (kN/m)** as shown.

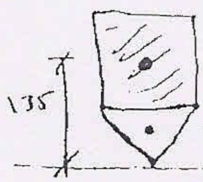
a) Determine the moment of inertia of the beam's cross section about the Neutral Axis.

b) Determine the maximum value of **W** that can be applied given the following information :

Safety Factor = 2
For tension σ_{ult} = 30 MPa
For compression σ_{ult} = 40 MPa.

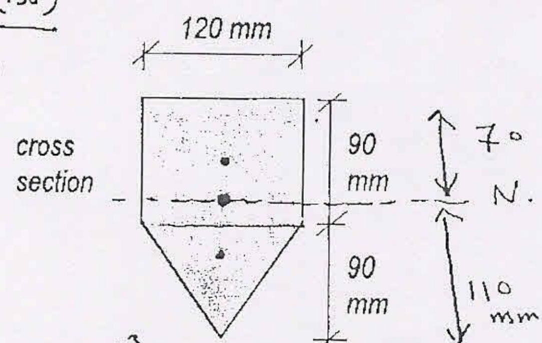
a) Find location of Centroid





$$\bar{y} = \frac{\Sigma A\tilde{y}}{\Sigma A} = \frac{\frac{1}{2}(120)(90)(60) + (120)(90)(135)}{\frac{1}{2}(120)(90) + (120)(90)}$$

$\boxed{\bar{y} = 110\text{ mm}}$ from the bottom



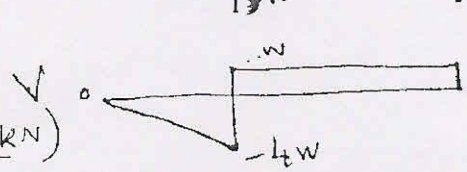
$$I_{NA} = (I_1)_{NA} + (I_2)_{AA}$$

$$I_{NA} = \left[\frac{(120)(90)^3}{36} + \frac{1}{2}(120)(90)(50)^2\right] + \left[\frac{(120)(90)^3}{12} + (120)(90)(25)^2\right]$$

$$I_{NA} = 15.93\times10^6 + 14.04\times10^6 = \boxed{29.97\times10^6\ \text{mm}^4}$$

b)

$M_{max} = -8w$ (kN.m), So $\left\{\begin{matrix} T & u \\ C & d\end{matrix}\right.$



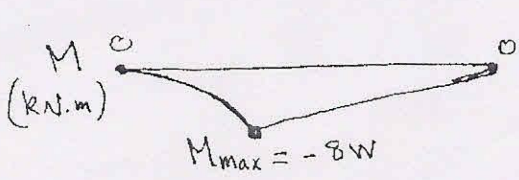
* Check tension $\sigma_{all} = \frac{30}{2} = 15$

$$15 = \frac{(8w)\times10^6\,(70)}{I} \rightarrow w = .8c$$

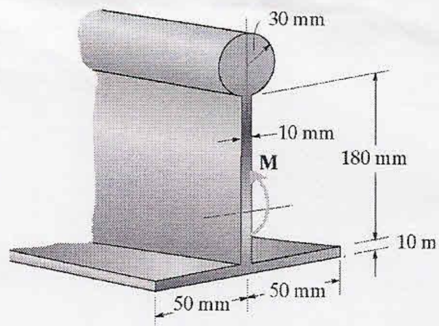
* check compression $\sigma_{all} = \frac{40}{2} = 20$

$$20 = \frac{(8w)\times10^6\,(110)}{I} \rightarrow w = .681\text{ kN}$$

$\therefore$ $\boxed{\text{Max } w = 0.681\text{ kN/m}}$



6–59. Determine the largest bending stress developed in the member if it is subjected to an internal bending moment of $M = 40\ \text{kN}\cdot\text{m}$.



Section Properties:

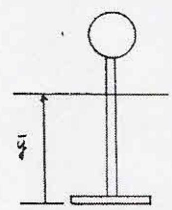
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

$$= \frac{0.005(0.1)(0.01) + 0.1(0.18)(0.01) + 0.22(\pi)(0.03^2)}{(0.1)(0.01) + (0.18)(0.01) + (\pi)(0.03^2)}$$

$$= 0.143411\ \text{m}$$

$$I = \frac{1}{12}(0.1)\left(0.01^3\right) + (0.1)(0.01)(0.143411 - 0.005)^2 + \frac{1}{12}(0.01)\left(0.18^3\right) + (0.01)(0.18)(0.143411 - 0.1)^2 + \frac{1}{4}\pi\left(0.03^4\right) + \pi\left(0.03^2\right)(0.22 - 0.143411)^2$$

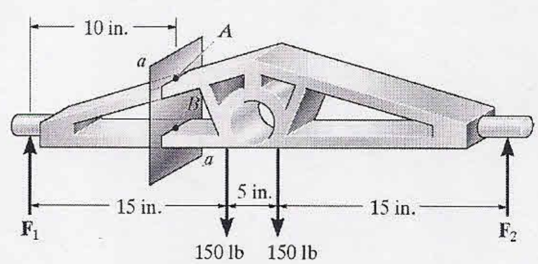
$$= 44.64\left(10^{-6}\right)\ \text{m}^4$$

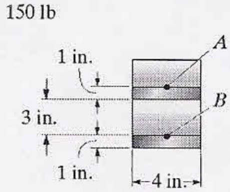


Maximum Bending Stress: The maximum bending stress occurs at the bottom fiber of the section which is subjected tensile stress. Applying the flexure formula.

$$\sigma_{max} = \frac{Mc}{I} = \frac{40(10^3)(0.143411)}{44.64(10^{-6})} = 129\ \text{MPa} \qquad \textbf{Ans}$$

***6–60.** The tapered casting supports the loading shown. Determine the bending stress at points A and B. The cross section at section a–a is given in the figure.



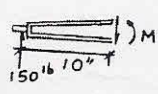


Casting:

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_1(35) - 150(20) - 150(15) = 0$$

$$F_1 = 150\ \text{lb}$$

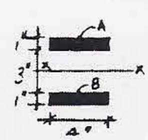
Section:



$$\circlearrowleft + \Sigma M = 0; \qquad M - 150(10) = 0$$

$$M = 1500\ \text{lb}\cdot\text{in.}$$

$$I_x = \frac{1}{12}(4)(5^3) - \frac{1}{12}(4)(3)^3 = 32.67\ \text{in}^4$$



$$\sigma_A = \frac{Mc}{I} = \frac{1500(2.5)}{32.67} = 115\ \text{psi (C)} \qquad \textbf{Ans}$$

$$\sigma_B = \frac{My}{I} = \frac{1500(1.5)}{32.67} = 68.9\ \text{psi (T)} \qquad \textbf{Ans}$$

Problem # (4)

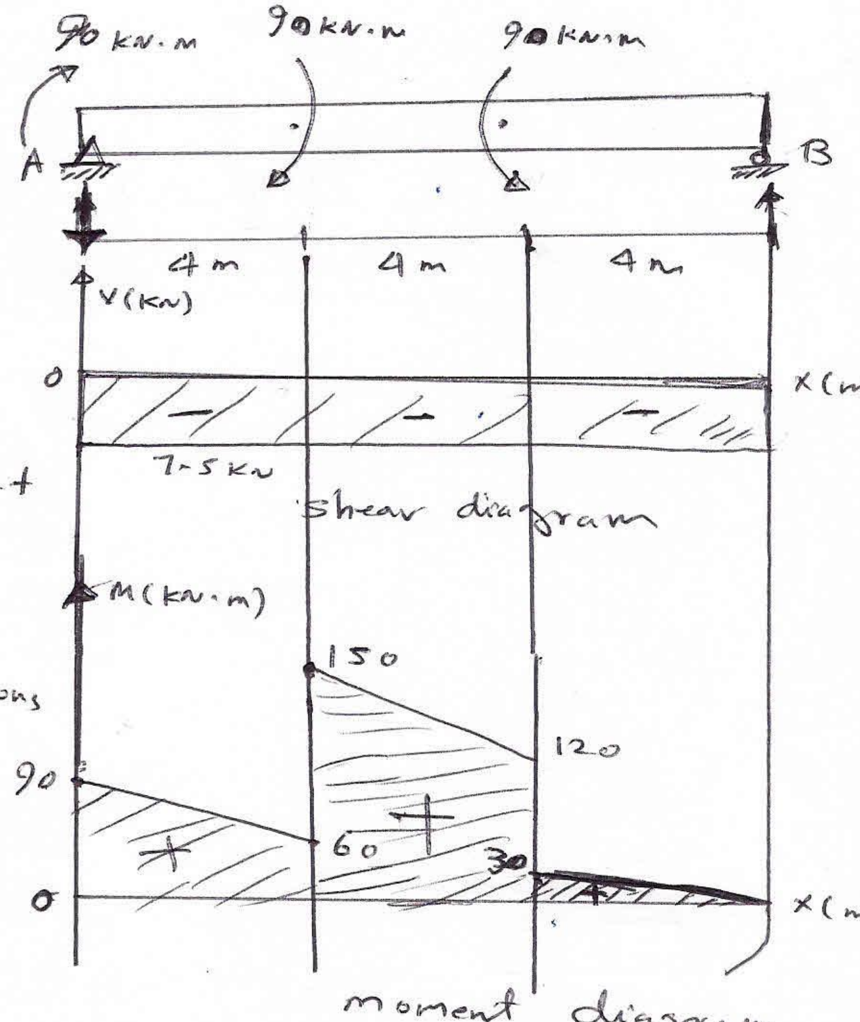
Given :-

The shown loaded beam. The cross-section is square ($h \times h$)

$\sigma_{allow} = 100$ MPa

Needed :-

The smallest value of the dimension h that can be safely used



solution

1) Finde the support reactions

$\sum M_A = 0$,

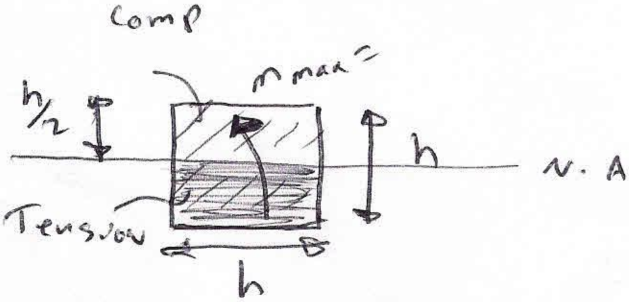
$$90 - 90 - 90 + B_y(12) = 0$$

$B_y = 7.5$ kN ↑

$A_y = 7.5$ kN ↓

$$\sigma_{max} = \frac{Mc}{I} = 100 \times 10^6 = \frac{(150 \times 10^3)\left(\frac{h}{2}\right)}{\frac{h^4}{12}}$$

$$\Rightarrow \boxed{h \approx 0.208 \text{ m} \approx 208 \text{ mm}}$$



$$I = \frac{h(h^3)}{12} = \frac{h^4}{12}$$

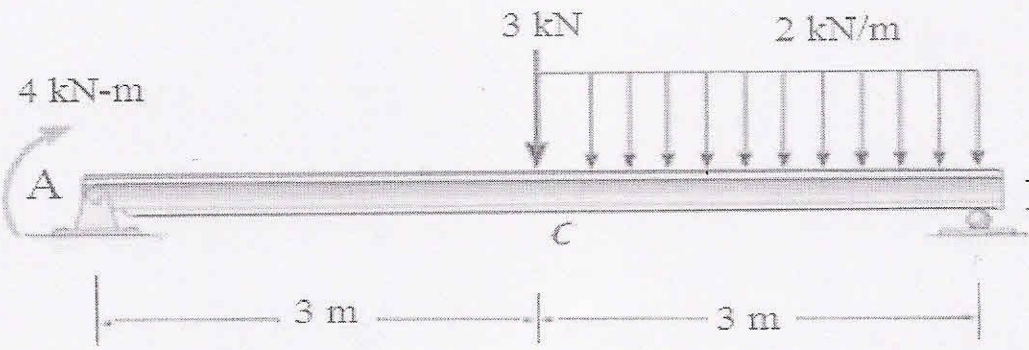
Problem # 4

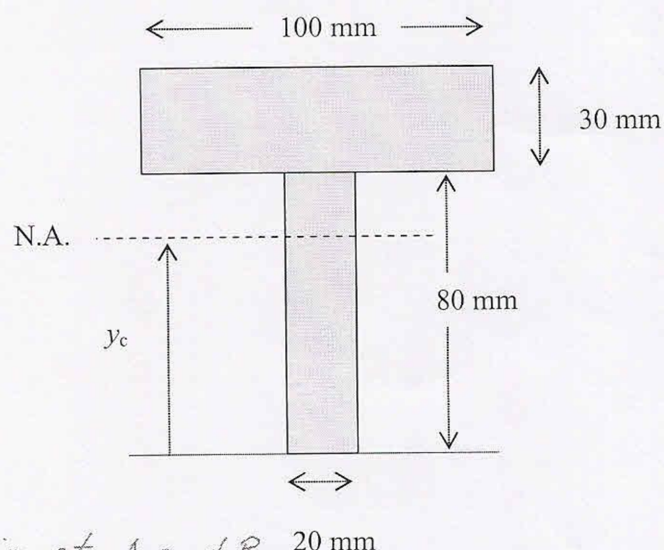
The simply supported T-beam is subjected to the loading shown:

a) Verify that $y_c = 75.87$ **mm** from the bottom of the T- cross-section and that $I_{NA} = 4.235 \times 10^{-6}$ **m**^{4}.

b) Determine the maximum tensile and the maximum compressive bending stress in the T-cross-section at point C in the beam.

c) Plot the bending stress distribution along the height of the T-cross-section at point C in the beam.





Solution:

a) First determine reactions at A and B.

$\sum F_y^{+\uparrow} = 0; \quad R_A + R_B = 3 + 2 \times 3 = 9 \text{ kN}$

$\sum M_A^{(+)} = 0; \quad -4 - 3 \times 3 - 2 \times 3 (3 + 3/2) + R_B(6) = 0$

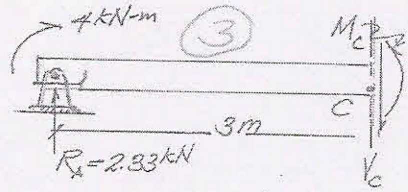
$\therefore \quad R_B = \underline{6.67 \text{ kN}\uparrow}$

and $R_A = \underline{2.33 \text{ kN}\uparrow}$

2

$$\Sigma M_c = 0; \quad M_c - 4 - 2.33 \times 3 = 0$$

$$\therefore M_c = 10.99 \cong \underline{11.0\ kN\text{-}m}$$



b) $$y_{o\ \text{from bottom}} = \frac{\Sigma y'A}{\Sigma A}$$

2

$$= \frac{(40)(20 \times 80) + (95)(30 \times 100)}{20 \times 80 + 30 \times 100} = \underline{75.87\ mm}$$

$$I_{NA} = \Sigma(I_{x'} + Ad^2)$$

$$= \left[\tfrac{1}{12}(0.02)(0.08)^3 + (0.02 \times 0.08)(0.07587 - 0.04)^2\right]$$

$$+ \left[\tfrac{1}{12}(0.1)(0.03)^3 + (0.03 \times 0.1)(0.11 - 0.07587 - 0.015)^2\right]$$

$$= 0.853 \times 10^{-6} + 2.059 \times 10^{-6} + 0.225 \times 10^{-6} + 1.098 \times 10^{-6}$$

$$= \underline{4.235 \times 10^{-6}\ m^4}$$

3

c) Since the bending moment is positive, then the max tensile bending stress is in the bottom fiber of the cross-section at $C_{bottom} = 75.87\ mm$,

1

or

$$(\sigma_{max})_{Ten} = \frac{M_c C_{bottom}}{I_{NA}} = \frac{(11 \times 10^3\ N\text{-}m)(0.07587\ m)}{4.235 \times 10^{-6}\ m^4}$$

$$= \underline{197.065\ MPa}$$

2

and

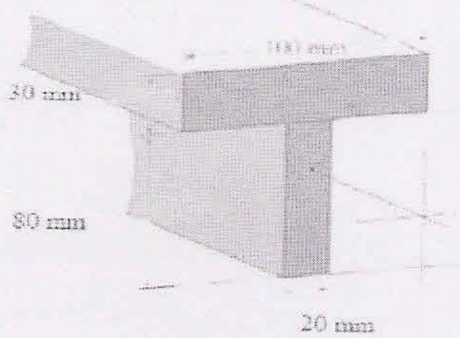
$$(\sigma_{max})_{Comp} = \frac{M_c C_{top}}{I_{NA}} = \frac{(11 \times 10^3\ N\text{-}m)(0.03413\ m)}{4.235 \times 10^{-6}\ m^4}$$

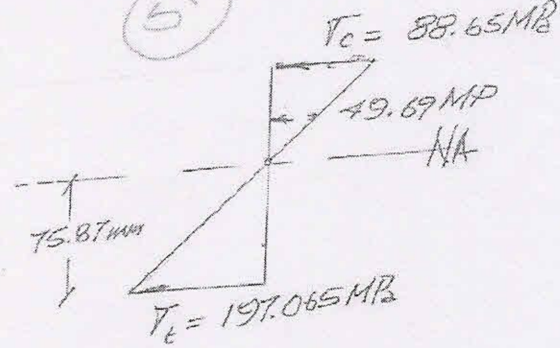
$$= \underline{88.65\ MPa}$$

2

d) Bending Stress distribution:

5





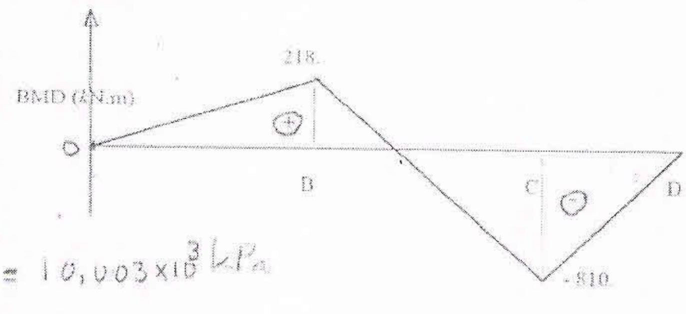
Problem # 4 **Key Solution**

The beam ABCD shown has a cross-section that is composed of two identical vertical boards and one horizontal board (all with common thickness $t = 200$ mm) and has the given bending moment diagram.

a) Compute the maximum *tensile* stress σ_t and *clearly* specify its location in the beam.

b) Compute the resultant force on the left vertical board at the location of maximum positive moment.

Given: $\bar{y} = 325$ mm; and $I_{CA} = 7.083 \times 10^{-3}$ m^4



a)

$$\sigma(y) = -\frac{M\,y}{I_{CA}}$$

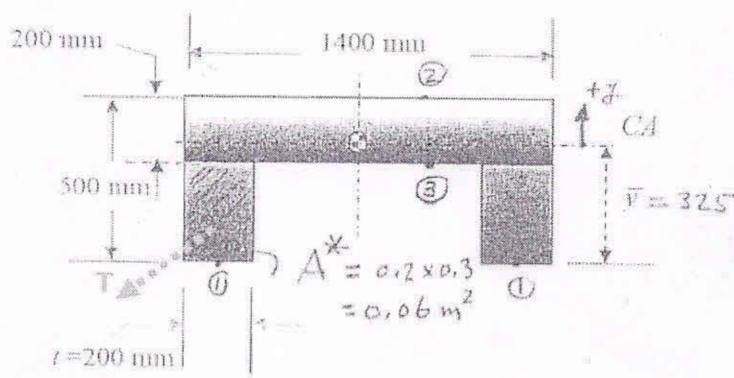
$$\sigma_t^{①} = -\frac{218(-0.325)}{7.083\times10^{-3}} = 10{,}003\times10^{3}\ \text{kPa}$$

$$\therefore\ \sigma_t^{①} \simeq 10.\ \text{MPa}$$

$$\sigma_t^{②} = -\frac{-810(0.175)}{7.083\times10^{-3}}$$

$$\simeq 20.0\ \text{MPa}$$

$$\therefore\ \sigma_t^{(max)} = 20.\ \text{MPa}$$



Cross-section

Caused by negative moment in cross section at C. This tensile stress is acting on all material points at $y = +0.175$ m from C.A.

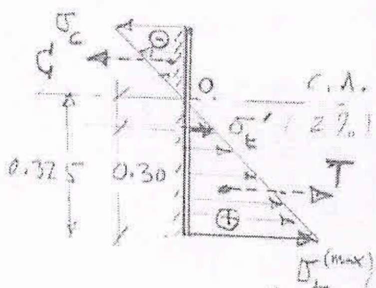
b) Since $N = \int \sigma\, dA$, then on area A^* with

$M(x) = 218$ kN.m

$$\sigma_t' = \sigma_t^{③} = \sigma(y = -0.025)$$

$$= -\frac{218(-0.025)}{7.083\times10^{-3}}$$

$$\simeq 0.769\ \text{MPa}$$



Resultant force on left board with area $A^* = T^*$

$$\therefore\ T = \tfrac{1}{2}\left[\sigma_t^{①} + \sigma_t^{③}\right] * A^* = \tfrac{1}{2}\left[10. + 0.769\right] * 0.06$$

$$= 0.3231\ \text{MN} \Rightarrow T^* \cong 323.1\ \text{kN}$$

Problem # 5

The bending moment diagram (BMD) and the cross-section of a beam are shown.

a) Sketch the bending stress variation along the y-axis at location B indicating critical values. 6

b) Determine the resultant force that the bending stresses produce on the flange at location B. 6

c) Determine the maximum tensile stress and compressive stress in the whole beam and indicate where each one acts. 8

Take I = 3 x 10⁴ mm⁴

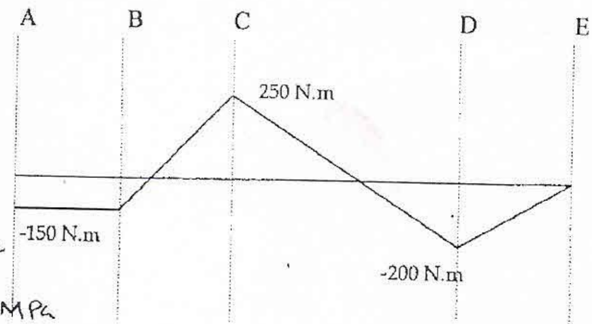
a) $\sigma_{(y)} = -\dfrac{My}{I}$ (2)

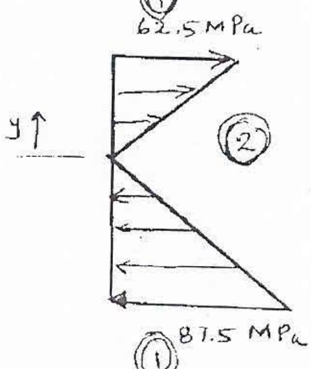
$\sigma_{(y)} = -\dfrac{(-150\times10^3)\,y}{3\times10^4}$

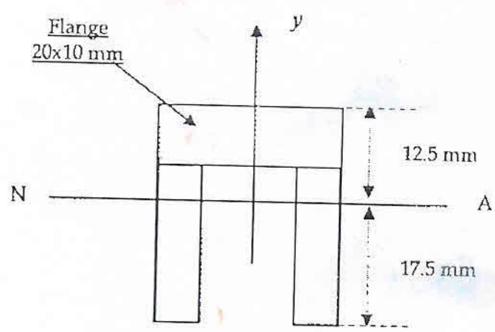
$\sigma_{(y)} = 5y$ MPa

$\sigma_{top} = 5(12.5) = \underline{62.5\text{ MPa}}$

$\sigma_{bottom} = 5(-17.5) = \underline{-87.5\text{ MPa}}$







Cross section

b) $F = \sigma_{avg} * A_{flew} = \dfrac{(\sigma_{top} + \sigma_{bot})}{2}(20\times10)$ (4)

$\sigma_{bottom} = 5y\Big|_{y=12.5-10} = 12.5\text{ MPa}$

$\therefore F = \dfrac{62.5 + 12.5}{2} * (20*10) = \underline{7500}$ N (2)

c) Consider two locations: max +ve moment (C) & max -ve moment (D) (2)
Since section is not symmetric wrt N.A.

<u>At C</u>

$\sigma_{top} = -\dfrac{(250\times10^3)\times12.5}{3\times10^4} = -104.2$ MPa (2)

$\sigma_{bottom} = -250\times10^3\times(-17.5)/3\times10^4 = 145.8$ MPa ← $\max_T$ (1) bottom C

<u>At D</u>

$\sigma_{top} = -\dfrac{(-200\times10^3)(12.5)}{3*10^4} = +83.3$ MPa (2)

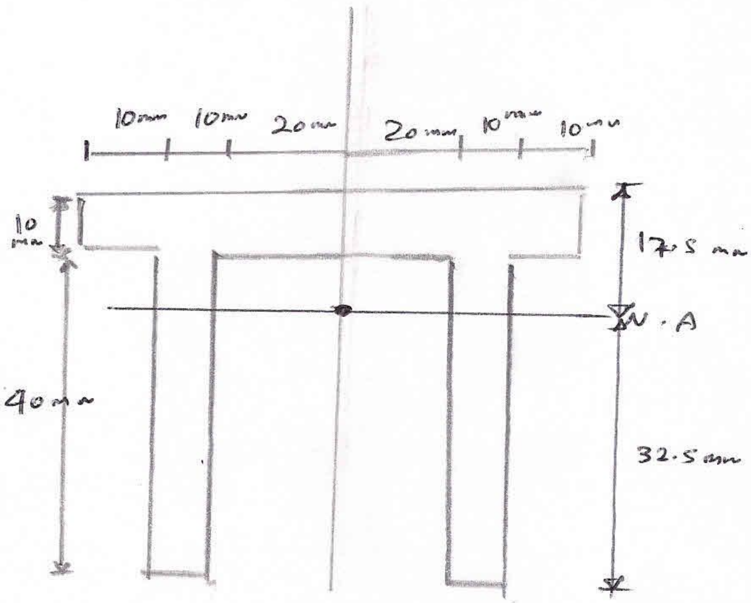
$\sigma_{bottom} = -(-200\times10^3)(-17.5)/3\times10^4 = -116.7$ MPa ← $\max_C$ (1) bottom D problem # 1

Given :-

* the cross-section as shown

* $\sigma_{(allow)\,Tensile} = 20\ MPa$

* $\sigma_{(allow)\,comp} = 40\ MPa$



Needed :-

1. largest positive moment
&
2. largest negative moment

that can be applied safely to the cross-section

Solution

$$\sigma_{max} = \frac{Mc}{I}$$

$$\bar{y} = \frac{[0.005(.08)(0.01) + 2(0.03)(.04)(.01)]}{(0.08)(.01) + 2(0.04)(0.01)} = .0175\ m$$

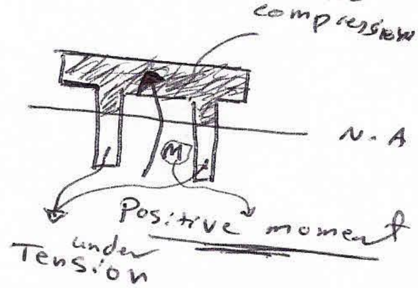
$$I = \frac{1}{12}(.08)(.01)^3 + 0.08(0.01)(.0125^2) + 2\left[\frac{1}{12}(.01)(0.04)^3 + (0.01)(.04)(.0125^2)\right]$$

$$I = 0.3633 \times 10^{-6}\ m^4.$$

1. largest positive moment

$$\sigma_{max\,(Ten)} = 20 \times 10^6 = \frac{M_1(.0325)}{0.3633 \times 10^{-6}}$$

ANS $\Rightarrow \boxed{M_1 = 223.57\ N.m}$

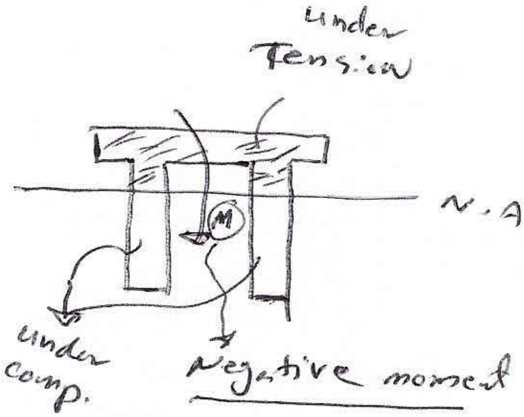


$$\sigma_{max\,(Comp)} = 40 \times 10^6 = \frac{M_2(.0175)}{0.3633 \times 10^{-6}}$$

$$M_2 = 830.4\ N.m$$

continue on problem (1)

(2) largest negative moment



$$\underset{(Ten)}{\sigma_{max}} = 20 \times 10^{6} = \frac{M_1(0.0175)}{0.3633 \times 10^{-6}}$$

ANS $\Rightarrow$ $\boxed{M_1 = 415.2\ N.m}$

$$\underset{(Comp)}{\sigma_{max}} = 40 \times 10^{6} = \frac{M_2(.0325}{0.3633 \times 10^{-6}}$$

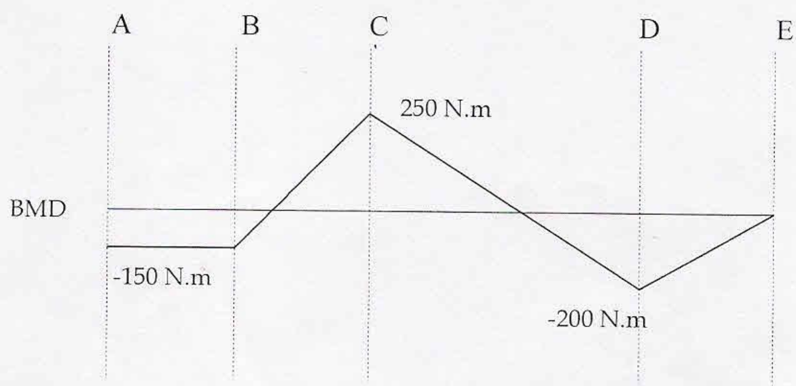
$$M_2 = 447.17\ N.m$$

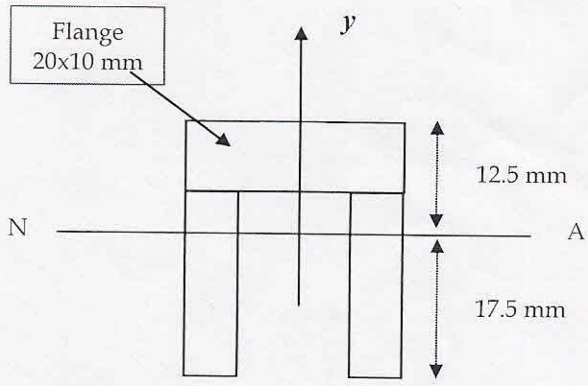
Problem # 5

The bending moment diagram (BMD) and the cross-section of a beam are shown.

a) Sketch the bending stress variation along the y-axis at location B.
b) Determine the resultant force bending stresses produce on the flange at location B.
c) Determine the maximum tensile and compressive stresses in the whole beam and indicate where they act.

Take I = 3 x 10^4 mm^4





Cross section

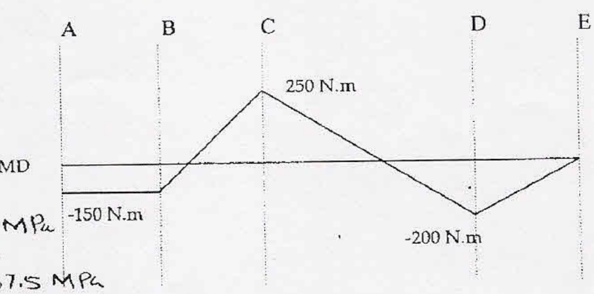
Problem # 5

The bending moment diagram (BMD) and the cross-section of a beam are shown.

a) Sketch the bending stress variation along the y-axis at location B indicating critical values. 6
b) Determine the resultant force that the bending stresses produce on the flange at location B. 6
c) Determine the maximum tensile stress and compressive stress in the whole beam and indicate where each one acts. 8

Take I = 3 x 10⁴ mm⁴

a) $\sigma_{(y)} = -\frac{My}{I}$ (2)

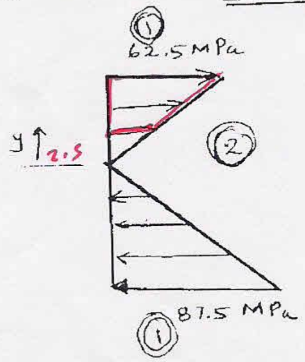


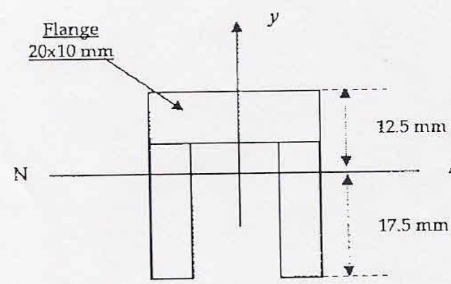
$\sigma_{(y)} = -\frac{(-150 \times 10^3)\,y}{3 \times 10^4}$

$\sigma_{(y)} = 5y$ MPa

$\sigma_{top} = 5(12.5) = \underline{62.5}$ MPa

$\sigma_{bottom} = 5(-17.5) = \underline{-87.5 \text{ MPa}}$





Cross section

b) $F = \sigma_{avg} * A_{flew} = \frac{(\sigma_{top} + \sigma_{bot})}{2}(20 \times 10)$ (4)

$\sigma_{bottom} = 5y\Big|_{y = 12.5 - 10} = 12.5$ MPa

$\therefore F = \frac{62.5 + 12.5}{2} * (20 * 10) = \underline{7500}$ N (2)

c) consider two locations: (2) max +ve moment (C) & max -ve moment (D)
Since section is not symmetric wrt N.A.

At C

$\sigma_{top} = -\frac{(250 \times 10^3) \times 12.5}{3 \times 10^4} = -104.2$ MPa (2)

$\sigma_{bottom} = -250 \times 10^3 \times (-17.5)/3 \times 10^4 = 145.8$ MPa ← max T

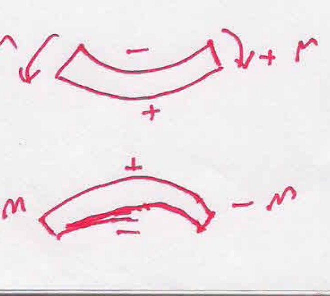
(1) bottom C

At D

$\sigma_{top} = -\frac{(-200 \times 10^3)(12.5)}{3 \times 10^4} = +83.3$ MPa (2)

$\sigma_{bottom} = -(-200 \times 10^3)(-17.5)/3 \times 10^4 = -116.7$ MPa ← max C

(1) bottom D



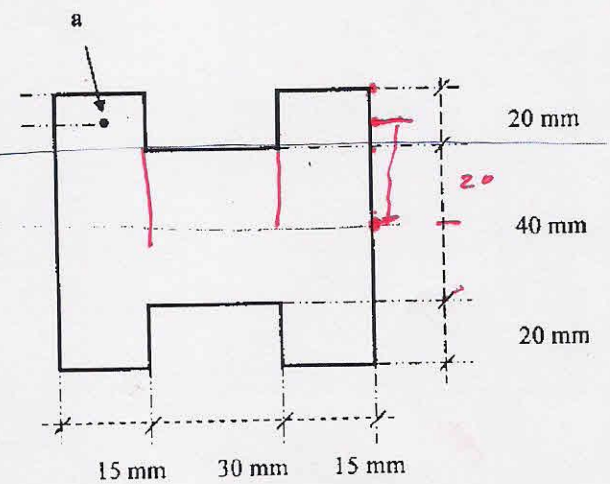
Problem # 5

The beam with the shown cross-section is subjected to a vertical shear force of 20 kN.

a) Determine the moment of inertia about the neutral axis.
b) Determine the shear stress at point a.
c) Determine the maximum shear stress and indicate where it acts.

$2 \times 15 \times 40 \times \frac{40}{2}$

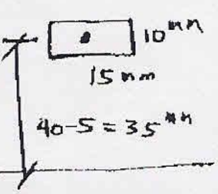
ⓐ divide into 3 rectangles



$$I_z = 2\left(\tfrac{1}{12} \times 15 \times 80^3\right) \quad (3)$$
$$+ \tfrac{1}{12} \times 30 \times 40^3$$
$$I_z = 144 \times 10^4 \ mm^4 \quad (2)$$

ⓑ $\tau_a = \frac{VQ}{It}$ (2)

$Q = 15 \times 10 \times 35$ (1)
$= 5250 \ mm^3$

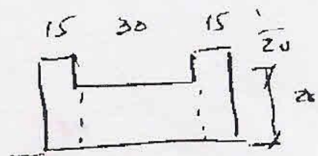


$\therefore \tau_a = \frac{20 \times 10^3 \times 15 \times 10 \times 35}{144 \times 10^4 \times 15}$ (1)

$\tau_a = 4.9 \ MPa$ (1)

ⓒ realizing that (2)
τ varies with $\frac{Q}{t}$
we have to check two points (locations)
① max Q → at N/A
② min t → at 20 mm above N. A

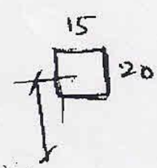
$\tau_{N/A} = \frac{VQ}{It}$ (2)



$Q = 2 \times 15 \times \frac{40^2}{2}$ (1)
$+ 30 \times \frac{20^2}{2} = 30{,}000 \ mm^3$

$\therefore \tau_{N/A} = \frac{20 \times 10^3 (\quad)}{144 \times 10^4 \times 60}$ (1) $= 6.9 \ MPa$ (1)

$\tau_{20^+ mm \text{ above N.A.}} = \frac{VQ}{It}$



$Q = 15 \times 20 \times 30$ (1)
$t = 15 \ mm$

$\therefore \tau_{20^+ mm} = \frac{20 \times 10^3 (9000)}{144 \times 10^4 \times 15} = 8.3 \ MPa$

$\therefore \tau_{max} = 8.3 \ MPa$ acts 20 mm from top of section (2)

Problem ②

Given:- The following beam has a rectangular cross-section as shown. P = 1.5 KN.

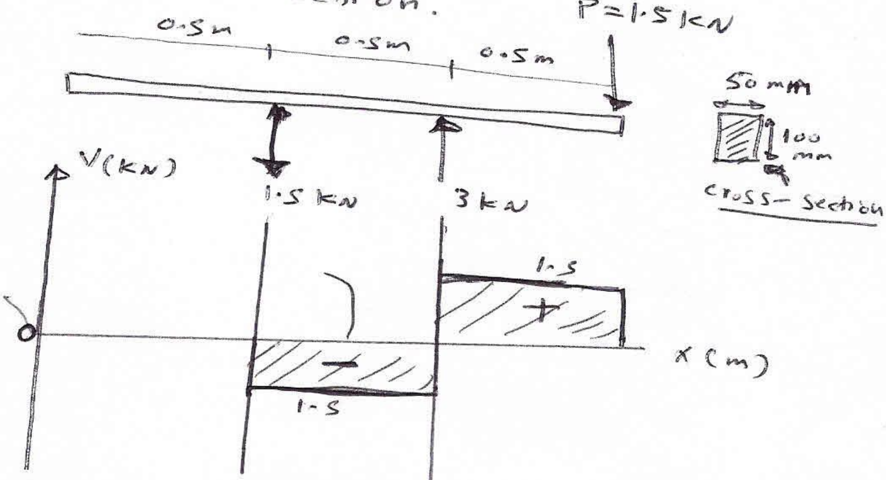
Needed

① Determine the maximum bending stress in the beam.

② Sketch the stress distribution acting over the cross-section.

Solution

① Draw S & M diagram to get the maximum moment as shown.



shear diagram

$M_{max} = 0.75$ KN.m

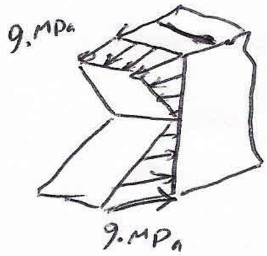
$I_{N.A} = \frac{1}{12}(0.05)(0.1^3)$

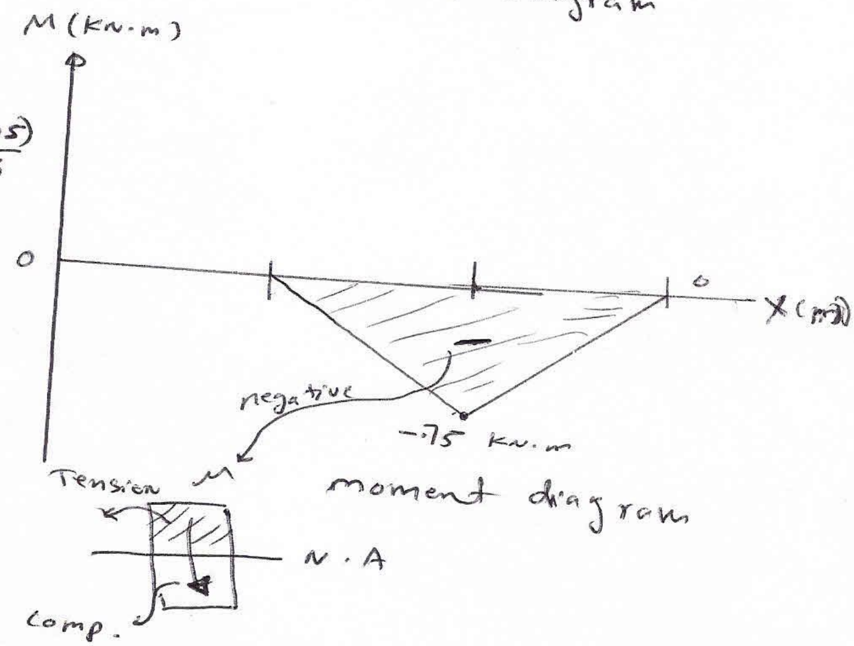
$I_{N.A} = 4.167 \times 10^{-6}\ m^4$

$\sigma_{max} = \frac{M_{max}\, C}{I}$

$\sigma_{max} = \frac{0.75 \times 10^3 (0.05)}{4.167 \times 10^{-6}}$

$\sigma_{max} = 9$ MPa



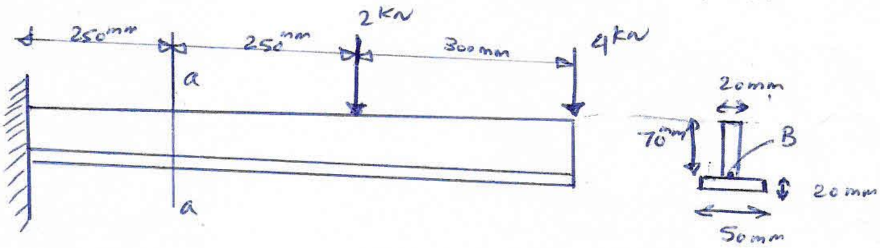


moment diagram

Contine on lecture 24

Problem 7-29

Determine the shear stress at Point B on the Web of the cantilevered strut at section a-a.

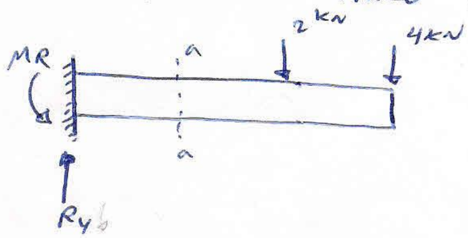


Solution

$$\tau = \frac{VQ}{It}$$

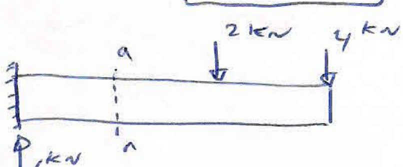
Step #1

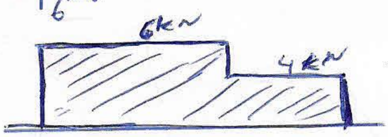
To find V, we should draw the shear diagram and Pick the shear Value at location a-a or 250 mm from the fixed support.



$\Sigma F_y = 0$, $R_y - 2 - 4 = 0$

$$\boxed{R_y = 6\ kN}$$



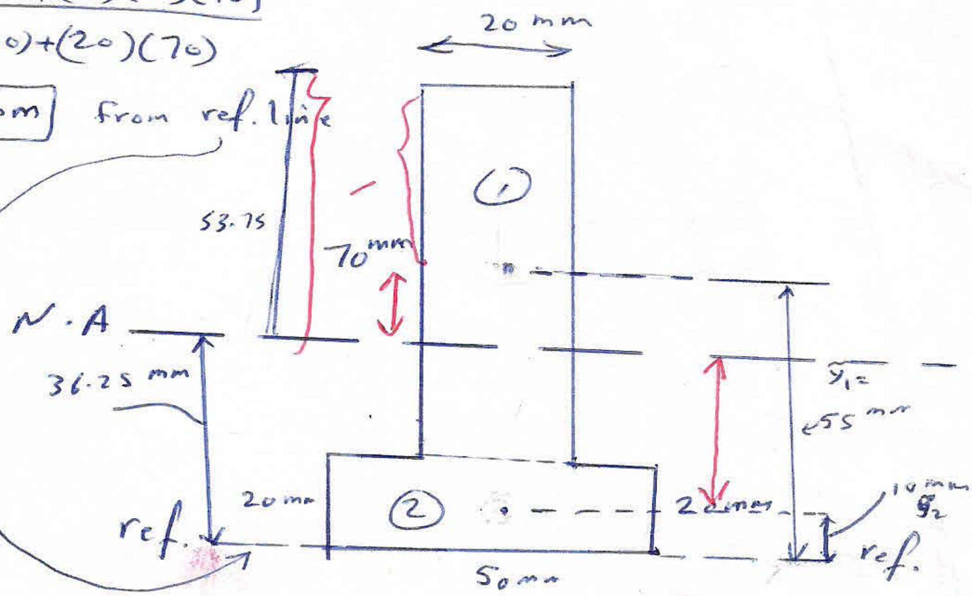


V-diagram

at section a-a, $\boxed{V_{a-a} = 6\ kN}$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{(20)(70)(55)+(20)(50)(10)}{(20)(50)+(20)(70)}$$

$N \cdot A = \boxed{36.25\ mm}$ from ref. line



$$I_{N.A} = \Sigma I_x + A d_y^2$$

$$I_{N.A} = \left[\frac{(20)(70)^3}{12} + (20)(70)(18.75)^2\right] + \left[\frac{(50)(20)^3}{12} + (20)(50)(26.25)^2\right]$$

$$I_{N.A} = 178625\ mm^4 = \boxed{1.78625 \times 10^{-6}\ m^4}$$

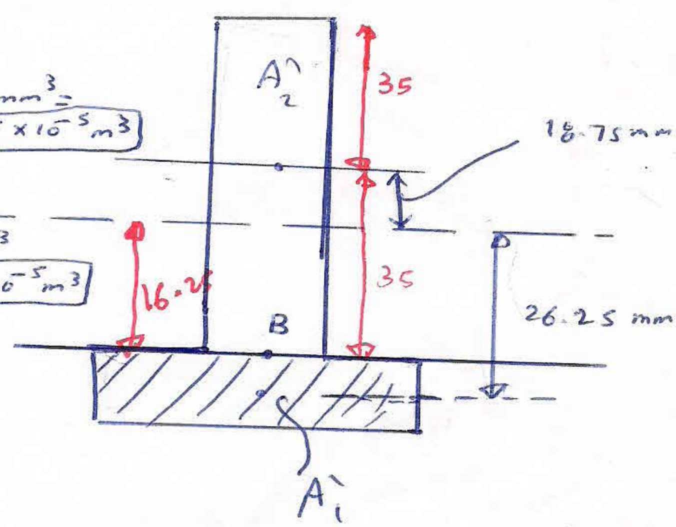
Find Q =

$$Q = (20)(50)(26.25) = 26250\ mm^3 = \boxed{2.625 \times 10^{-5}\ m^3}$$

or

$$Q = (70)(20)(18.75) = 26250\ mm^3 = \boxed{2.625 \times 10^{-5}\ m^3}$$

N.A



$$\tau = \frac{(6)(2.625 \times 10^{-5})}{(0.02)(1.78625 \times 10^{-6})}$$

$$= 4408.67\ kPa = \boxed{4.41\ MPa}$$