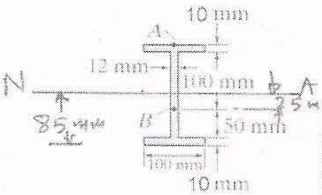
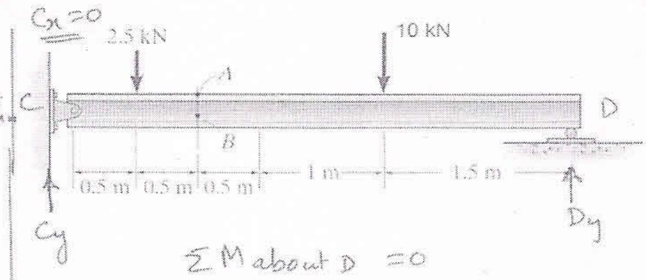
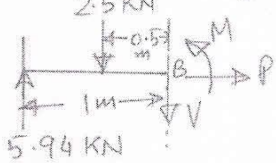
2 - The given I-beam is subjected to the loading shown. Determine the stress components (normal & shear) at points *A* and *B* and show the results on a differential element at each of these points. Use the shear formula (*VQ/Ib*) to compute the shear stress.





$$I_{NA} = \frac{100 \times 170^3}{12} - \frac{88 \times 150^3}{12}$$
$$= 1.619 \times 10^7 \ mm^4$$

$$\Sigma M_{about\ D} = 0$$
$$\Rightarrow -4C_y + 2.5 \times 3.5 + 10 \times 1.5 = 0$$
$$\Rightarrow C_y = 5.94 \ kN$$

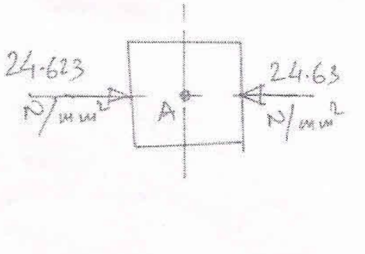


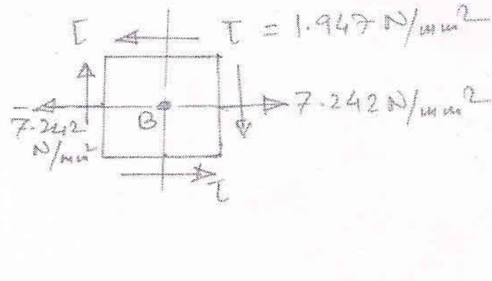
$$\Sigma F_x = 0 \Rightarrow P = 0$$
$$\Sigma F_y = 0 \Rightarrow 5.94 - 2.5 - V = 0$$
$$\Rightarrow V = 3.44 \ kN$$
$$\Sigma M = 0$$
$$\Rightarrow -5.94 \times 1 + 2.5 \times 0.5 + M = 0$$
$$\Rightarrow M = 4.69 \ kN\text{-}m$$

$$Q_B = 100 \times 10 \times (85 - 5) + 50 \times 12 \times (25 + 25)$$
$$= 11 \times 10^4 \ mm^3$$

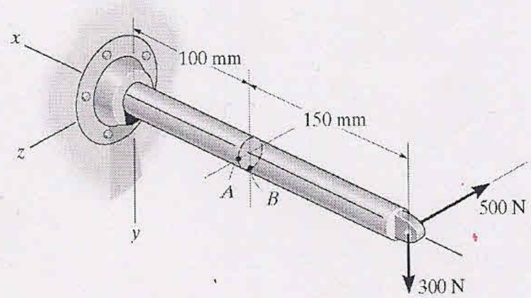
A:
$$\sigma_A = \frac{4.69 \times 10^6 \times 85}{1.619 \times 10^7} = 24.623 \ N/mm^2 \ (\text{Compressive})$$
$$\tau_A = 0$$

B:
$$\sigma_B = \frac{4.69 \times 10^6 \times 25}{1.619 \times 10^7} = 7.242 \ N/mm^2 \ (\text{Tensile})$$
$$\tau_B = \frac{3.44 \times 10^3 \times 11 \times 10^4}{1.619 \times 10^7 \times 12} = 1.947 \ N/mm^2$$



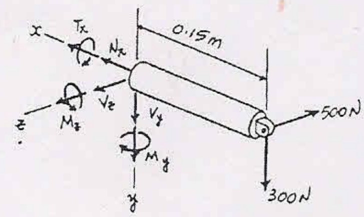


8–46. Solve Prob. 8–45 for point B.



Internal Forces and Moment :

$$\Sigma F_x = 0;\quad N_x = 0$$
$$\Sigma F_y = 0;\quad V_y + 300 = 0 \quad V_y = -300\text{ N}$$
$$\Sigma F_z = 0;\quad V_z - 500 = 0 \quad V_z = 500\text{ N}$$
$$\Sigma M_x = 0;\quad T_x = 0$$
$$\Sigma M_y = 0;\quad M_y - 500(0.15) = 0 \quad M_y = 75.0\text{ N}\cdot\text{m}$$
$$\Sigma M_z = 0;\quad M_z - 300(0.15) = 0 \quad M_z = 45.0\text{ N}\cdot\text{m}$$



Section Properties :

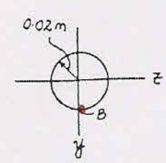
$$A = \pi\left(0.02^2\right) = 0.400\left(10^{-3}\right)\pi\text{ m}^2$$
$$I_x = I_y = \frac{\pi}{4}\left(0.02^4\right) = 40.0\left(10^{-9}\right)\pi\text{ m}^4$$
$$J = \frac{\pi}{2}\left(0.02^4\right) = 80.0\left(10^{-9}\right)\pi\text{ m}^4$$
$$(Q_B)_y = 0$$
$$(Q_B)_z = \frac{4(0.02)}{3\pi}\left[\frac{1}{2}\pi\left(0.02^2\right)\right] = 5.333\left(10^{-6}\right)\text{ m}^3$$

$(Q_B)_z = \checkmark$

$(Q_B)_y = 0$

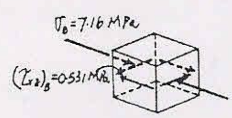
Normal Stress :

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$



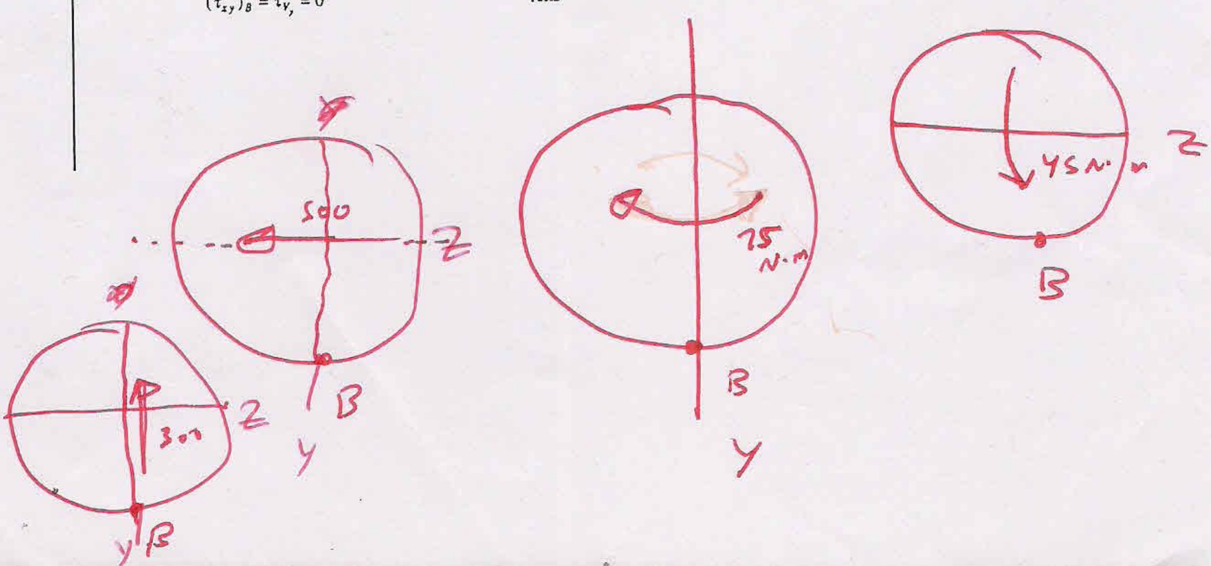
$$\sigma_B = 0 - \frac{45.0(0.02)}{40.0(10^{-9})\pi} + \frac{75.0(0)}{40.0(10^{-9})\pi}$$
$$= -7.16\text{ MPa} = 7.16\text{ MPa (C)}$$ **Ans**

Shear Stress : The tranverse shear stress in the z and y directions can be obtained using the shear formula, $\tau_V = \dfrac{VQ}{It}$.



$$(\tau_{xz})_B = \tau_{V_z} = \frac{500\left[5.333(10^{-6})\right]}{40.0(10^{-9})\pi\ (0.04)}$$
$$= 0.531\text{ MPa}$$ **Ans**

$$(\tau_{xy})_B = \tau_{V_y} = 0$$ **Ans**



CIVIL ENGINEERING DEPARTEMNT - KFUPM
CE 203 STRUCTURAL MECHANICS I
Second Semester: '09- '10
HOME WORK ASSIGNMENT NO. 13

- **Textbook Sections Covered:** 8.2, 9.1-9.3.
- **Subject material covered:** Generalized Hook's Law & Torsion Formula

• DUE DATE: **Wednesday, June 02, '10**

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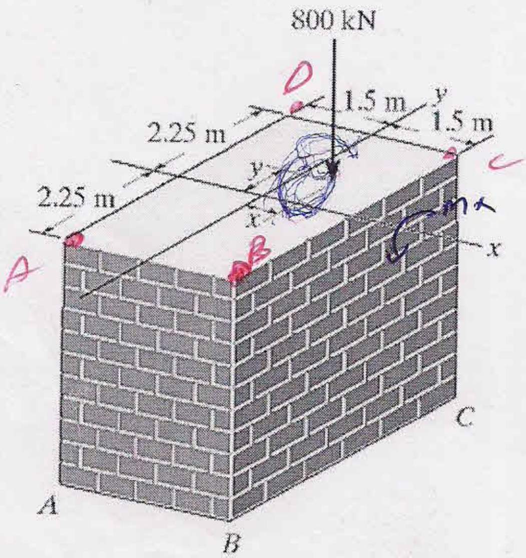
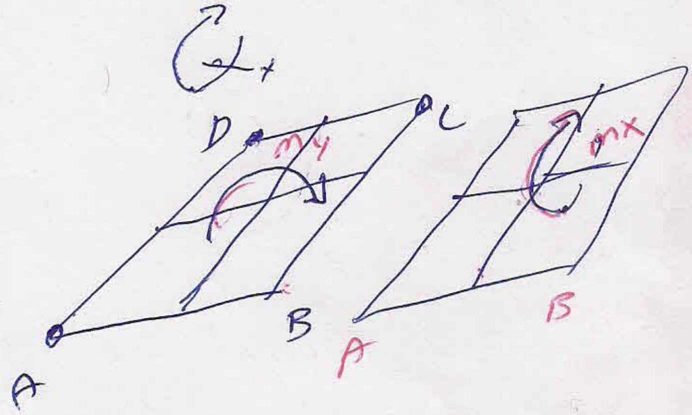
1. **Solve problem # 8-60, let x = 0.5 m, y = 0.75 m (page 461)**

Solution





$$A = 3(4.5) = 13.5\,m^2$$

$$I_x = \frac{1}{12}3(4.5)^3 = 22.78125\,m^4$$

$$I_y = \frac{1}{12} * 3^3 * 4.5 = 10.125\,m^4$$

$$M_x = 800 * 0.75 = 600\,kN.m$$
$$M_y = 800 * 0.5 = 400\,kN.m$$

$mx - 600 = 0$

$mx = 600$ N.m

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y y}{I_y}$$

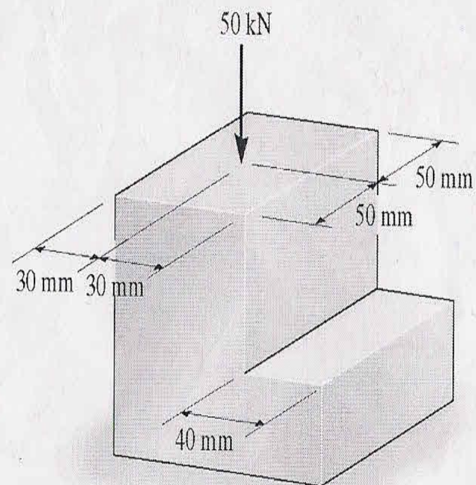
$$\sigma_A = \frac{-800(10^3)}{13.5} + \frac{600(10^3)(2.25)}{22.78125} + \frac{400(10^3)(1.5)}{10.125} = 59.259\,kPa\,(T)$$

$$\sigma_B = \frac{-800(10^3)}{13.5} + \frac{600(10^3)(2.25)}{22.78125} - \frac{400(10^3)(1.5)}{10.125} = -59.259\,kPa\,(C)$$

$$\sigma_C = \frac{-800(10^3)}{13.5} - \frac{600(10^3)(2.25)}{22.78125} - \frac{400(10^3)(1.5)}{10.125} = -177.78\,kPa\,(C)$$

$$\sigma_D = \frac{-800(10^3)}{13.5} - \frac{600(10^3)(2.25)}{22.78125} + \frac{400(10^3)(1.5)}{10.125} = -59.259\,kPa\,(C)$$

8–25. The stepped support is subjected to the bearing load of 50 kN. Determine the maximum and minimum compressive stress in the material.



Internal Force and Moment: As shown on FBD.

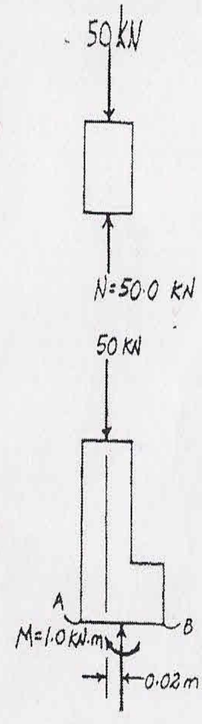
Section Properties: For the top portion of the stepped support.

$$A = 0.06(0.1) = 0.00600 \text{ m}^2$$

For the bottom portion of the stepped support.

$$A = 0.1(0.1) = 0.0100 \text{ m}^2$$

$$I = \frac{1}{12}(0.1)\left(0.1^3\right) = 8.333\left(10^{-6}\right) \text{ m}^4$$



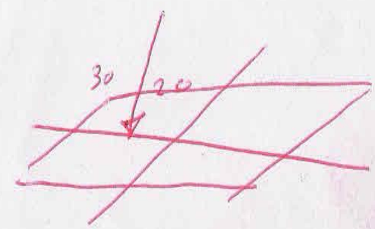
Normal Stress: For the top portion of the stepped support.

$$\sigma = \frac{N}{A} = \frac{-50.0(10^3)}{0.00600} = -8.33 \text{ MPa} = 8.33 \text{ MPa (C)}$$

For the bottom portion of the stepped support.

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

$$= \frac{-50.0(10^3)}{0.0100} \pm \frac{1.00(10^3)(0.05)}{8.333(10^{-6})}$$

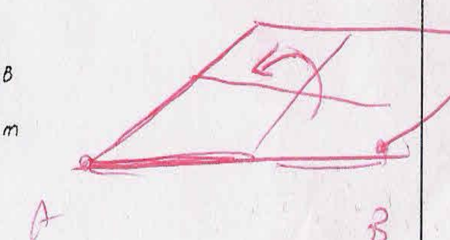


$$\sigma_A = -5.00\left(10^6\right) - 6.00\left(10^6\right)$$

$$= -11.0 \text{ MPa} = 11.0 \text{ MPa (C)}$$

$$\sigma_B = -5.00\left(10^6\right) + 6.00\left(10^6\right)$$

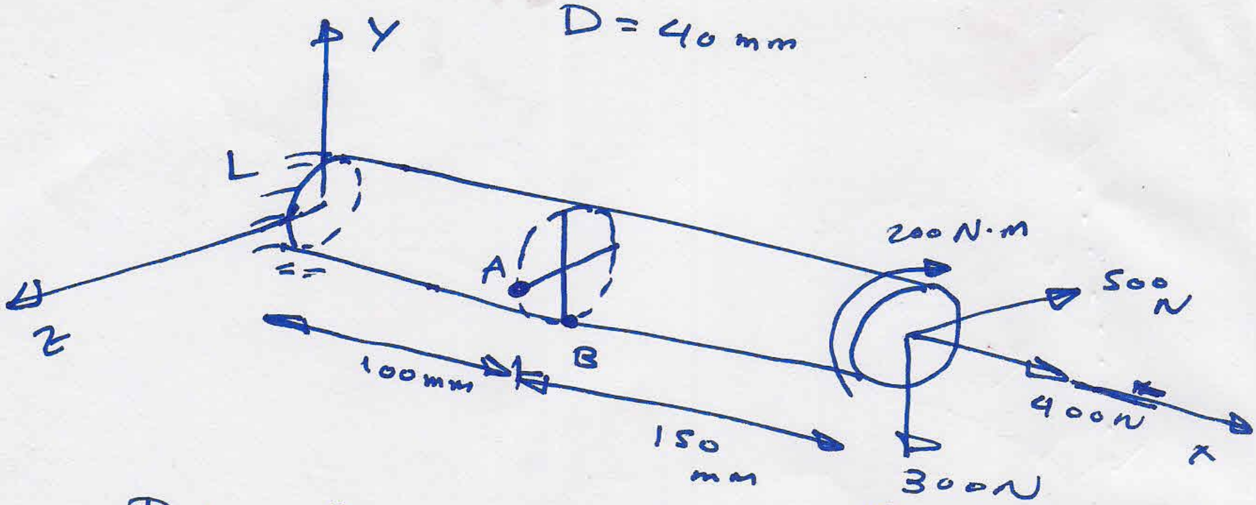
$$= 1.00 \text{ MPa (T)}$$



Therefore, the maximum and minimum compressive stress is

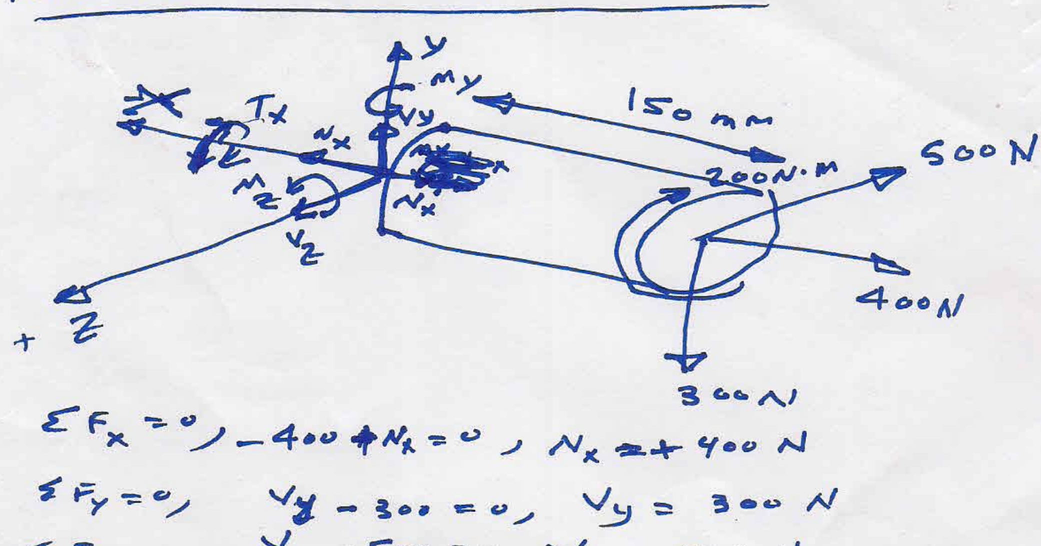
$(\sigma_C)_{max} = 11.$ MPa **Ans**

$(\sigma_C)_{min} = 0$ **Ans**



Determine States of stress at Point A, and at Point B. Show the results on an element located at each point separately.

Internal forces and moments



$\Sigma F_x = 0, \; -400 + N_x = 0, \; N_x = +400 \text{ N}$

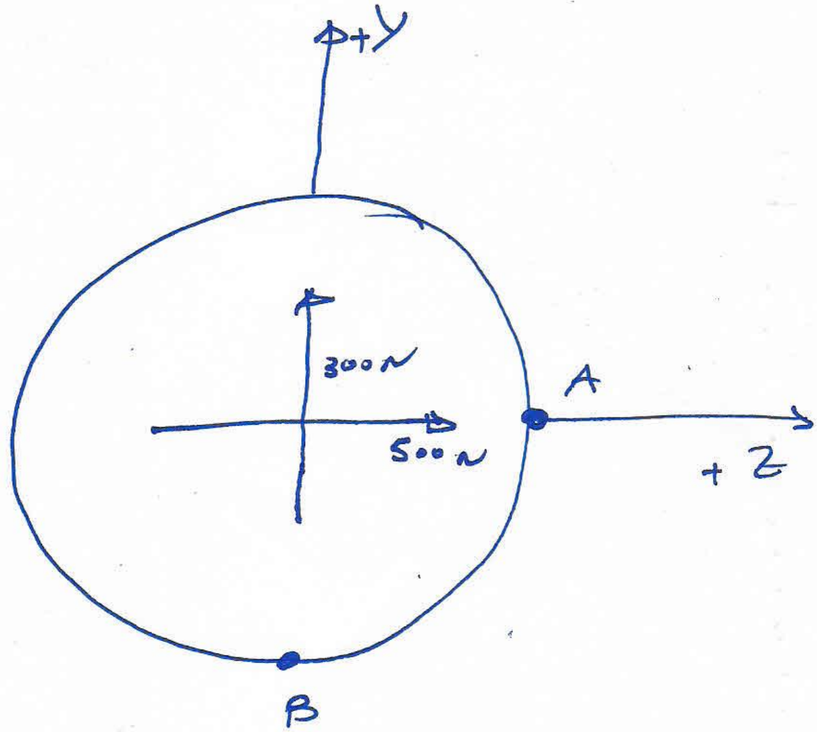
$\Sigma F_y = 0, \; V_y - 300 = 0, \; V_y = 300 \text{ N}$

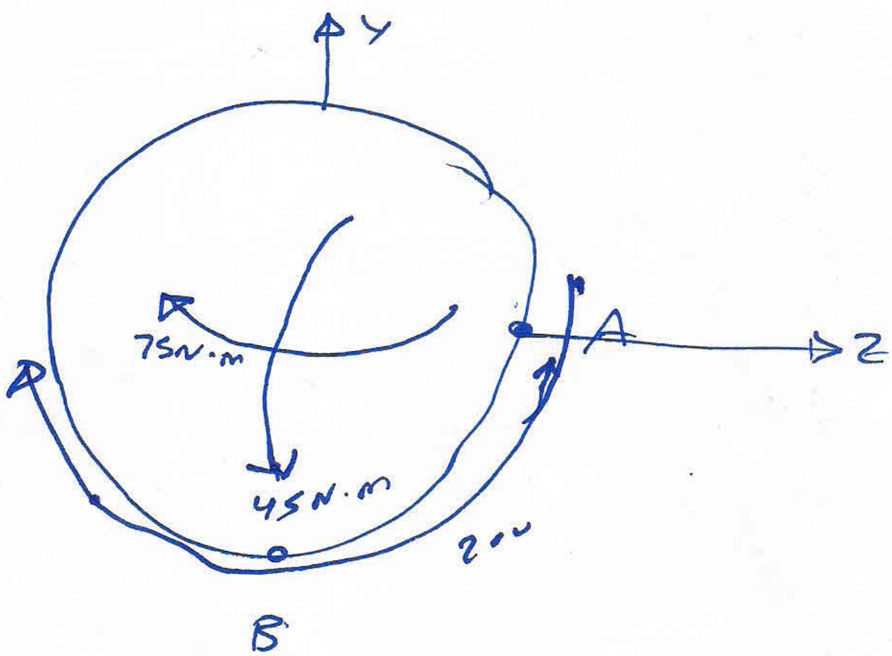
$\Sigma F_z = 0, \; V_z - 500 = 0, \; V_z = 500 \text{ N}$

$\Sigma M_x = 0, \; T_x - 200 = 0, \; T_x = 200 \text{ N·m}$

$\Sigma M_y = 0, \; M_y + (500)(.15) = 0, \; M_y = -75 \text{ N·m}$

$\Sigma M_z = 0, \; M_z - 300(.15) = 0, \; M_z = 45 \text{ N·m}$


+Y
300N
500N
A
+Z
B


Y
75N·m
A
Z
45N·m
200
B

$$\sigma = \frac{N}{A} + \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

Point A

$$Area = \frac{\pi}{4}(.04)^2 = 1.256 \times 10^{-3}\ m^2$$

$$I = \frac{\pi}{4} c^4 = \frac{\pi}{4}(.02)^4 = 1.257 \times 10^{-7}\ m^4$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(.02)^4 = 2.512 \times 10^{-7}\ m^4$$

① Normal stresses

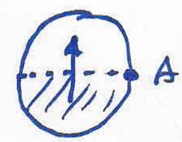
$$\sigma_N = \frac{400}{1.256 \times 10^{-3}} = 0.318\ MPa\ (T)$$

$$\sigma = \frac{(75)(.02)}{1.256 \times 10^{-7}} = 11.94\ MPa\ (T)$$

$$(\sigma_{TOT})_A = 0.318 + 11.94 = 12.258\ MPa\ (T)$$

② Shear stresses

$$(Q_A)_z = 0$$



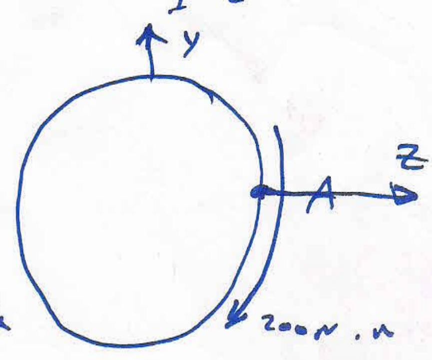
$$(Q_A)_y = \left(\frac{1.256 \times 10^{-3}}{2}\right)\left(\frac{4(.02)}{3\pi}\right) = 5.33 \times 10^{-6}\ m^3$$

$$\tau_{xy} = \frac{(300)(5.33 \times 10^{-6})}{(1.257 \times 10^{-7})(.04)}$$

$$\tau_{xy} = 0.316\ MPa$$

$$\frac{V_y (Q_A)_y}{I\,t}$$

$$\tau_{xy} = \frac{(200)(.02)}{2.512 \times 10^{-7}} = -15.92\ MPa$$



$$\tau_{xy} = 0.316 - 15.92 = -15.6\ MPa$$

$$\tau_{xz} = 0$$

" ~~$M_z(y) = M_y z$~~

Point B

Normal Stress :-

$$(\sigma_N)_B = \frac{400}{1.256 \times 10^{-3}} = 0.318 \text{ MPa (T)}$$

$$(\sigma_b)_B = \frac{(45)(0.02)}{1.256 \times 10^{-7}} = 7.16 \text{ MPa (C)}$$

$$(\sigma_{TOT})_B = 0.318 - 7.16 = -6.85 \text{ MPa (C)}.$$

Shear Stresses

$$(Q_B)_y = 0$$

$$(Q_B)_z = 5.33 \times 10^{-6}\ m^3$$

$$\tau_{xz} = \frac{V_z (Q_B)_z}{I_z t} = \frac{(500)(5.33 \times 10^{-6})}{1.257 \times 10^{-7}(.04)} = 0.53 \text{ MPa}$$

$$\tau_{xz} = \frac{Tc}{J} = \frac{(200)(.02)}{2.512 \times 10^{-7}} = -15.92 \text{ MPa}$$

$$\boxed{\tau_{xz} = -15.39 \text{ MPa}}$$

$$\tau_{xy} = 0$$