

Dividing by  $\Delta x$  and taking the limit as  $\Delta x \rightarrow 0$ ,  
 equation (1) and (2) became :-

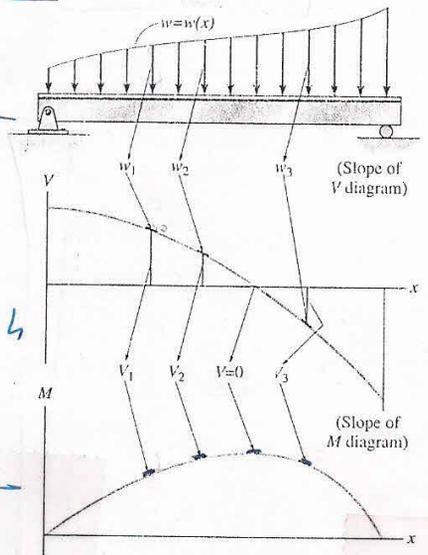
(2)

$$\frac{dV}{dx} = -w(x) \quad \text{--- (3)}$$

Slope of  
 shear diagram = - distributed  
 Load intensity  
 at each point

$$\frac{dM}{dx} = V \quad \text{--- (4)}$$

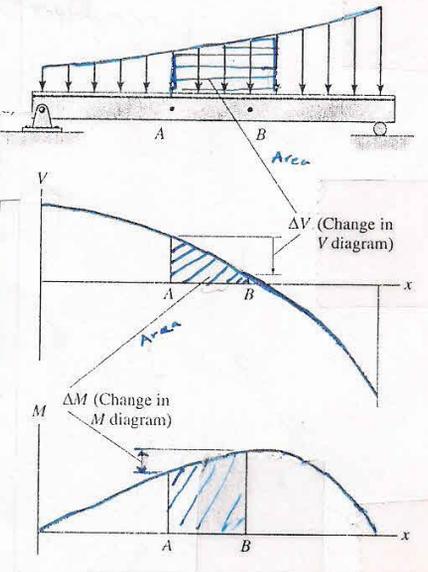
Slope of  
 moment diagram = Shear at each  
 Point



Now take integration for both  
 equations (3) and (4)

$$\Delta V = -\int w(x) dx$$

change in  
 shear = - area under  
 distributed Loading

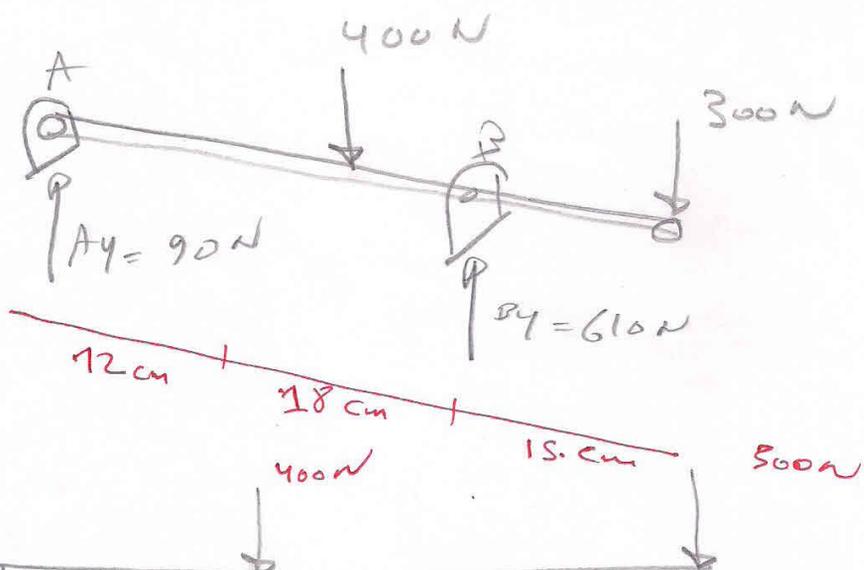


$$\Delta M = \int V(x) dx$$

change  
 in  
 moment = area under  
 Shear diagram

675

Determine the smallest allowable diameter of the shaft,  $\sigma_{allow} = 154 \text{ MPa}$



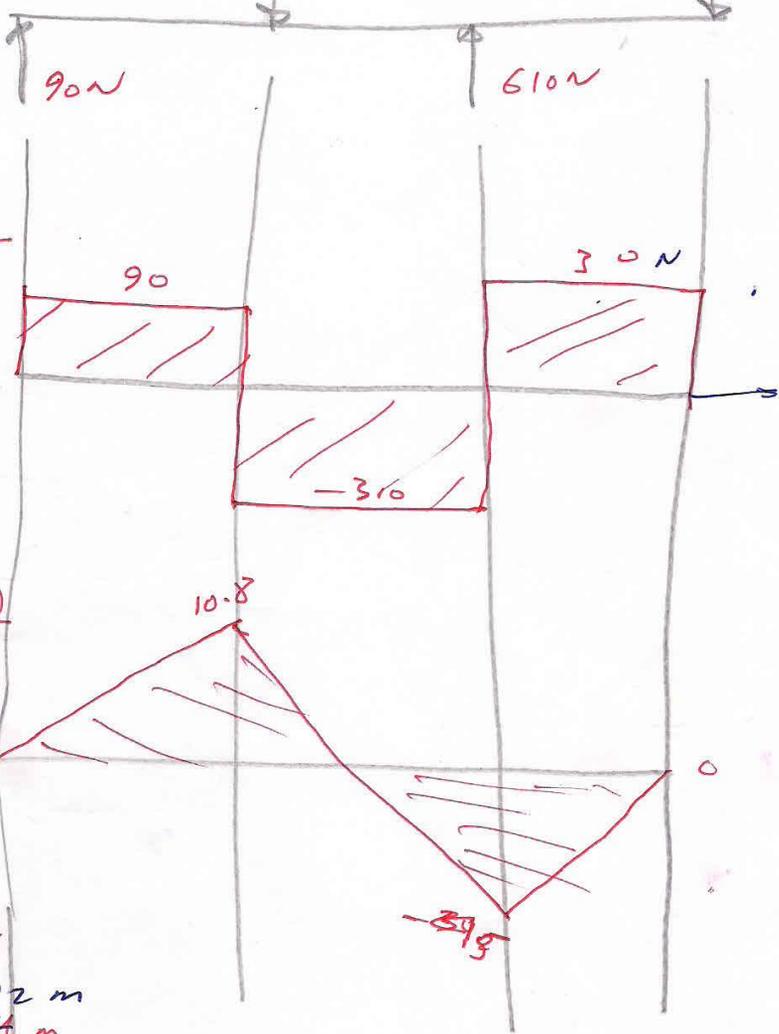
$$\sum M_A = 0$$

$$-400(-12) + B_y(.45) - 300(.45) = 0$$

$$B_y = 610 \text{ N}$$

$$\sum F_y = 0$$

$$A_y = 90 \text{ N}$$



$$\sigma = \frac{Mc}{I}$$

$$I = \frac{1}{4} \pi r^4$$

$$154 \times 10^6 = \frac{(45)(\checkmark)}{\frac{1}{4} \pi r^4 / 3}$$

$$120,951,317.2 r^3 = 45$$

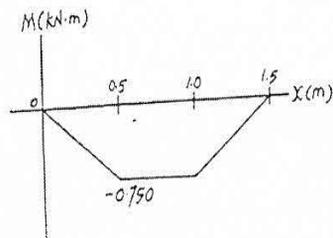
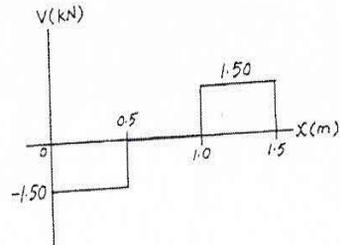
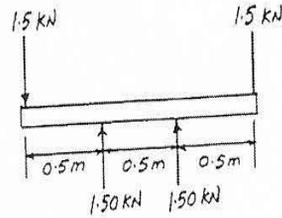
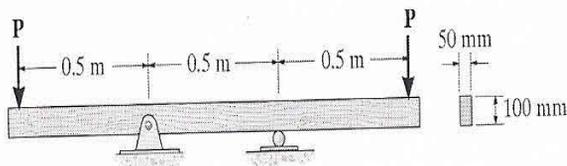
$$r^3 = \frac{45}{120,951,317.2}$$

$$r = 0.001792 \text{ m}$$

$$r = 0.015514 \text{ m}$$

$r = 15.514 \text{ mm}$ , diameter = 14.38 mm.

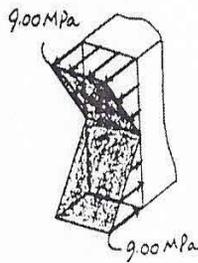
6-91. The beam has the rectangular cross section shown. If  $P = 1.5 \text{ kN}$ , determine the maximum bending stress in the beam. Sketch the stress distribution acting over the cross section.



$$\sigma_{\max} = \frac{M_{\max} c}{I}$$

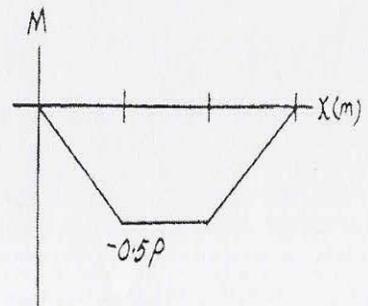
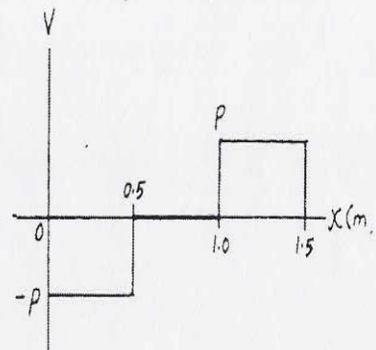
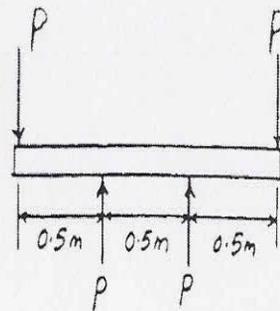
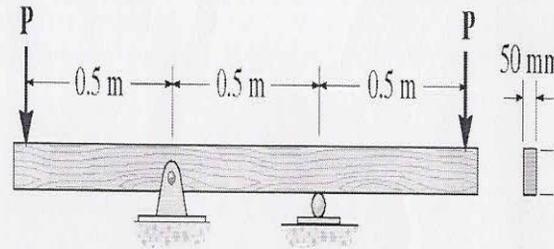
$$= \frac{-75 (10^3) (0.05)}{\frac{1}{12} (0.05) (0.1)^3}$$

$$= 9 \text{ MPa}$$



**Absolute Maximum Bending Stress:** The maximum moment is  $M_{\max} = 0.750 \text{ kN} \cdot \text{m}$  as indicated on moment diagram. Applying the flexure formula

6-90. The beam has a rectangular cross section as shown. Determine the largest load  $P$  that can be supported on its overhanging ends so that the bending stress in the beam does not exceed  $\sigma_{\max} = 10 \text{ MPa}$ .



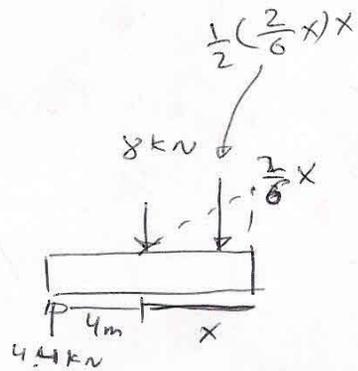
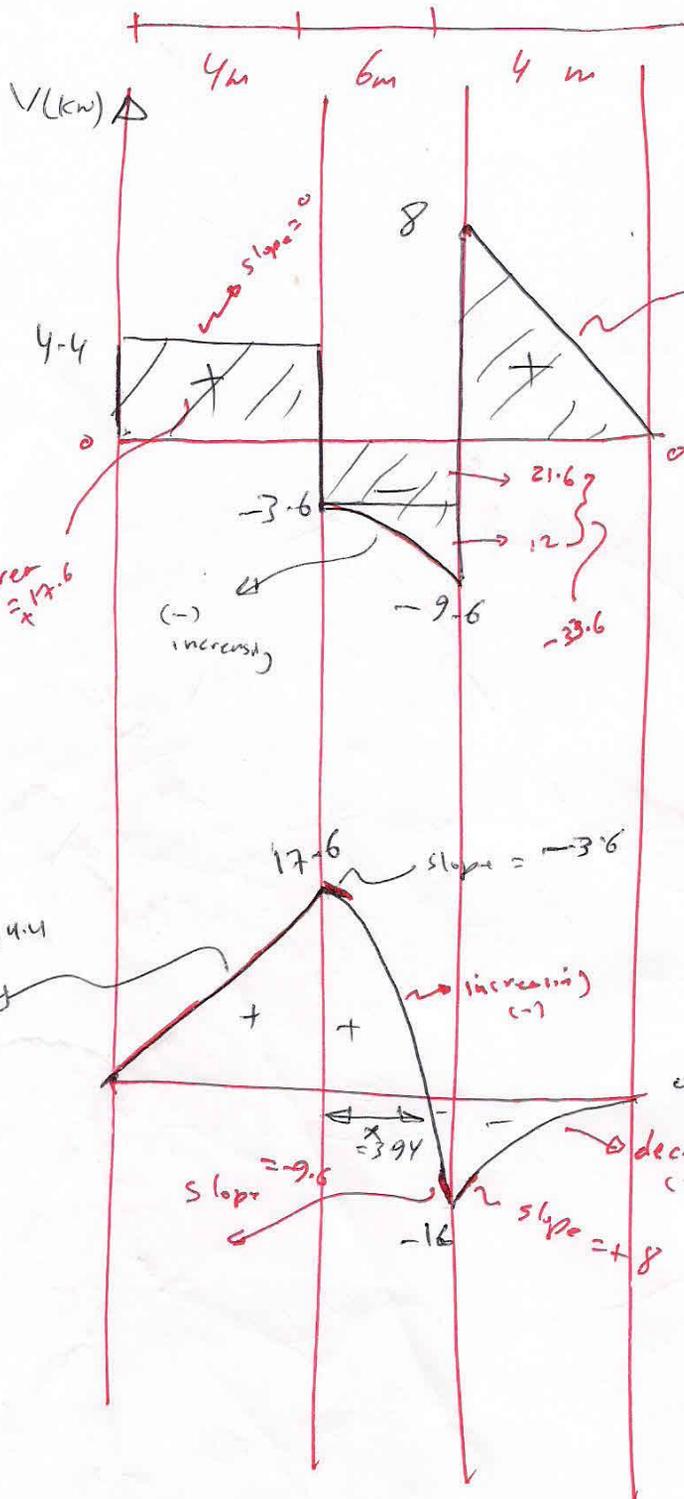
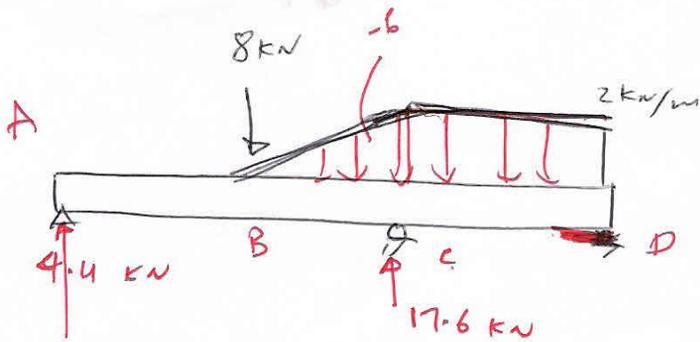
*Absolute Maximum Bending Stress:* The maximum moment is  $M_{\max} = 0.5P$  as indicated on the moment diagram. Applying the flexure formula

$$\sigma_{\max} = \frac{M_{\max} c}{I}$$

$$10(10^6) = \frac{0.5P(0.05)}{\frac{1}{12}(0.05)(0.1^3)}$$

$$P = 1666.7 \text{ N} = 1.67 \text{ kN}$$

Ans



$$\sum M = 0$$

$$-4.4(4+x) + 8x + \frac{1}{2} \left( \frac{2}{3} \right) x^2 \frac{x}{3} + M = 0$$

$$M = -\frac{1}{18} x^3 - 3.6x + 17.6 = 0$$

$$x = 3.94 \text{ m}$$

21.6  
12

over  
= 17.6

(-) increasing

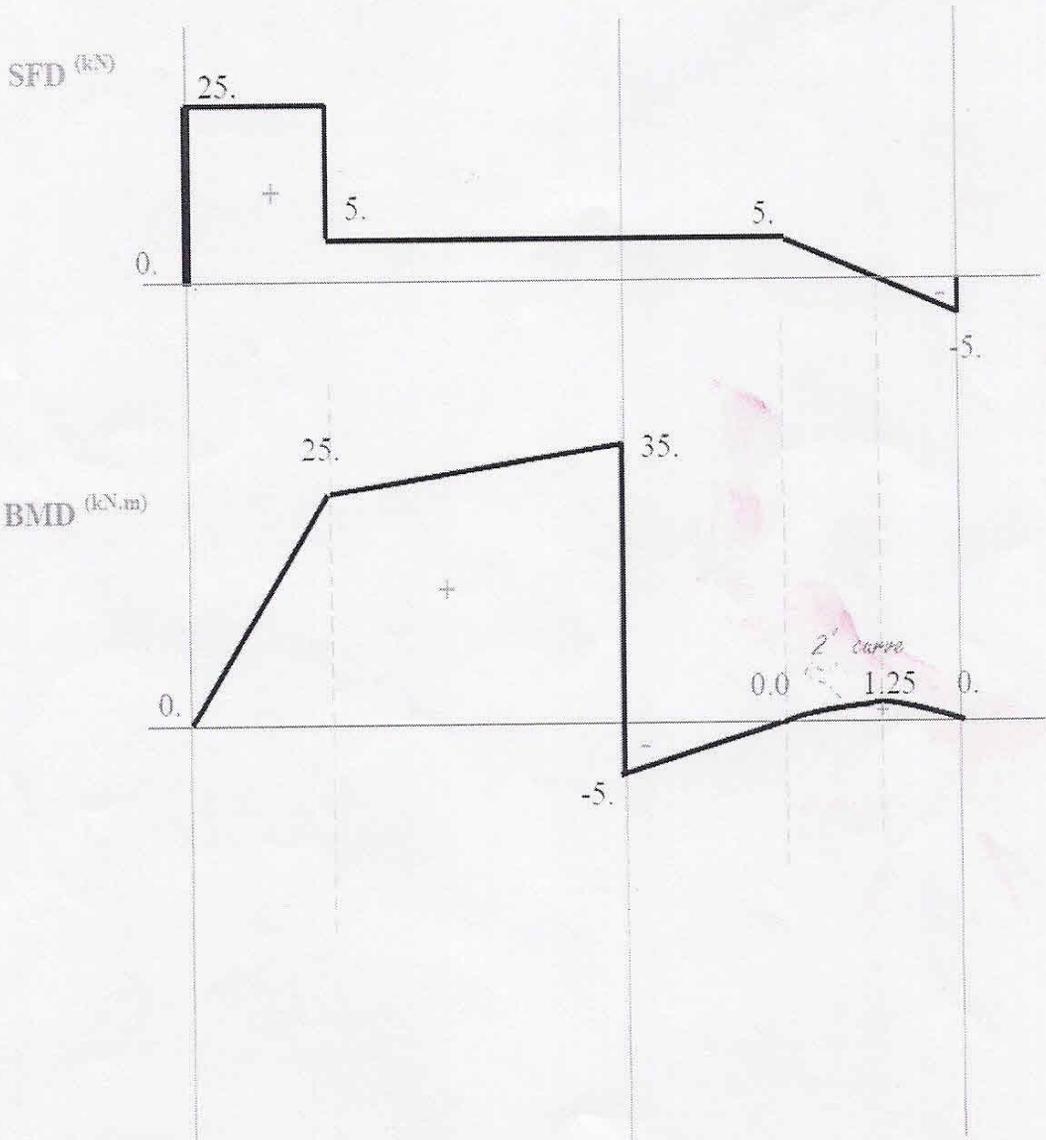
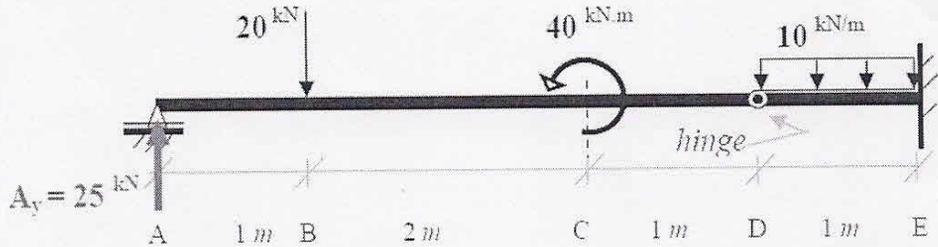
(-) increasing

(+) decreasing

### Problem # 3

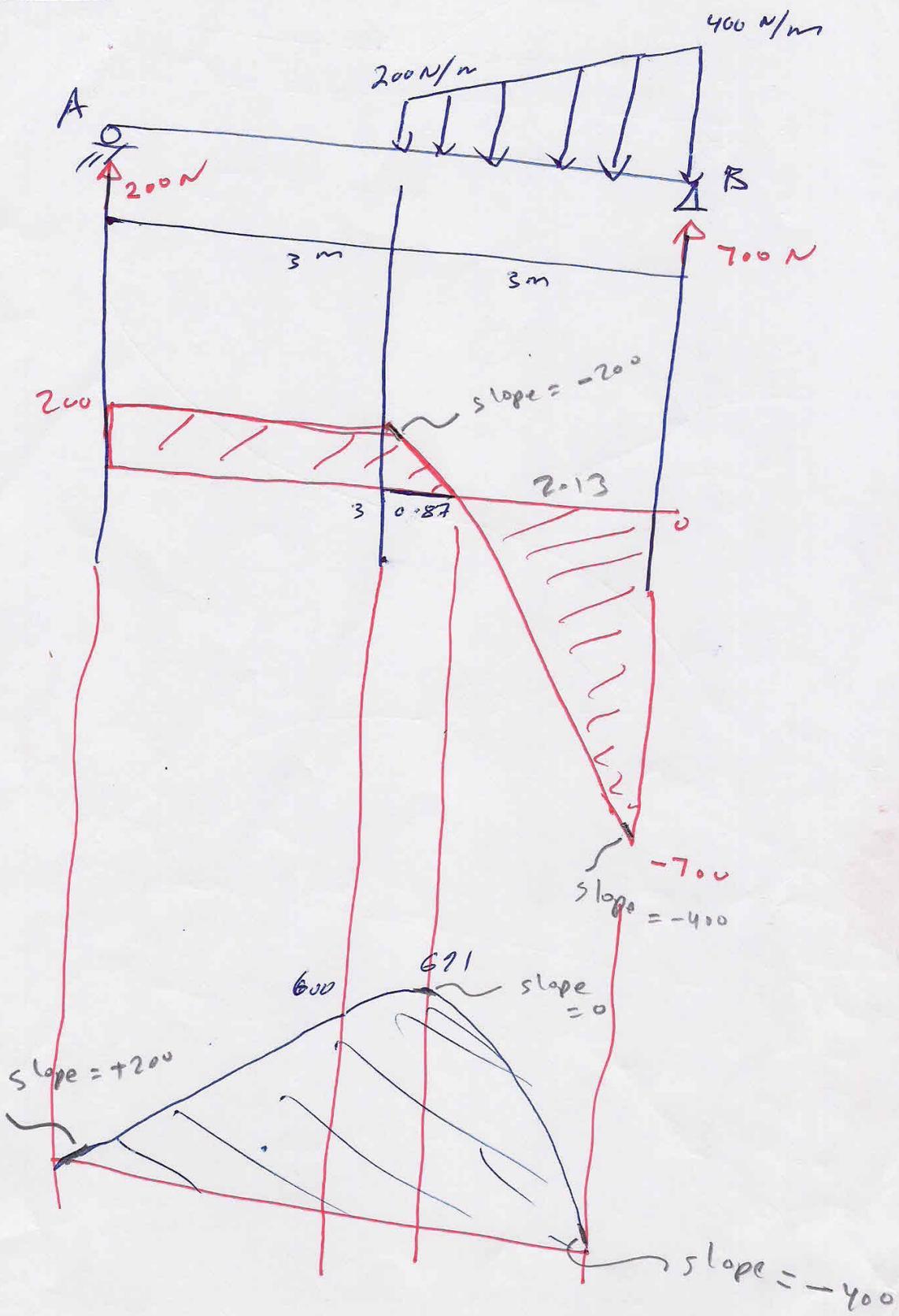
Use the graphical method (*i.e.* areas summation) to draw shear force diagram (SFD) and bending moment diagram (BMD) for beam ABCDE having a *hinge* at D.

**Note:** Reaction  $A_y = 25 \text{ kN}$  ( $\uparrow$ ).



**Note:**  $E_y = 5.0 \text{ kN}$  and  $M_E = 0.0 \text{ kN}\cdot\text{m}$ .

6-39



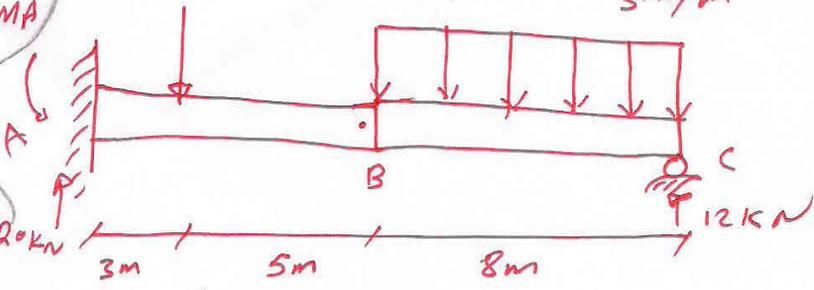
6-25

120 KN.m = MA

Max known from book

8 kN

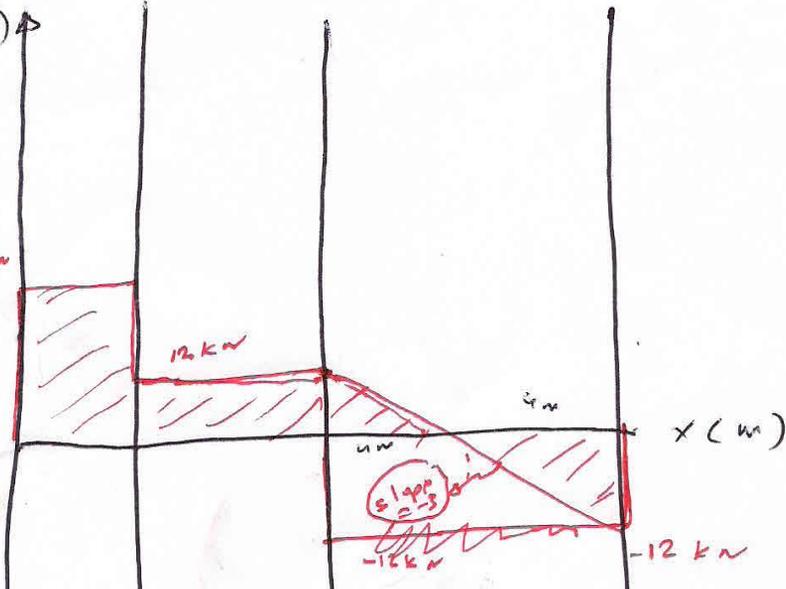
3 kN/m



dec (+)   
 inc (-)   
 dec (-)   
 inc (+)

V (kN)

20 kN



slope = -3

decreasing (+)

slope = 0

increasing (-)

slope = 12

slope = -12

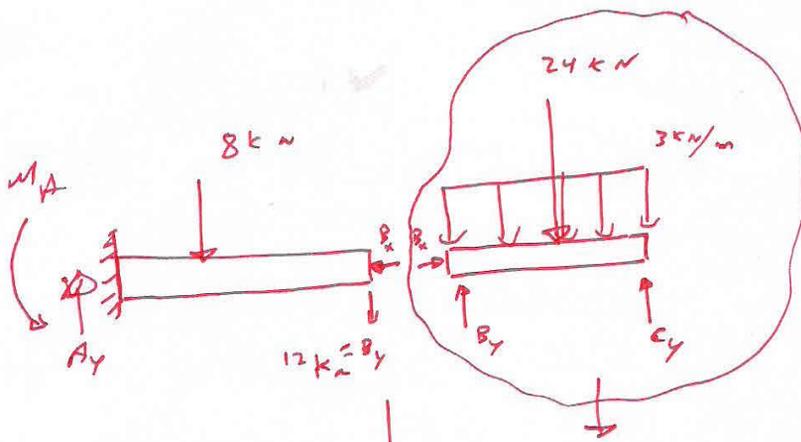
24 kN.m

slope = 12

slope = +20

-6

-120



$$+\uparrow \sum F_y = 0$$

$$A_y - 8 - 12 = 0$$

$$A_y = 20 \text{ kN}$$

$$+\circlearrowleft (\sum M_A = 0)$$

$$M_A - 8(3) - 12(8) = 0$$

$$M_A = 120 \text{ kN}\cdot\text{m}$$

$$+\uparrow \sum F_y = 0$$

$$B_y + C_y = 24 \text{ kN}$$

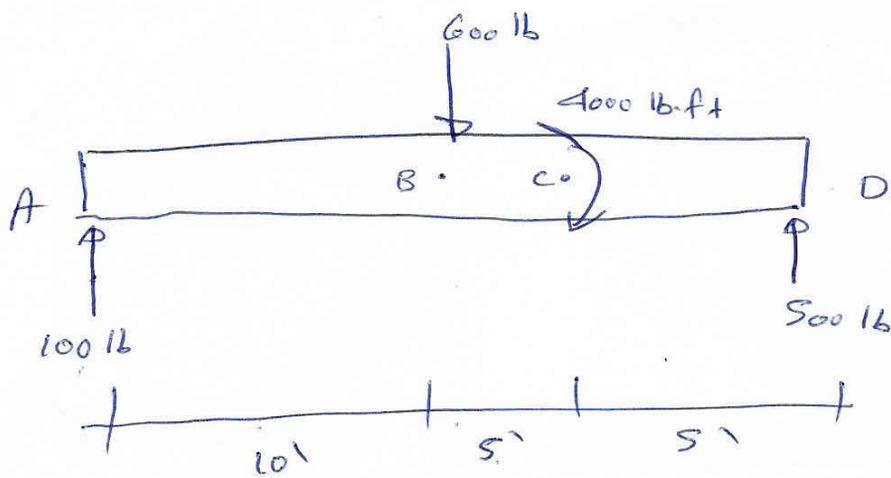
$$+\circlearrowleft (\sum M = 0); 24(4) - B_y(8) = 0$$

$$B_y = 12 \text{ kN}$$

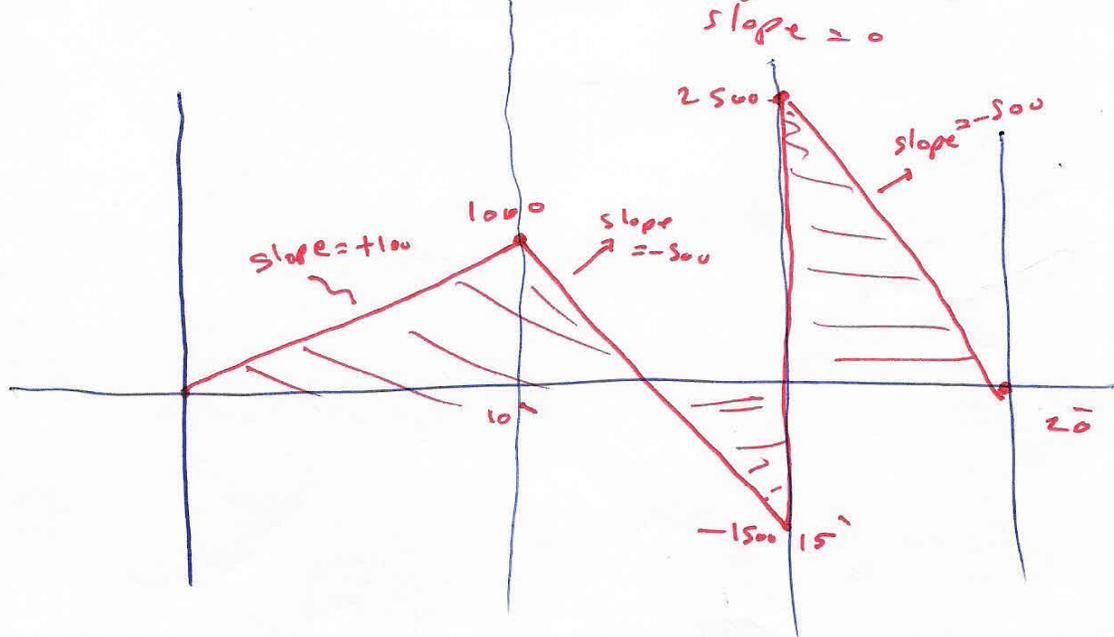
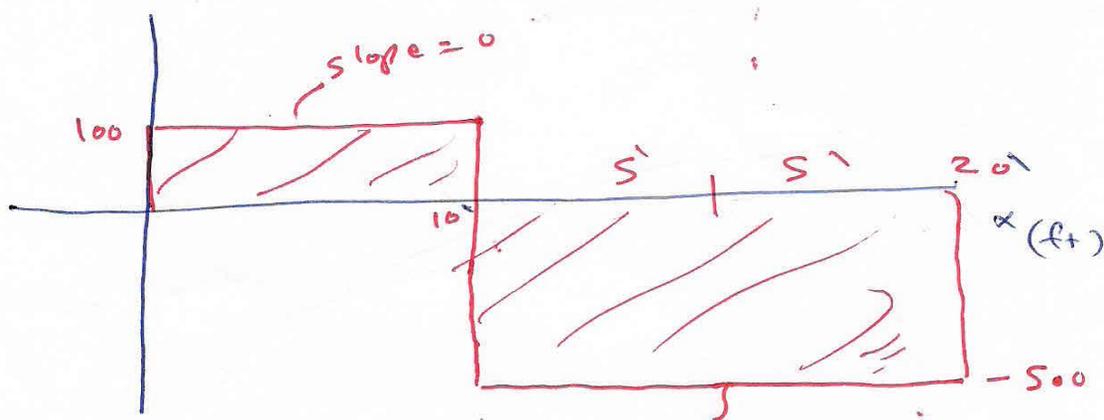
$$C_y = 12 \text{ kN}$$

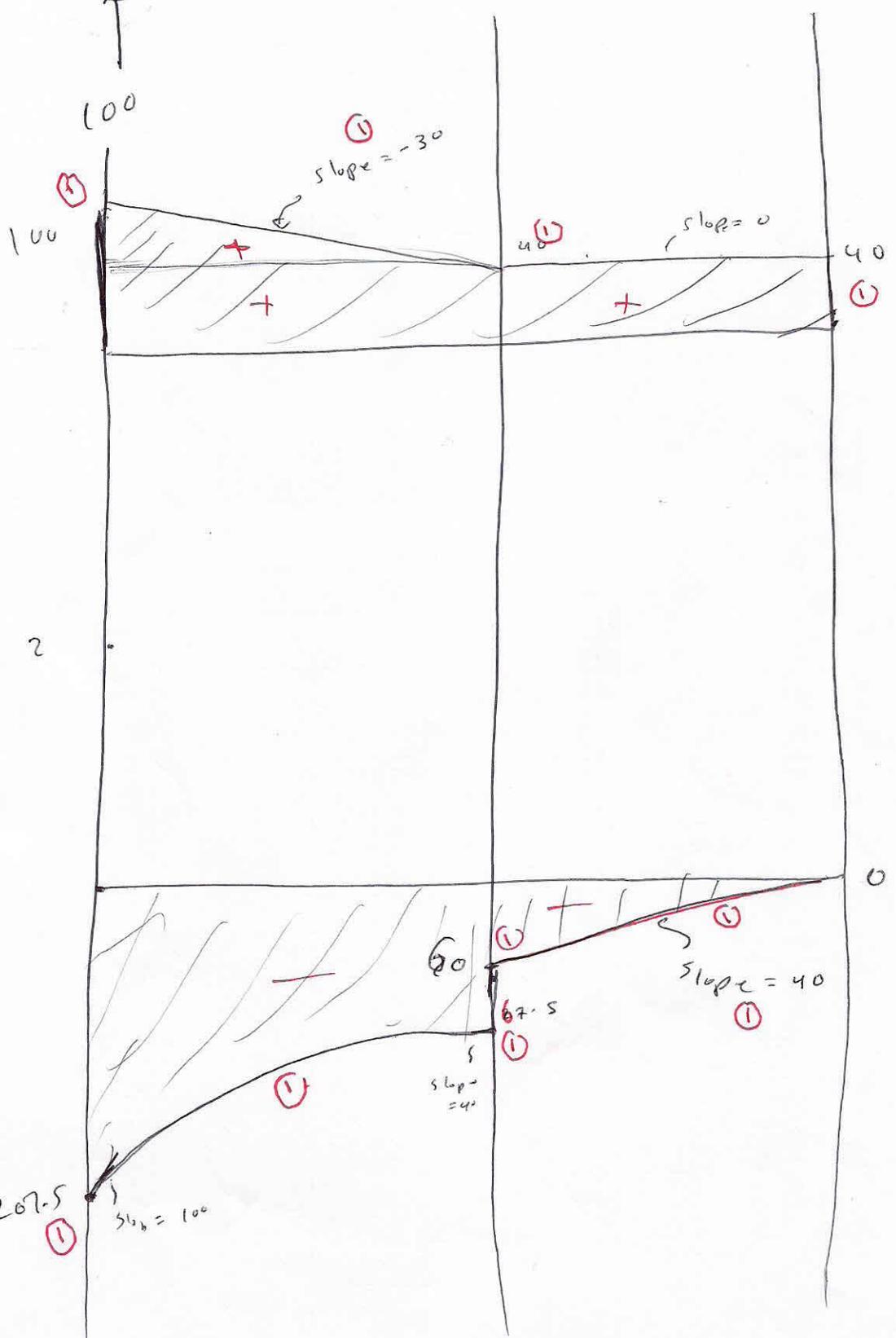
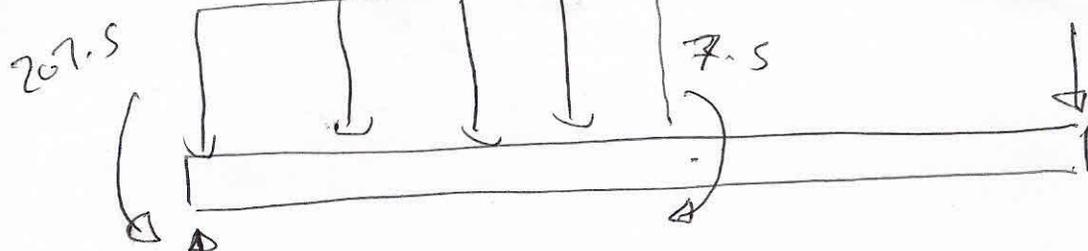
$$B_x = 0$$

Clockwise couple moment cause the jump upward in the moment diagram.

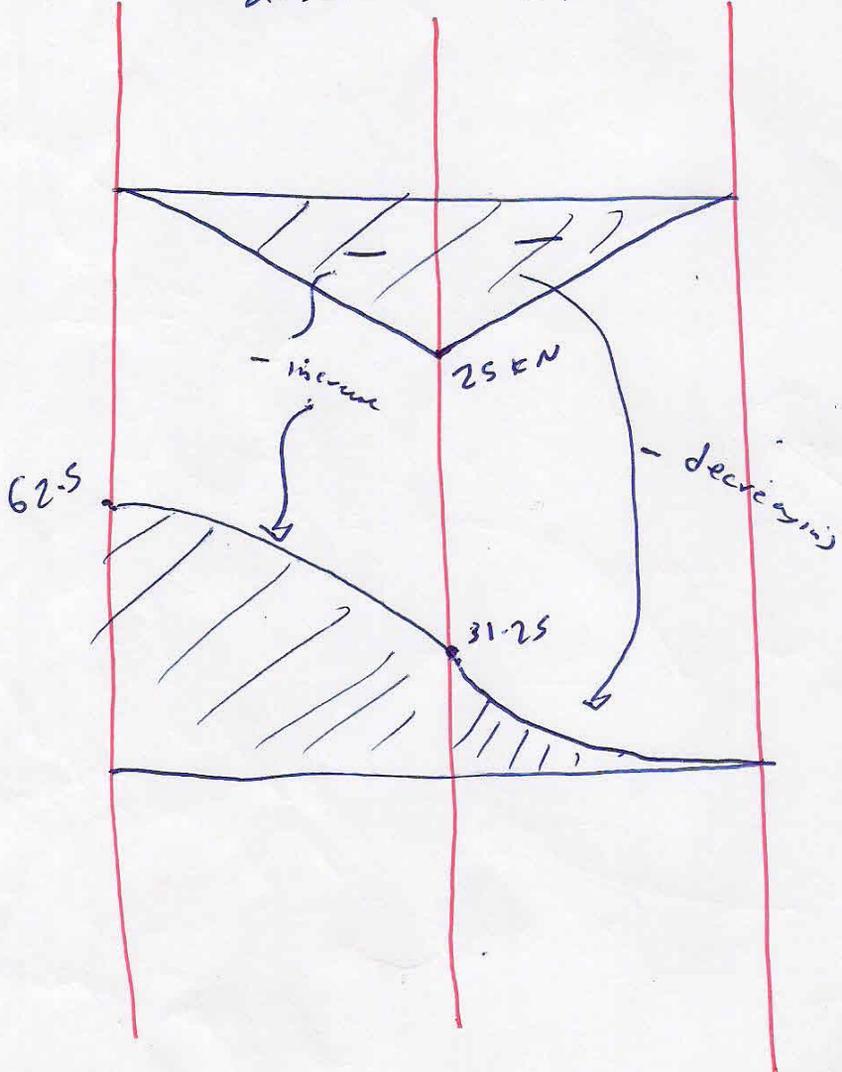
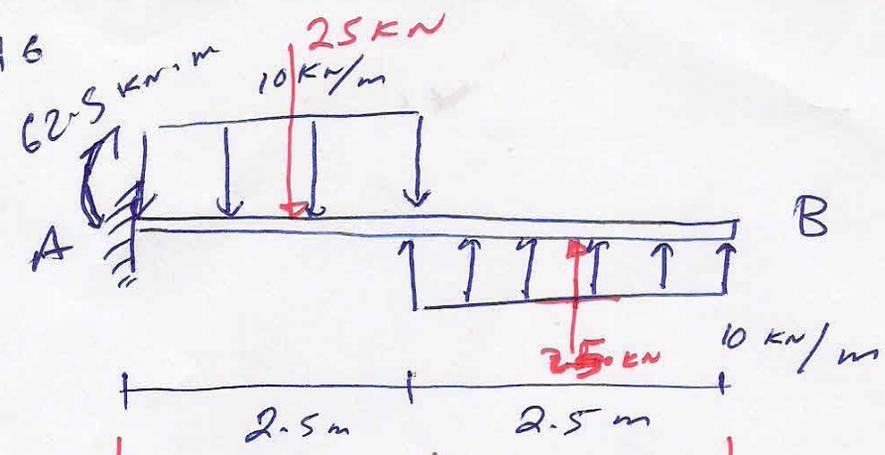


$V$  (lb)





6-16

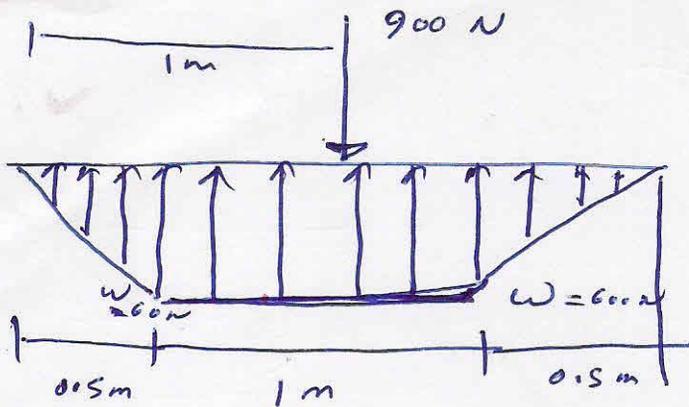


$$\sum \left( \begin{matrix} \curvearrowright \\ \uparrow \end{matrix} M_A = 0 \right) \rightarrow -25(1.25) + 25(3.75) + M_A = 0$$

$$M_A = 62.5 \text{ kN.m}$$

6-32

Determine  
the intensity  
 $w$ , draw  $V$  &  $M$



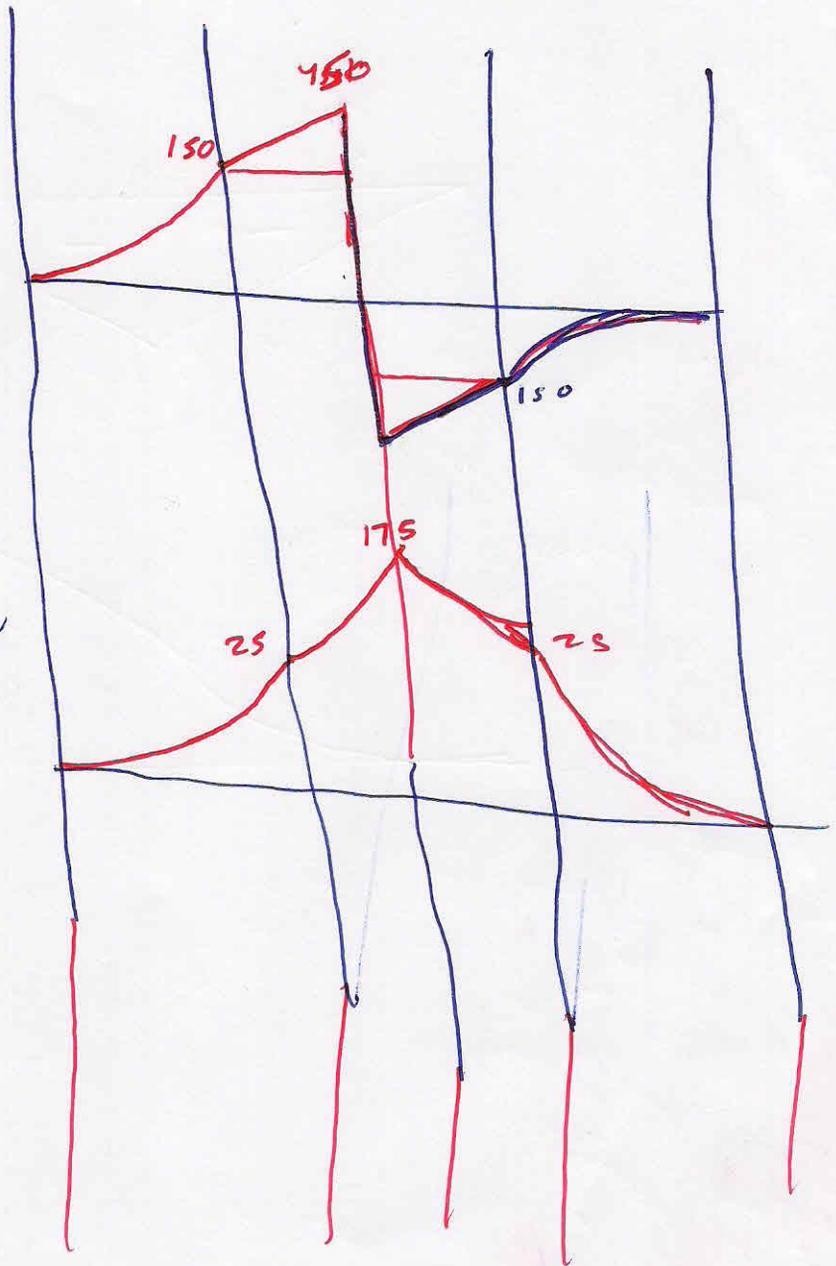
$$\sum F_y = 0,$$

$$\frac{1}{2} w(-.5) + w + \frac{1}{2} w(.5)$$

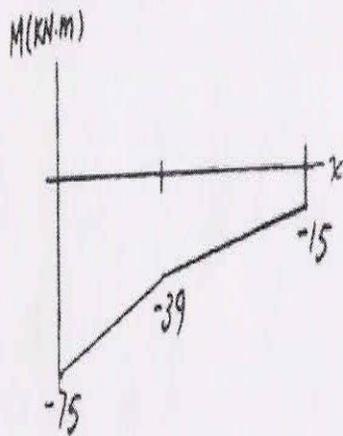
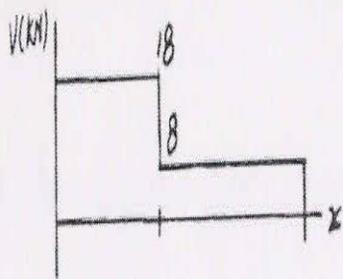
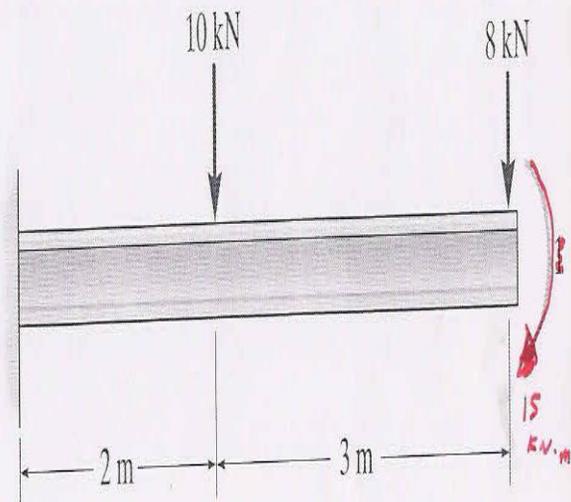
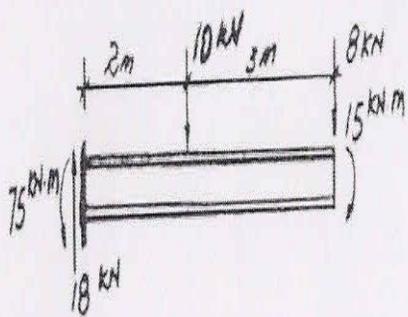
$$-900 = 0$$

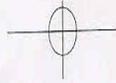
$$1.5w = 900$$

$$w = \frac{900}{1.5} = 600 \text{ N}$$



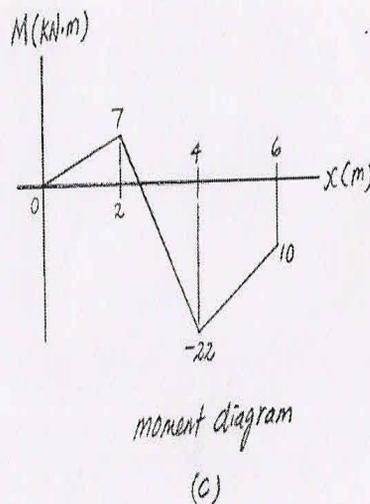
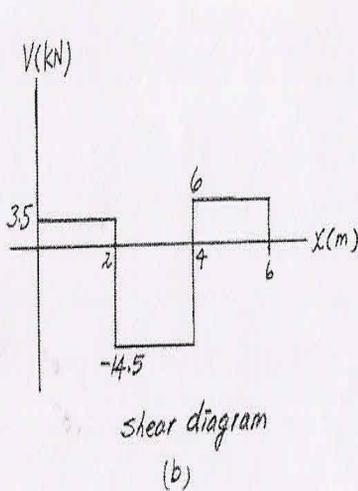
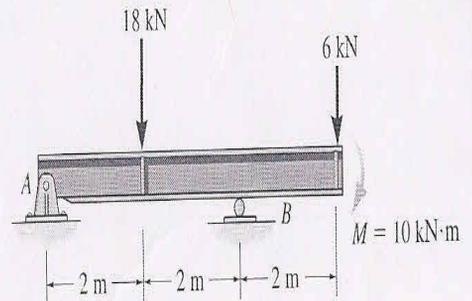
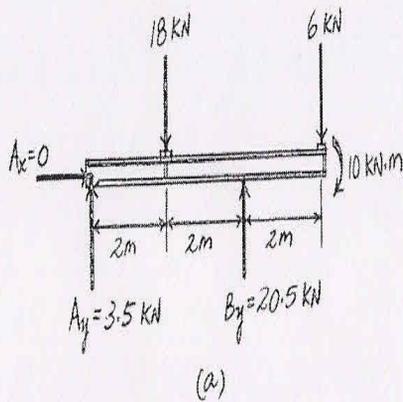
6-5. Draw the shear and moment diagrams for the beam.





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\*6-36. Draw the shear and moment diagrams for the overhang beam.

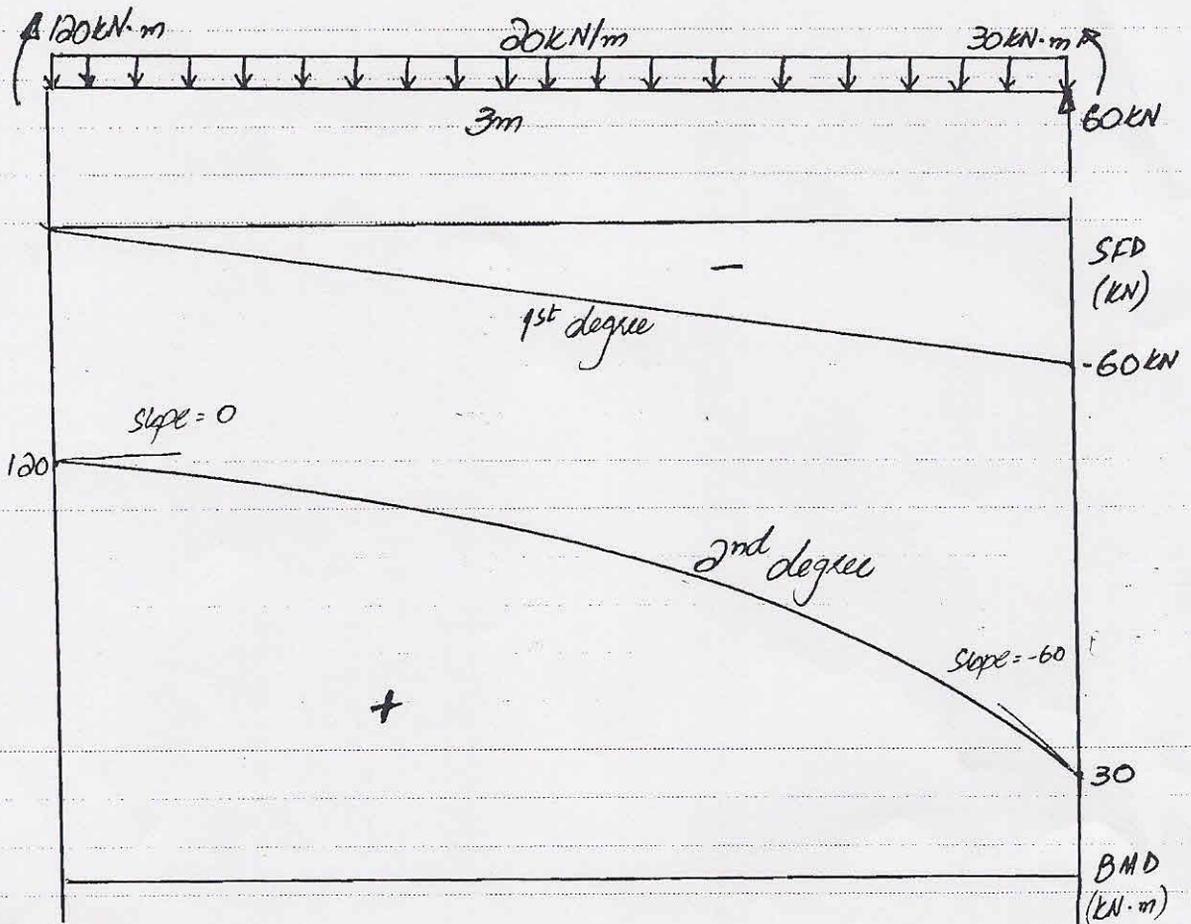


6-37. Draw the shear and moment diagrams for the beam.

50 kN/m

50 kN/m

## Solution of HW #9



Between A & B

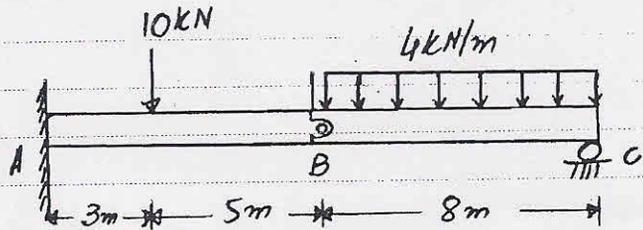
$$\text{Area of load} = \Delta V = -20(3) = -60 \text{ kN}$$

$$\text{Area of } \nabla = \Delta M = \frac{1}{2}(3)(-60) = -90 \text{ kN}\cdot\text{m}$$

Problem #03:

Given:

The beam shown



Required:

SFD & BMD by the Semi-graphical Method.

Solution:

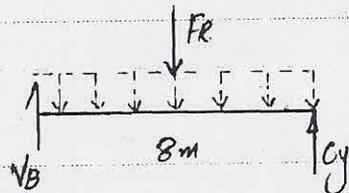
First, we need to calculate the reactions. (why?!)

To be able to find the reactions, we need to separate the beam from the pin at B. (why?!)

Without the "internal" pin at B, the problem is statically indeterminate. (How?!)

We take the right part from B (why?!)

$F_R = 4(8) = 32 \text{ kN}$   
 No M @ B. (why?!)

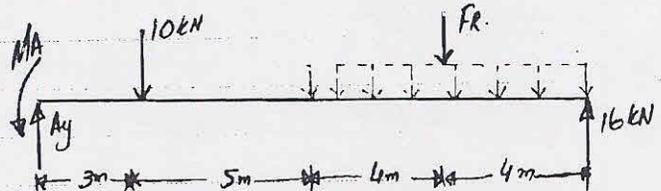


$\sum M_B = 0$  (why B?!)

$8C_y - 32(4) = 0 \Rightarrow$

$C_y = 16 \text{ kN} \uparrow$

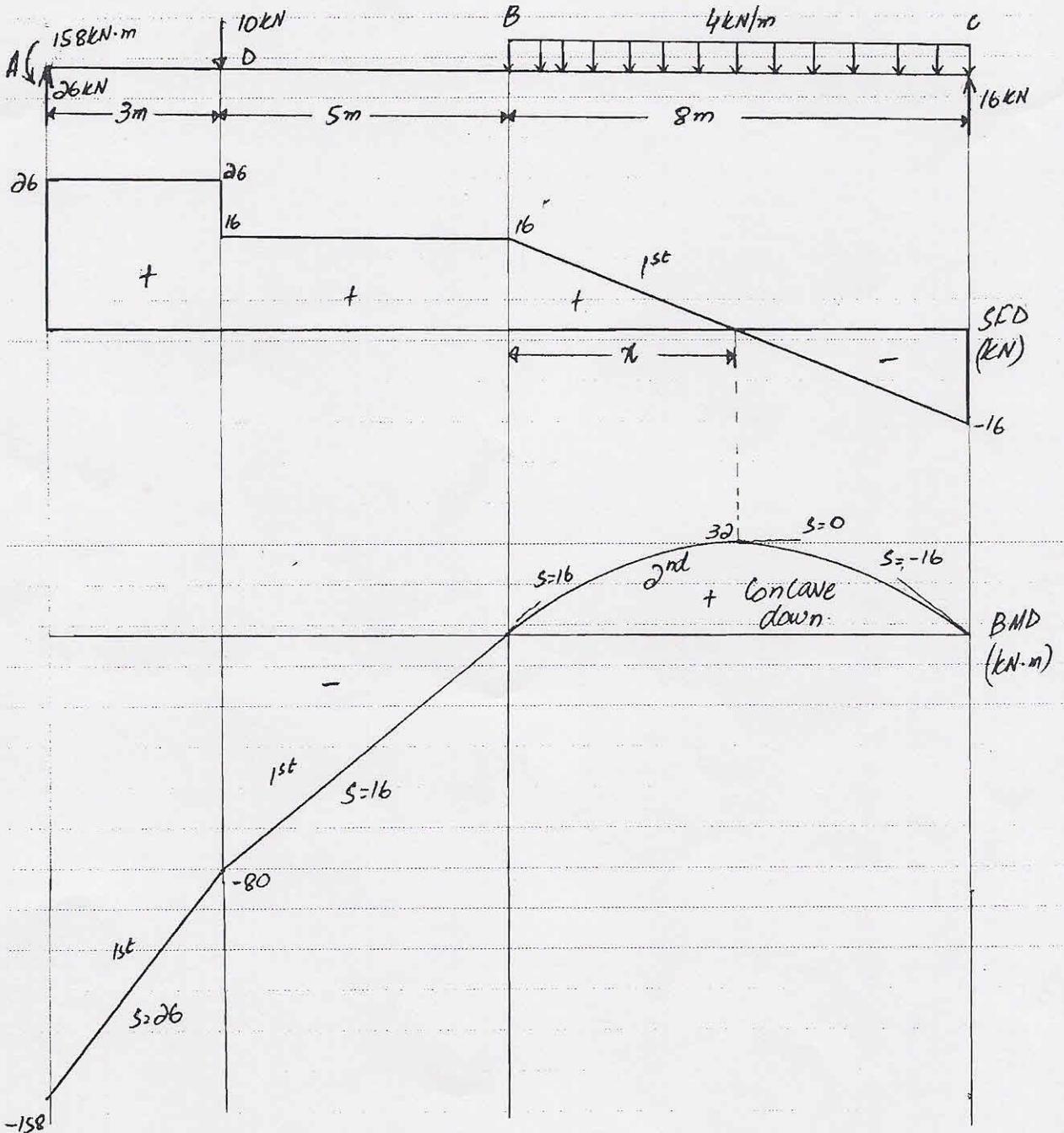
Now, the entire beam



$\sum F_y = 0 \Rightarrow A_y - 10 - 32 + 16 = 0 \Rightarrow A_y = 26 \text{ kN} \uparrow$

$\sum M_A = 0 \Rightarrow M_A - 10(3) - 32(12) + 16(16) = 0 \Rightarrow M_A = 158 \text{ kN}\cdot\text{m} \curvearrowleft$

Now, the SFD & BMD are drawn.



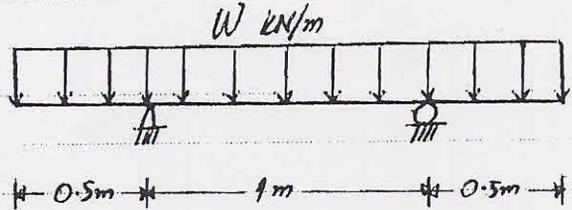
Explanation:

$S$  = Slope; 1<sup>st</sup>, 2<sup>nd</sup>, etc. = degree of curve

Even though, it is clear that  $x = 4$  m, we will use the general method to find it.

Problem #05:

Given:



The beam shown

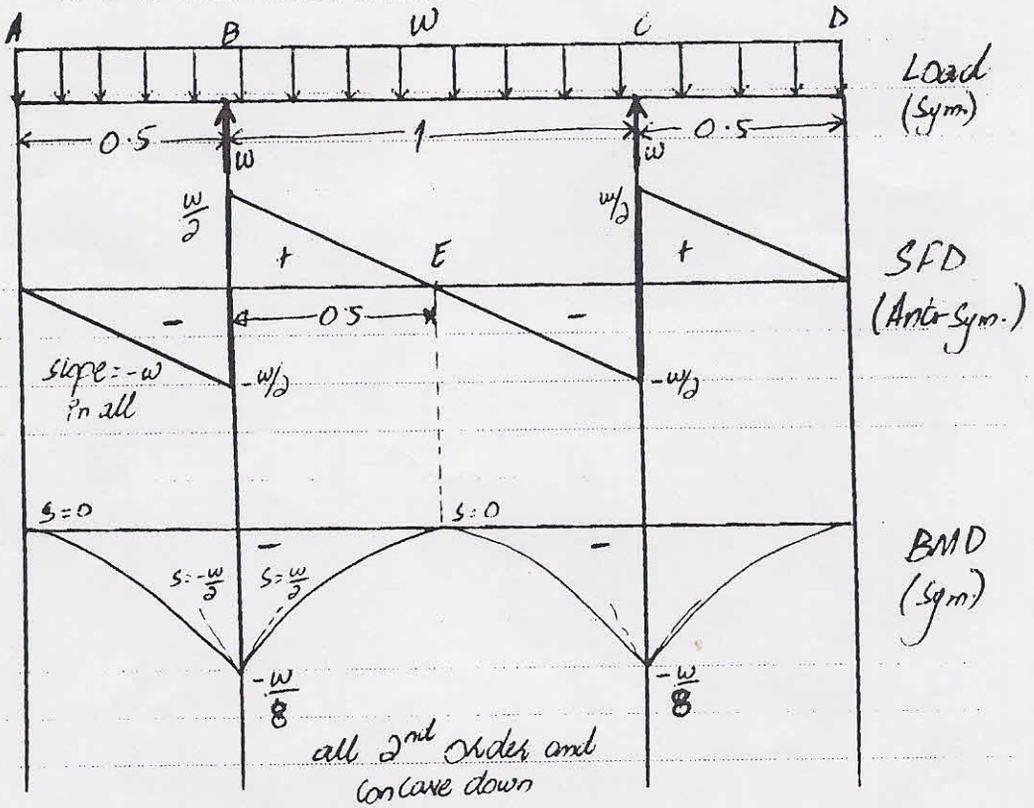
$V_{\max} = \pm 5 \text{ kN}$  ;  $M_{\max} = \pm 2 \text{ kN}\cdot\text{m}$

Required:

- SFD and BMD by the graphical method
- $W_{\max}$

Solution:

a) Due to symmetry,  $B_y = C_y = \frac{\Sigma W}{2} = \frac{2W}{2} = W \text{ kN}$



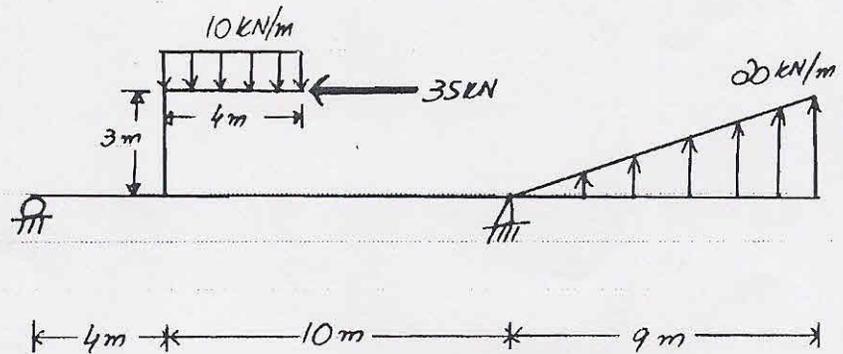
## Solution of HW #9

Problem #06:

Given:

The beam shown

Required:

SFD and BMD by  
the graphical method.

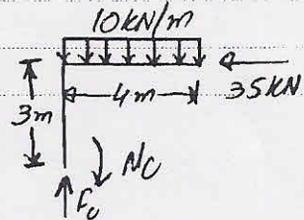
Solution:

First we need to determine the effect of the "extension" at C on the beam.

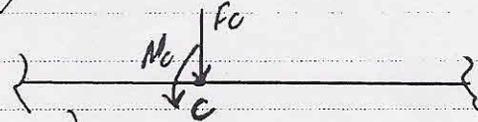
\* Remember

"equal and opposite!"

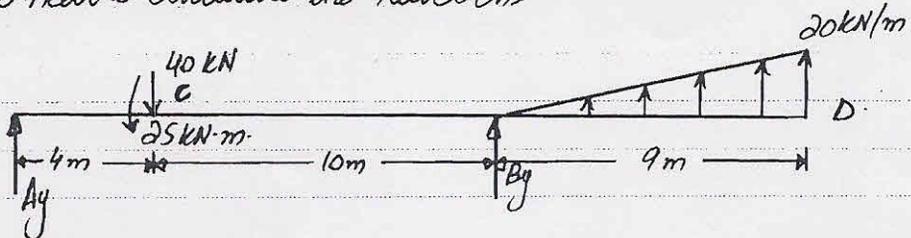
$$\uparrow \sum F_y = 0 \Rightarrow F_C = 40 \text{ kN} \quad \begin{matrix} \uparrow \text{ on Extension} \\ \downarrow \text{ on beam} \end{matrix}$$



$$\uparrow \sum M = 0 \Rightarrow M = -10(4)(2) + 35(3) = 25 \text{ kN}\cdot\text{m} \quad \begin{matrix} \downarrow \text{ on Extension} \\ \downarrow \text{ on beam} \end{matrix}$$



Now we need to calculate the reactions.



$$\sum M_B = 0 \Rightarrow -14A_y + 25 + 40(10) + \frac{(20)(9)}{2} \left(\frac{2}{3} \times 9\right) = 0$$

$$\Rightarrow A_y = 68.929 \text{ kN} \approx 68.93 \text{ kN} (\uparrow)$$

$$\sum F_y = 0 \Rightarrow 68.929 - 40 + \frac{20(9)}{2} + B_y = 0 \Rightarrow$$

$$B_y = -118.93 \text{ kN} = 118.93 \text{ kN} (\downarrow)$$

