b) Use Mohr's circle to determine the principal stresses acting on this element and show them on a properly oriented element.

c) Use Mohr's circle to determine the state of stress on an element oriented 30° **counterclockwise** from the position shown and show them on a properly oriented element.

$C = (\sigma_{avg}, 0)$

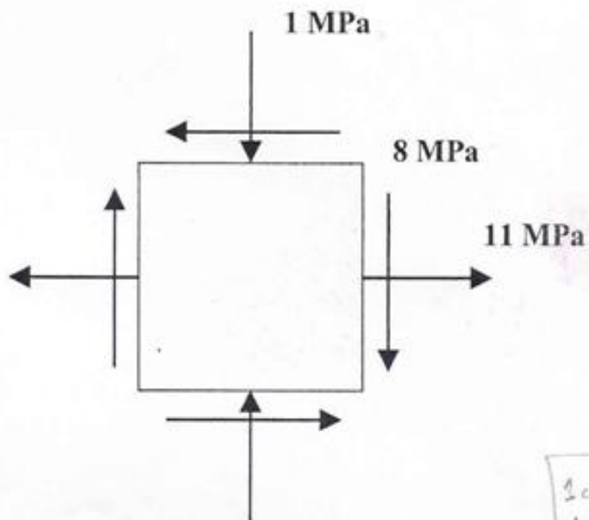
$A = (\sigma_x, \tau_{xy})$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$

$C = (5, 0)$

$A = (11, -8)$



$\sigma_x = 11$ MPa

$\sigma_y = -1$ MPa

$\tau_{xy} = -8$ MPa

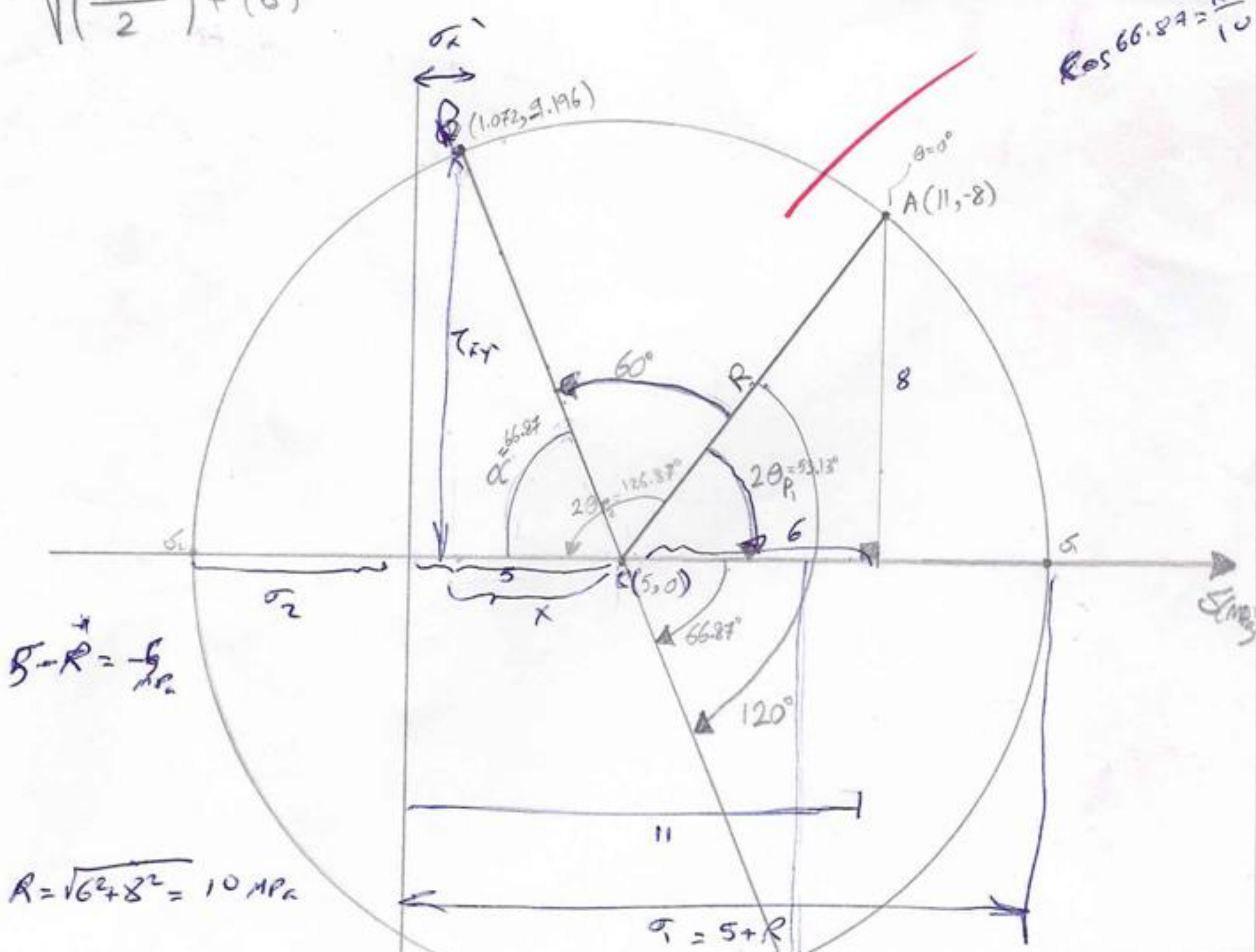
$\sigma_{avg} = \frac{11-1}{2} = 5$ MPa

$R = \sqrt{\left(\frac{12}{2}\right)^2 + (-8)^2} = 10$

1 cm = 1.428 MPa
1 MPa = 0.7 cm

a) Draw mohr's circle ?

$\cos 66.87 = \frac{x}{10}$



$\sigma_1 > \sigma_2$

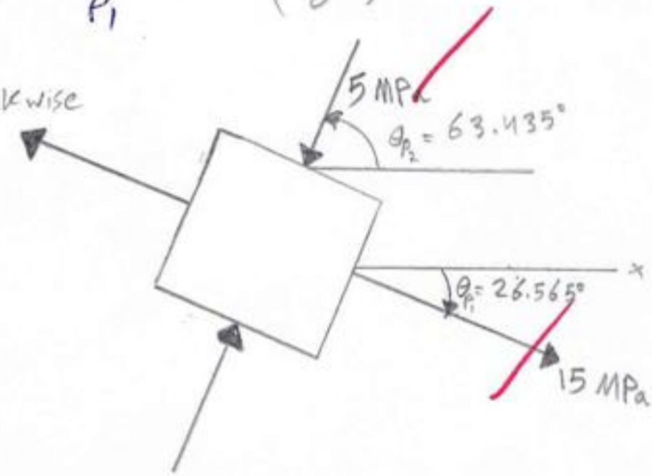
$\sigma_1 = 5 + 10 = 15 \text{ MPa}$

$\sigma_2 = 5 - 10 = -5 \text{ MPa}$

$\tan 2\theta_{p_1} = \frac{8}{6} \Rightarrow 2\theta_{p_1} = \tan^{-1}\left(\frac{8}{6}\right) = 53.13^\circ$

$\theta_{p_1} = 26.565^\circ$ clockwise

$\theta_{p_2} = 90 - 26.565^\circ$
$= 63.435^\circ$



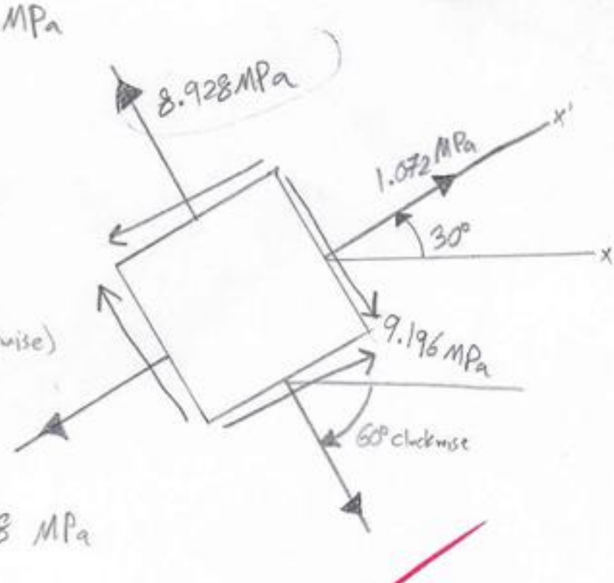
c) $30^\circ = 60^\circ$ in mohr's Circle (counterclockwise)

$\alpha = 126.87^\circ - 60^\circ = 66.87^\circ$

$P(A, B)$

$A = 5 - 10\cos(66.87^\circ) = 1.072 \text{ MPa}$

$B = -10 \sin(66.87^\circ) = -9.196 \text{ MPa}$



$60^\circ = 120^\circ$ in mohr's Circle (Clockwise)

$Q(C, D)$

$C = 5 + 10\cos(66.87^\circ) = 8.928 \text{ MPa}$

a) Determine the normal and shear stresses acting on the inclined plane AB shown in the figure. Show the results on a properly oriented element, indicating orientation and all stresses.

b) Using the original element (*i.e. ignoring AB*), determine the angle at which the shear stresses on the element will be equal to Zero. Show the results on a properly oriented element, indicating orientation and all stresses.

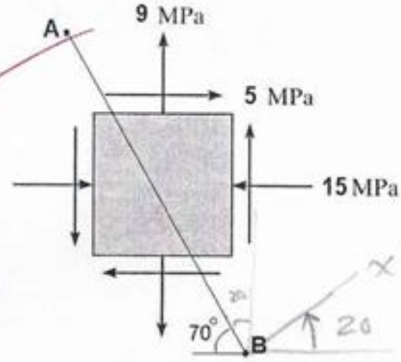
You may use any method that you prefer.

$\sigma_x = -15$ MPa

$\sigma_y = 9$ MPa

A

$\tau_{xy} = 5$ MPa



At $\boxed{\theta = 20}$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{x'} = \frac{-15+9}{2} + \frac{-15-9}{2}\cos(2\times 20) + (5)(\sin(2\times 20))$$

$$= -8.979 \text{ MPa ('C')}$$

At $\theta = 110°$

$$\sigma_{y'} = \frac{-15+9}{2} + \frac{-15-9}{2}\cos(220) + (5)\sin(220)$$

$$= 2.979 \text{ MPa ('T')}$$

At $\theta = 20°$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

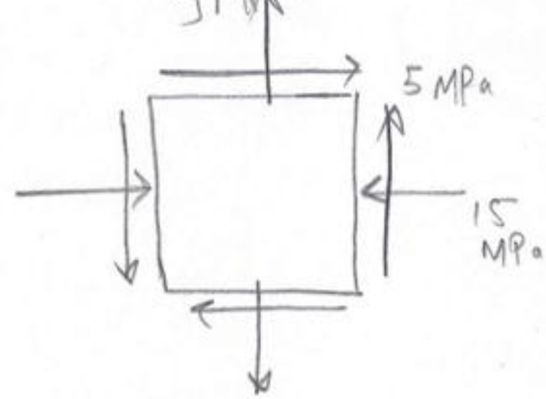
$$= -\frac{-15-9}{2}\sin 40 + 5\cos 40 = 11.54 \text{ MPa}$$

(B)

$\sigma_x = -15$ MPa

$\sigma_y = 9$ MPa

$\tau_{xy} = 5$ MPa



$$\cancel{\tau_{x'y'}} \xrightarrow{} 0 = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$-\frac{-15-9}{2}\sin 2\theta + 5\cdot\cos 2\theta = 0$$

$$12\sin 2\theta + 5\cos 2\theta = 0$$

$$\frac{\cancel{12}\sin 2\theta}{\cos 2\theta} = \frac{-5}{12}\frac{\cancel{\cos 2\theta}}{}$$

$$\tan 2\theta = -\frac{5}{12} \Rightarrow \underline{\underline{\theta = -11.31^\circ}}$$

Same

<u>or</u> By Principle Stress

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)} = \frac{5}{\left(\frac{-15-9}{2}\right)} \Rightarrow \theta_{p_2} = -11.31^\circ$$

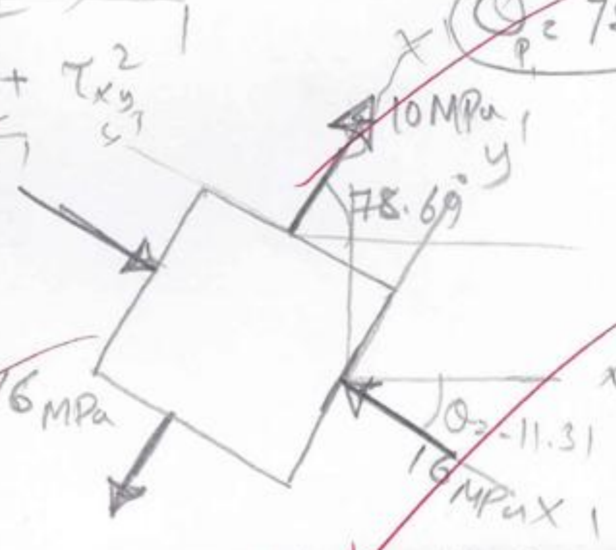
$\theta_{p_1} = 78.69^\circ$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_2}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-15+9}{2} \pm \sqrt{\left(\frac{-15-9}{2}\right)^2 + 5^2}$$

$$= -3 \pm 13$$

$\sigma_1 = 10$ MPa ; $\sigma_2 = -16$ MPa



~~At~~

$\theta = -11.31$

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)} = \frac{\quad}{\left(\frac{-15-9}{2}\right)} \Rightarrow \theta_{p_2} = -11.31$$

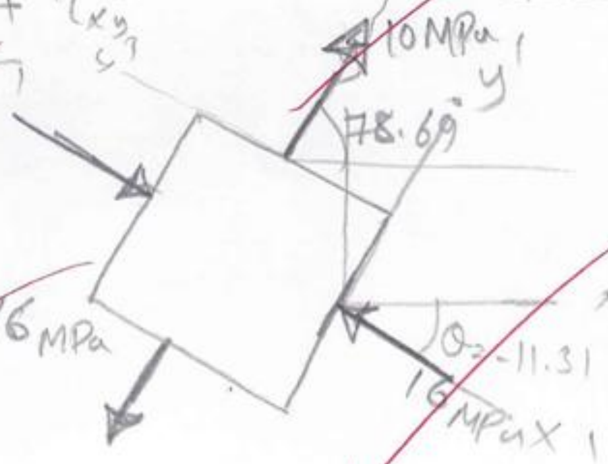
$$\boxed{\theta_{p_1} = 78.69^\circ}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_2}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-15+9}{2} \pm \sqrt{\left(\frac{-15-9}{2}\right)^2 + 5^2}$$

$$= -3 \pm 13$$

$$\sigma_1 = 10\ \text{MPa}\ ; \ \sigma_2 = -16\ \text{MPa}$$

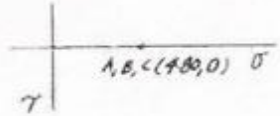


At $\boxed{\theta = -11.31}$

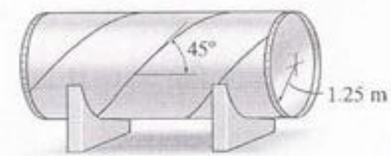
$$\sigma_{x'} = \frac{-15+9}{2} + \frac{-15-9}{2} \cos(2 \times -11.31) + (5)(\sin(2 \times -11.31))$$

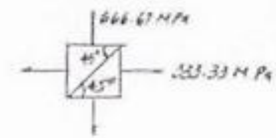
$$= -16\ \text{MPa}$$

$\boxed{\text{No shear stress}}$



9–78. The cylindrical pressure vessel has an inner radius of 1.25 m and a wall thickness of 15 mm. It is made from steel plates that are welded along the 45° seam. Determine the normal and shear stress components along this seam if the vessel is subjected to an internal pressure of 8 MPa.





$$\sigma_x = \frac{pr}{2t} = \frac{8(1.25)}{2(0.015)} = 333.33 \text{ MPa}$$

$$\sigma_y = 2\sigma_x = 666.67 \text{ MPa}$$

$A(333.33, 0) \qquad B(666.67, 0) \qquad C(500, 0)$

$$\sigma_{x'} = \frac{333.33 + 666.67}{2} = 500 \text{ MPa}$$ **Ans.**

$$\tau_{x'y'} = R = 666.67 - 500 = 167 \text{ MPa}$$ **Ans.**