Problem 1: (20 points)

The beam shown is made of wood and loaded with a uniformly distributed load w. The beam cross section is symmetric and consists of a T-beam with 2 rectangles added to it using nails (top) and glue (bottom), as shown. Determine the largest value of the load w that can be safely used.

Given:

Moment of Inertia for the **whole cross section** $I = 361.35X10^{6}\ mm^{4}$

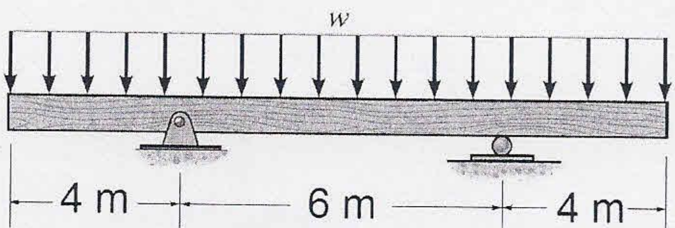
Maximum allowable shear stress for wood = $4\ MPa$.

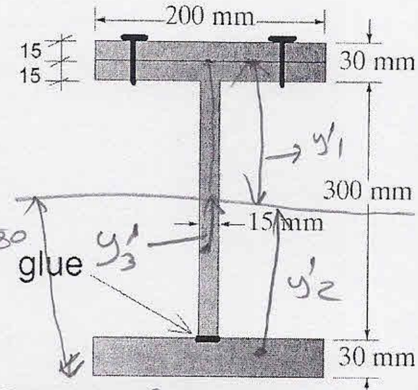
Capacity of each nail = 3 kN : Nail Spacing = 300 mm

Glue Strength = 2 MPa

ch7 quastio

$F = \cancel{w}\ 14w$ equation ①





$I = 361.35\times10^{6}\ mm^4$ / $\tau_{max\ wood} = 4\ MPa$ / $s = 300mm$ / $F_{nail} = 3\ kN$

glue strength = 2 MPa

$F_{nail} = q_{nail} \times spacing \rightarrow q_{nail} = \frac{2\times(F_{nail})}{Spacing} = \frac{2\times3\times10^3 N}{300mm} = 20\frac{N}{mm}$

$\tau_{max} = \frac{V_{max}\ Q_{max}}{I\ t}$ / we will find V_{max} once in term of glue and other in term of nail

for nail: $q = \frac{VQ}{I}$ / $Q = A'y'_1 = (30)(200)(165) = 990\times10^3\ mm^3$

$V_{max\ nail} = \frac{q_{nail} I}{Q} = \frac{20\frac{N}{mm}\times361.35\times10^{6}\ mm^4}{990\ \cancel{\times10^3}\ mm^3} = \boxed{7\cancel{300}\ N}$ units

No

for glue: Glue strength $= \frac{q_{glue}}{b}$ → thickness of glue

q_{glue} = glue strength × thickness of glue

$= 2\frac{N}{mm^2}\times 15\ mm = 30\frac{N}{mm}$

No

$q_{glue} = \frac{VQ}{I}$ & where $Q = y''_3 A'_3 + \cancel{A}\ y'_2 A'_2$

Stress Components

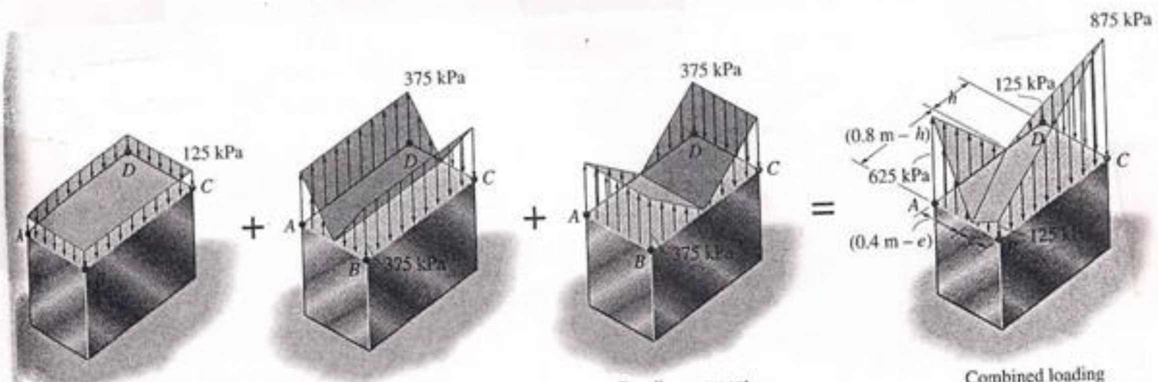
① Normal force

$$\sigma = \frac{P}{A} = \frac{40}{(0.8)(0.4)} = 125 \text{ kPa} \quad (\text{Fig. } 3)$$

② Bending Moment

$$\sigma_{max} = \frac{M_x c_y}{I_x} = \frac{8\,(0.2)}{\frac{1}{12}(0.8)(0.4)^3} = 375 \text{ kPa} \quad (\text{Fig. } 4)$$

$$\sigma_{max} = \frac{M_y c_x}{I_y} = \frac{(16)(0.4)}{\frac{1}{12}(0.4)(0.8)^3} = 375 \text{ kPa} \quad (\text{Fig. } 5)$$



Normal force (40 kN) | Bending moment (8 kN·m) | Bending moment (16 kN·m) | Combined loading

<u>superposition</u> The normal stress at each corner point can be determined by algebraic addition. Assuming tensile stress is positive, we have

$$\sigma_A = -125 + 375 + 375 = 625 \text{ kPa}$$

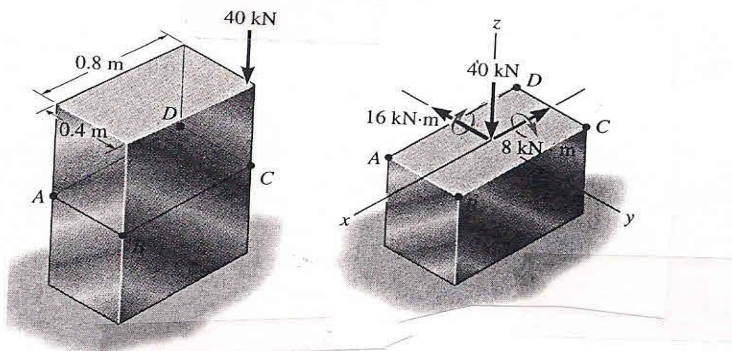
$$\sigma_B = -125 - 375 + 375 = -125 \text{ kPa}$$

$$\sigma_C = -125 - 375 - 375 = -875 \text{ kPa}$$

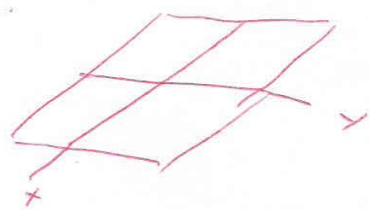
$$\sigma_d = -125 + 375 - 375 = -125 \text{ kPa}$$

Example

The rectangular block of negligible weight is subjected to vertical force of 40 kN, which is applied to its corner, Determine the normal-stress distribution acting on section through ABCD



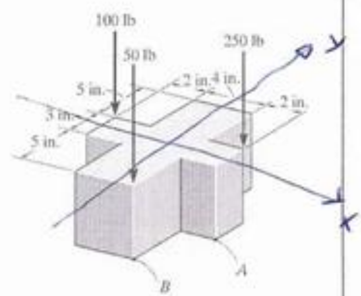
Solution



Internal loadings :-

if we consider the equilibrium of the bottom segment of the block, Fig 2, it is seen that the 40-kN force must act through the centriod of the cross-section and two bending-moment components must also act about the centroidal or principal axes of the inertia for the section

8–74. The block is subjected to the three axial loads shown. Determine the normal stress developed at points A and B. Neglect the weight of the block.



$M_x = -250(1.5) - 100(1.5) + 50(6.5) = -200 \text{ lb} \cdot \text{in.}$

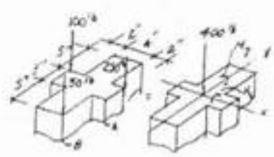
$M_y = 250(4) + 50(2) - 100(4) = 700 \text{ lb} \cdot \text{in.}$

$$I_x = \frac{1}{12}(4)(13^3) + 2\left(\frac{1}{12}\right)(2)(3^3) = 741.33 \text{ in}^4$$

$$I_y = \frac{1}{12}(3)(8^3) + 2\left(\frac{1}{12}\right)(5)(4^3) = 181.33 \text{ in}^4$$

$A = 4(13) + 2(2)(3) = 64 \text{ in}^2$

$$\sigma = \frac{P}{A} - \frac{M_y x}{I_y} + \frac{M_x y}{I_x}$$



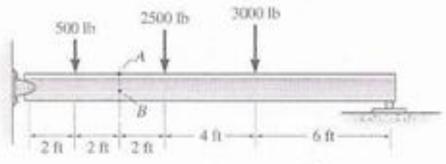
$$\sigma_A = -\frac{400}{64} - \frac{700(4)}{181.33} + \frac{-200(-1.5)}{741.33}$$

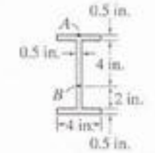
$= -21.3 \text{ psi}$ **Ans.**

$$\sigma_B = -\frac{400}{64} - \frac{700(2)}{181.33} + \frac{-200(-6.5)}{741.33}$$

$= -12.2 \text{ psi}$ **Ans.**

8–35. The wide-flange beam is subjected to the loading shown. Determine the stress components at points A and B and show the results on a volume element at each of these points. Use the shear formula to compute the shear stress.

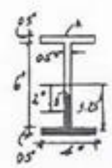




$$I = \frac{1}{12}(4)(7^3) - \frac{1}{12}(3.5)(6^3) = 51.33 \text{ in}^4$$

$$A = 2(0.5)(4) + 6(0.5) = 7 \text{ in}^2$$

$$Q_B = \Sigma \bar{y}' A' = 3.25(4)(0.5) + 2(2)(0.5) = 8.5 \text{ in}^3$$



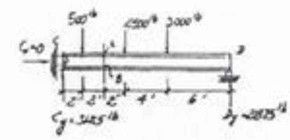
$$Q_A = 0$$

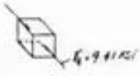
$$\sigma_A = \frac{-Mc}{I} = \frac{-11500\,(12)(3.5)}{51.33} = -9.41 \text{ ksi} \qquad \textbf{Ans.}$$

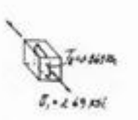
$$\tau_A = 0 \qquad \textbf{Ans.}$$

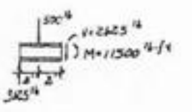
$$\sigma_B = \frac{My}{I} = \frac{11500(12)(1)}{51.33} = 2.69 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{2625(8.5)}{51.33(0.5)} = 0.869 \text{ ksi} \qquad \textbf{Ans.}$$

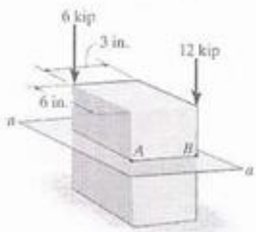








•8–44. Determine the normal stress developed at points A and B. Neglect the weight of the block.



Referring to Fig. a,

$\Sigma F_x = (F_R)_x;\quad -6 - 12 = F \quad F = -18.0 \text{ kip}$

$\Sigma M_y = (M_R)_y;\quad 6(1.5) - 12(1.5) = M_y \quad M_y = -9.00 \text{ kip}\cdot\text{in}$

$\Sigma M_z = (M_R)_z;\quad 12(3) - 6(3) = M_z \quad M_z = 18.0 \text{ kip}\cdot\text{in}$

The cross-sectional area and moment of inertia about the y and z axes of the cross-section are

$$A = 6(3) = 18 \text{ in}^2$$

$$I_y = \frac{1}{12}(6)(3)^3 = 13.5 \text{ in}^4$$

$$I_z = \frac{1}{12}(3)(6^3) = 54.0 \text{ in}^4$$

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{F}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point A, $y = 3$ in. and $z = -1.5$ in.

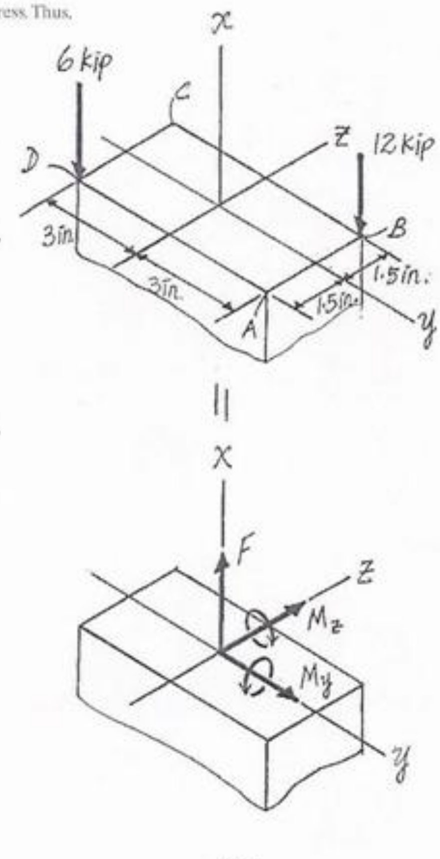
$$\sigma_A = \frac{-18.0}{18.0} - \frac{18.0(3)}{54.0} + \frac{-9.00(-1.5)}{13.5}$$

$$= -1.00 \text{ ksi} = 1.00 \text{ ksi (C)} \qquad \textbf{Ans.}$$

For point B, $y = 3$ in and $z = 1.5$ in.

$$\sigma_B = \frac{-18.0}{18.0} - \frac{18.0(3)}{54} + \frac{-9.00(1.5)}{13.5}$$

$$= -3.00 \text{ ksi} = 3.00 \text{ ksi (C)} \qquad \textbf{Ans.}$$

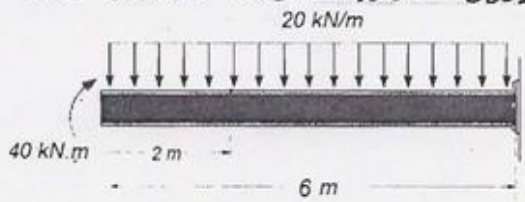


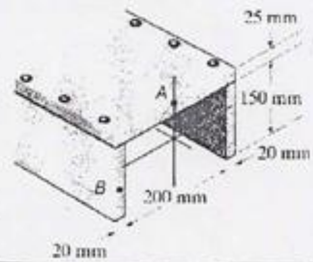
(a)

Problem #2 :-

Given:-

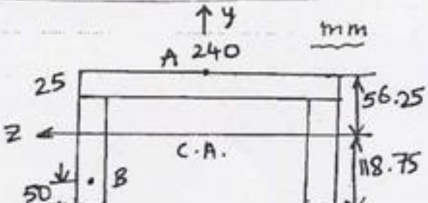
The beam with the cross-section shown.





Required:

σ and τ @ A and B



Solution :-

$\bar{y} = 118.75$ mm $= 0.11875$ m
$I_z = 3.453125(10)^{-5}$ m^4. } See HW#12

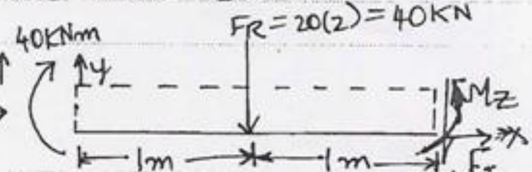
To calculate σ and τ, we need to determine the internal forces at a section 2-m from left that passes through A and B, as shown in the FBD below.

$\Sigma F_x = 0 \Rightarrow F_x = 0$

$+\uparrow \Sigma F_y = 0 \Rightarrow V = -40KN = 40KN\uparrow$

$+\circlearrowleft \Sigma M_A = 0 \Rightarrow -40 + 40(1) + M_z = 0 \Rightarrow$

$M_z = 0$



$M_y = 0$ ((2-D))

$$\sigma_x = \pm \frac{\cancel{F_x}^{\,0}}{A_x} \pm \frac{\cancel{M_z}^{\,0}\, y}{I_z} \pm \frac{\cancel{M_y}^{\,0}\, z}{I_y} \Rightarrow$$

σ is zero throughout the section $\rightarrow$ $\boxed{\sigma_A = \sigma_B = 0}$

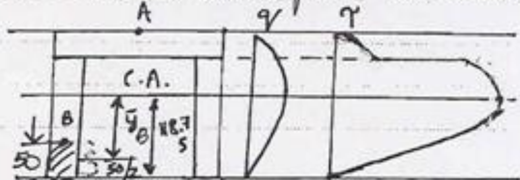
$$\tau = \pm \frac{\vec{V_y}\, Q_z}{I_z t_z} \pm \frac{\cancel{\vec{V_z}}^{\,0}\, Q_y}{I_y t_y} \pm \cancel{T}^{\,0}(*)$$

↑ not taken by students.

Only V_y is present. q and τ-distribution throughout the section are as shown.

$Q_z^A = A\bar{y} = 0 \Rightarrow$

$\boxed{\tau_A = 0}$

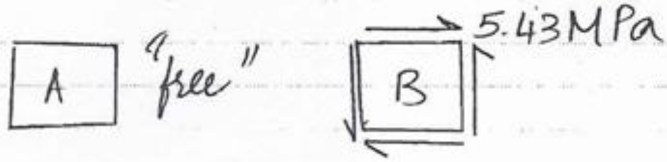


$$Q_z^B = (A\bar{y})_B = 20(50)\left(118.75 - \frac{50}{2}\right) = 93750\,\text{mm}^3$$

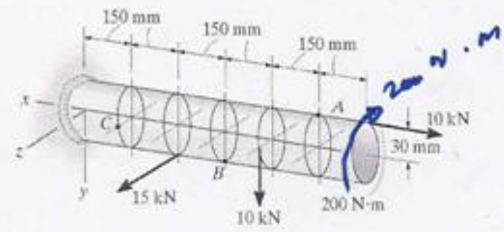
$$\tau_B = \left| \frac{V_y Q_z}{I_z t_z} \right| = \frac{40(10)^{-3}(9.375)(10)^{-5}}{3.453125(10)^{-5}(20)(10)^{-3}}$$

$$\boxed{\tau_B = 5.43\ \text{MPa}}$$

The states of stress at A and B are shown.

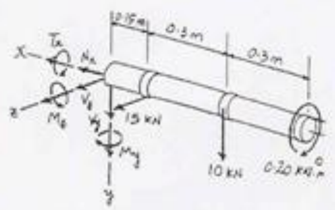


8–55. The solid rod is subjected to the loading shown. Determine the state of stress at point C, and show the results on a differential volume element located at this point.



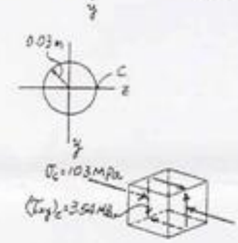
Internal Forces and Moment:

$$\Sigma F_x = 0;\quad N_x - 10 = 0 \quad N_x = 10.0\ \text{kN}$$
$$\Sigma F_y = 0;\quad V_y + 10 = 0 \quad V_y = -10.0\ \text{kN}$$
$$\Sigma F_z = 0;\quad V_z + 15 = 0 \quad V_z = -15.0\ \text{kN}$$
$$\Sigma M_x = 0;\quad T_x + 0.200 - 10(0.03) + 15(0.03) = 0$$
$$T_x = -0.350\ \text{kN}\cdot\text{m}$$
$$\Sigma M_y = 0;\quad M_y + 15(0.15) = 0 \quad M_y = -2.25\ \text{kN}\cdot\text{m}$$
$$\Sigma M_z = 0;\quad M_z - 10(0.03) - 10(0.45) = 0$$
$$M_z = 4.80\ \text{kN}\cdot\text{m}$$



Section Properties:

$$A = \pi\left(0.03^2\right) = 0.900\left(10^{-3}\right)\pi\ \text{m}^2$$
$$I_z = I_y = \frac{\pi}{4}\left(0.03^4\right) = 0.2025\left(10^{-6}\right)\pi\ \text{m}^4$$
$$J = \frac{\pi}{2}\left(0.03^4\right) = 0.405\left(10^{-6}\right)\pi\ \text{m}^4$$
$$(Q_C)_z = 0$$
$$(Q_C)_y = \frac{4(0.03)}{3\pi}\left[\frac{1}{2}\pi\left(0.03^2\right)\right] = 18.0\left(10^{-6}\right)\ \text{m}^3$$



Normal Stress:

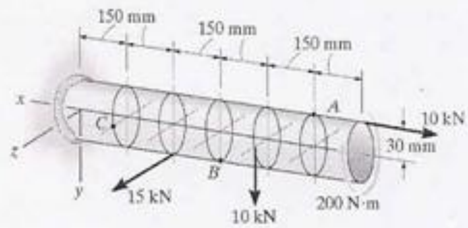
$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$
$$\sigma_C = \frac{10.0(10^3)}{0.900(10^{-3})\pi} - \frac{4.80(10^3)(0)}{0.2025(10^{-6})\pi} + \frac{-2.25(10^3)(0.03)}{0.2025(10^{-6})\pi}$$
$$= -103\ \text{MPa} = 103\ \text{MPa (C)} \qquad \textbf{Ans}$$

Shear Stress: The tranverse shear stress in the z and y directions and the torsional shear stress can be obtained using the shear formula and the torsion formula, $\tau_V = \frac{VQ}{It}$ and $\tau_{twist} = \frac{T\rho}{J}$, respectively.

$$(\tau_{xy})_C = \tau_{twist} - \tau_{V_y}$$
$$= \frac{0.350(10^3)(0.03)}{0.405(10^{-6})\pi} - \frac{10.0(10^3)\,18.0(10^{-6})}{0.2025(10^{-6})\pi(0.06)}$$
$$= 3.54\ \text{MPa} \qquad \textbf{Ans}$$

$$(\tau_{xz})_C = \tau_{V_z} = 0 \qquad \textbf{Ans}$$

8–54. The solid rod is subjected to the loading shown. Determine the state of stress at point B, and show the results on a differential volume element located at this point.



Internal Forces and Moments:

$\Sigma F_x = 0;\quad N_x - 10 = 0 \quad N_x = 10.0\text{ kN}$

$\Sigma F_y = 0;\quad V_y + 10 = 0 \quad V_y = -10.0\text{ kN}$

$\Sigma F_z = 0;\quad V_z = 0$

$\Sigma M_x = 0;\quad T_x + 0.200 - 10(0.03) = 0$

$T_x = 0.100\text{ kN}\cdot\text{m}$

$\Sigma M_y = 0;\quad M_y = 0$

$\Sigma M_z = 0;\quad M_z - 10(0.03) - 10(0.15) = 0$

$M_z = 1.80\text{ kN}\cdot\text{m}$

Section Properties:

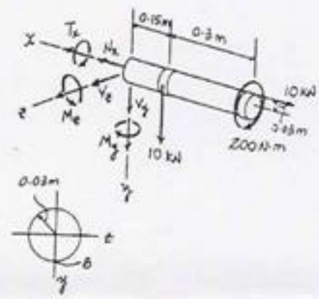
$$A = \pi(0.03^2) = 0.900(10^{-3})\,\pi\text{ m}^2$$

$$I_z = I_y = \frac{\pi}{4}(0.03^4) = 0.2025(10^{-6})\,\pi\text{ m}^4$$

$$J = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\,\pi\text{ m}^4$$

$$(Q_B)_y = 0$$

$$(Q_B)_z = \frac{4(0.03)}{3\pi}\left[\frac{1}{2}\pi(0.03^2)\right] = 18.0(10^{-6})\text{ m}^3$$

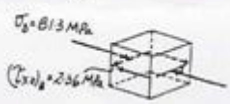


Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_B = \frac{10.0(10^3)}{0.900(10^{-3})\,\pi} - \frac{1.80(10^3)(0.03)}{0.2025(10^{-6})\,\pi} + 0$$

$$= -81.3\text{ MPa} = 81.3\text{ MPa (C)} \qquad \textbf{Ans}$$



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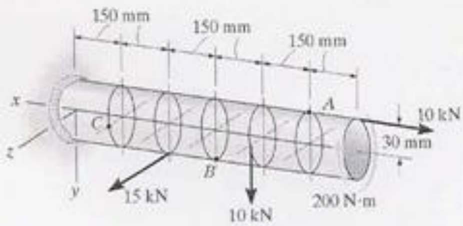
$$(\tau_{xz})_B = \tau_{twist} + \tau_{V_z}$$

$$= \frac{0.100(10^3)(0.03)}{0.405(10^{-6})\,\pi} + 0 = 2.36\text{ MPa} \qquad \textbf{Ans}$$

$$(\tau_{xy})_B = \tau_{V_y} = 0 \qquad \textbf{Ans}$$

8–53. The solid rod is subjected to the loading shown. Determine the state of stress developed in the material at point A, and show the results on a differential volume element at this point.



Internal Forces and Moments:

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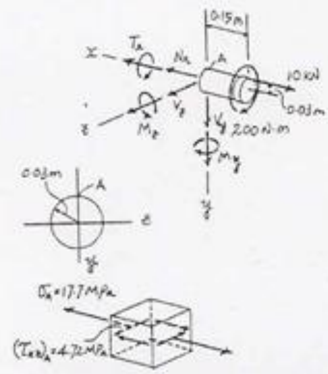
$\Sigma F_y = 0;\quad V_y = 0$
$\Sigma F_z = 0;\quad V_z = 0$
$\Sigma M_x = 0;\quad T_x + 0.200 = 0 \quad T_x = -0.200\text{ kN}\cdot\text{m}$
$\Sigma M_y = 0;\quad M_y = 0$
$\Sigma M_z = 0;\quad M_z - 10(0.03) = 0 \quad M_z = 0.300\text{ kN}\cdot\text{m}$



Section Properties:

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$$J = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\,\pi\ \text{m}^4$$
$$(Q_A)_y = 0$$
$$(Q_A)_z = \frac{4(0.03)}{3\pi}\left[\frac{1}{2}\pi(0.03^2)\right] = 18.0(10^{-6})\ \text{m}^3$$

Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = \frac{10.0(10^3)}{0.900(10^{-3})\pi} - \frac{0.300(10^3)(-0.03)}{0.2025(10^{-6})\pi} + 0$$
$$= 17.7\text{ MPa (T)} \qquad \textbf{Ans}$$

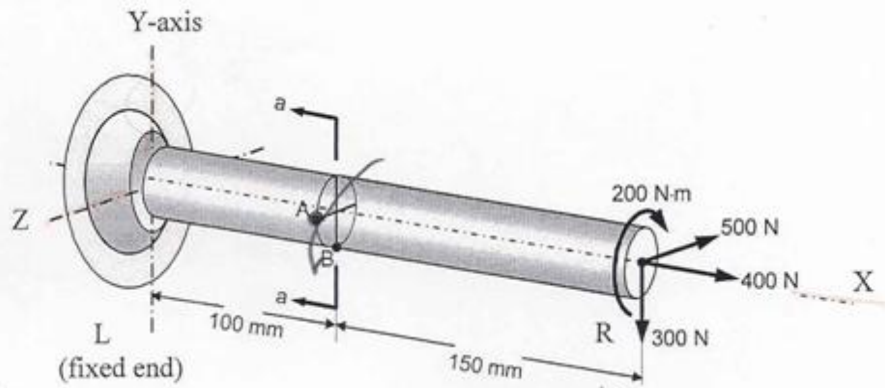
Shear Stress: The tranverse shear stress in the z and y directions and the torsional shear stress can be obtained using the shear formula and the torsion formula, $\tau_v = \frac{VQ}{It}$ and $\tau_{twist} = \frac{T\rho}{J}$, respectively.

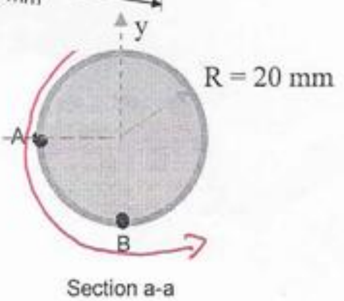
$$(\tau_{xz})_A = \tau_{twist} + \tau_{v_z}$$
$$= \frac{0.200(10^3)(0.03)}{0.405(10^{-6})\pi} + 0 = 4.72\text{ MPa} \qquad \textbf{Ans}$$

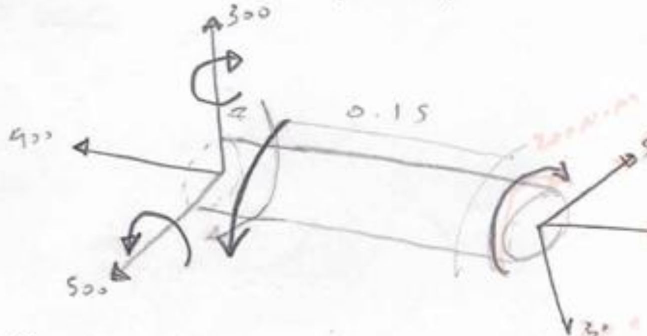
$$(\tau_{xy})_A = \tau_{v_y} = 0 \qquad \textbf{Ans}$$

section has a diameter D = 40 mm. If it is subjected to the loads shown at end R, determine the states of stress at point A, and at point B. Then show the results on an element located at each point separately.

Fig. P-4: A cantilever shaft LR with three forces and one torque at end R.





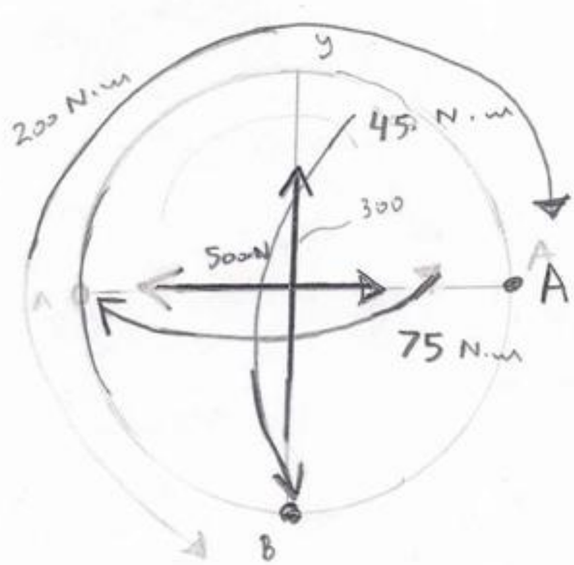


$$\Sigma M = 0 = M_x + M_y + M_z + 0.15\hat{i} \times (500\hat{k} + 300\hat{j}) - 200\hat{i}$$

$$= M_x + M_y + M_z - 75\hat{j} + 45\hat{k} - 200\hat{i}$$

$M_x = 200$ N·m $\quad$ $M_y = 75$ N·m $\quad$ $M_z = 45$ N·m

P



$$\sigma_N = \frac{400}{1.256\times10^{-3}} = 0.318 \text{ MPa (T)}$$

$$\sigma_{yz}b = \frac{(75)(0.02)}{1.256\times10^{-7}} = 11.94 \text{ MPa (T)}$$

point A

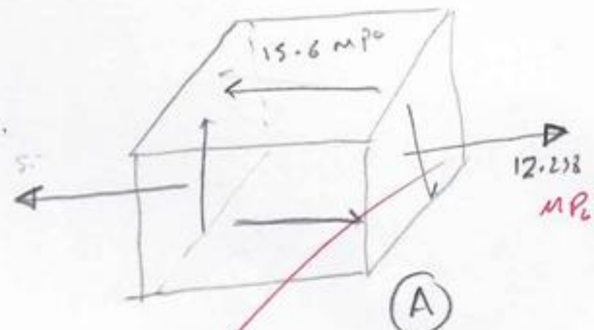
$$\text{so } \boxed{\sigma = \sigma_N + \sigma_{yz} \cdot b = 12.258 \text{ MPa (T)}}$$

$$\tau_{yz} = \frac{VQ}{It} \longrightarrow Q = \frac{4(0.02)}{3\pi}\left(\frac{1}{2}\right)(1.256\times10^{-3}) = 5.33\times10^{-6}\ m^3$$

$$V = 300 \text{ N}$$

$$\tau_{yz} = \frac{(300)(5.33\times10^{-6})}{(1.257\times10^{-7})(0.04)} = 0.316 \text{ MPa}$$

$$\tau_{yz} = \frac{Tc}{J} = \frac{(200)(0.02)}{2.512\times10^{-7}} = -15.92 \text{ MPa}$$



$$\boxed{\tau_{yz} = 0.316 - 15.92 = -15.6 \text{ MPa}}$$

$$\boxed{\tau_{xz} = 0}$$:)

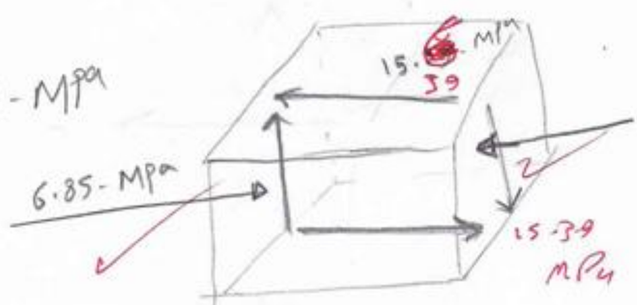
Point B

$$\boxed{\text{Point B}} \quad \boxed{\sigma_N = 0.318 \text{ MPa}} \quad \sigma_b = \frac{(45)(0.02)}{1.256\times10^{-7}} = 7.16 \text{ MPa (C)}$$

$$\boxed{\sigma_B = \sigma_N - \sigma_b = -6.85 \text{ MPa (C)}}$$

$$\tau_{xz} = \frac{VQ}{It} = \frac{(500)(5.33\times10^{-6})}{(1.256\times10^{-7})(0.04)} = 0.53 \text{ MPa}$$

$$\tau_{xz} = \frac{Tc}{J} = -\frac{(200)(0.02)}{2.512\times10^{-7}} = -15.92 \text{ MPa}$$



$$\boxed{\tau_{xz} = -15.39 \text{ MPa}}$$

$$\boxed{\tau_{yz} = 0}$$:)