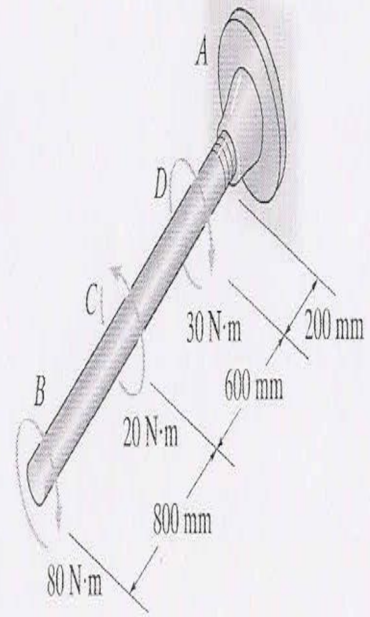
•5–53. The 20-mm-diameter A-36 steel shaft is subjected to the torques shown. Determine the angle of twist of the end B.



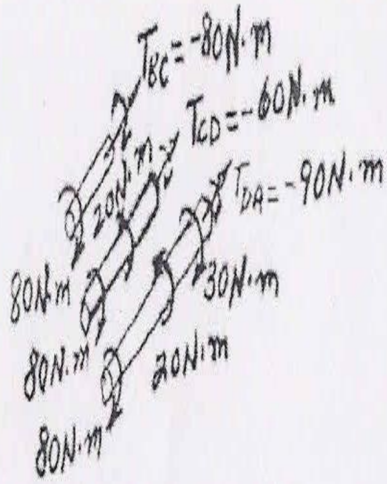
Internal Torque: As shown on FBD.

Angle of Twist:

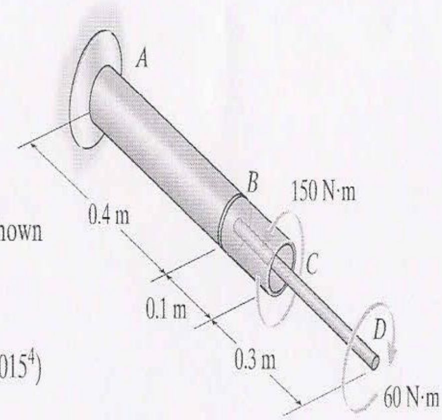
$$\phi_B = \sum \frac{TL}{JG}$$

$$= \frac{1}{\frac{\pi}{2}(0.01^4)(75.0)(10^9)}\left[-80.0(0.8) + (-60.0)(0.6) + (-90.0)(0.2)\right]$$

$$= -0.1002 \text{ rad} = \left|5.74^\circ\right| \qquad \textbf{Ans.}$$



5-54. The assembly is made of A-36 steel and consists of a solid rod 20 mm in diameter fixed to the inside of a tube using a rigid disk at B. Determine the angle of twist at D. The tube has an outer diameter of 40 mm and wall thickness of 5 mm.



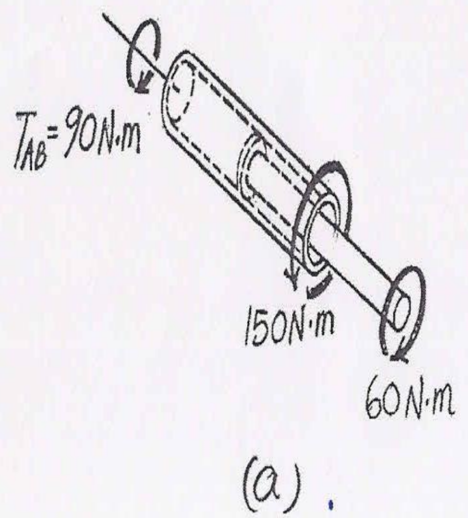
The internal torques developed in segments AB and BD of the assembly are shown in Fig. a and b

The polar moment of inertia of solid rod and tube are $J_{AB} = \frac{\pi}{2}(0.02^4 - 0.015^4) = 54.6875(10^{-9})\pi \text{ m}^4$ and $J_{BD} = \frac{\pi}{2}(0.01^4) = 5(10^{-9})\pi \text{ m}^4$. Thus,

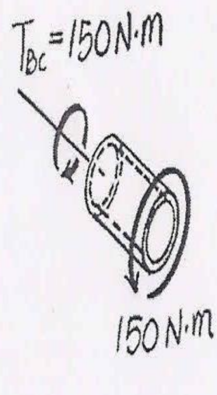
$$\phi_D = \Sigma \frac{T_i L_i}{J_i G_i} = \frac{T_{AB} L_{AB}}{J_{AB} G_{st}} + \frac{T_{BD} L_{BD}}{J_{BD} G_{st}}$$

$$= \frac{90(0.4)}{54.6875(10^{-9})\pi\,[75(10^9)]} + \frac{-60\,(0.4)}{5(10^{-9})\pi\,[75(10^9)]}$$

$$= -0.01758 \text{ rad} = 1.01^\circ \qquad \textbf{Ans.}$$

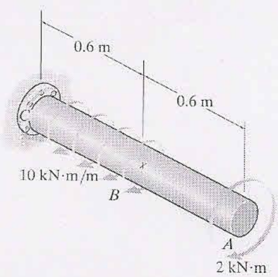


(a)



(b)

***5–72.** The 80-mm diameter shaft is made of 6061-T6 aluminum alloy and subjected to the torsional loading shown. Determine the angle of twist at end A.



Equilibrium: Referring to the free - body diagram of segment AB shown in Fig. a,

$\Sigma M_x = 0;\quad -T_{AB} - 2(10^3) = 0 \qquad\qquad T_{AB} = -2(10^3)\text{N}\cdot\text{m}$

And the free - body diagram of segment BC, Fig. b,

$\Sigma M_x = 0;\quad -T_{BC} - 10(10^3)x - 2(10^3) = 0 \qquad\qquad T_{BC} = -\left[10(10^3)x + 2(10^3)\right]\text{N}\cdot\text{m}$

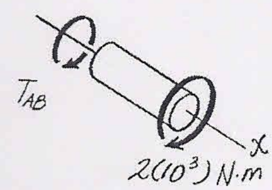
Angle of Twist: The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(0.04^2\right) = 1.28(10^{-6})\pi\ \text{m}^4$. We have

$$\phi_A = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{AB}L_{AB}}{JG_{al}} + \int_0^{L_{BC}}\frac{T_{BC}\,dx}{JG_{al}}$$

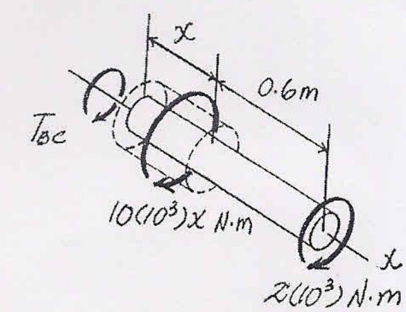
$$= \frac{-2(10^3)(0.6)}{1.28(10^{-6})\pi(26)(10^9)} + \int_0^{0.6\text{ m}}\frac{-\left[10(10^3)x + 2(10^3)\right]dx}{1.28(10^{-6})\pi(26)(10^9)}$$

$$= -\frac{1}{1.28(10^{-6})\pi(26)(10^9)}\left\{1200 + \left[5(10^3)x^2 + 2(10^3)x\right]\Big|_0^{0.6\text{m}}\right\}$$

$$= -0.04017\text{ rad} = 2.30^\circ$$ **Ans.**



(a)

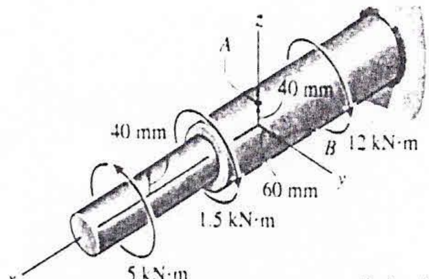


(b)

5.19

Given :-

The figure shown.



Probs. 5–18/19

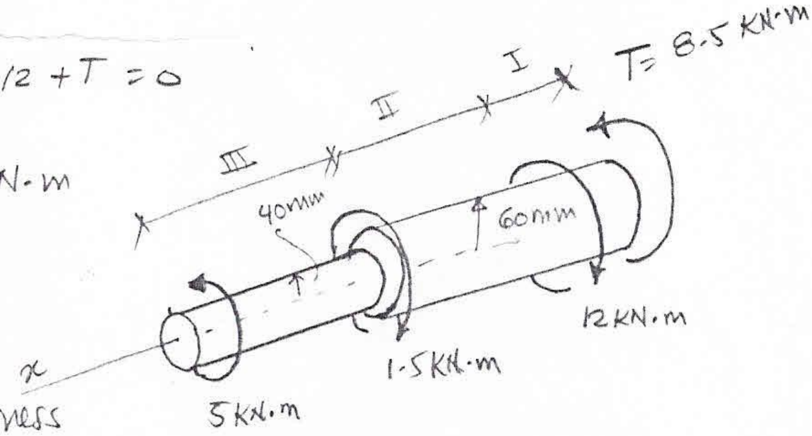
Req'd :-

τ_{max} .

Sol'n :-

$\sum M_x = 0 ; \quad 5 - 1.5 - 12 + T = 0$

$\Rightarrow \quad T = 8.5 \text{ KN·m}$



from The F.B.D

The maximum shear stress should be either in region I or III,

for region "I" $\quad \tau_{max_{(I)}} = \dfrac{T * c}{J} = \dfrac{8.5 \times 10^6 * 60}{\left(\frac{\pi}{2}\right)(60)^4} = 25 \text{ MPa}$

for region III $\quad \tau_{max_{III}} = \dfrac{T * c}{J} = \dfrac{5 \times 10^6 \times 40}{\left(\frac{\pi}{2}\right)(40)^4} = 49.7 \text{ MPa}$

$\therefore \quad \tau_{max} = \underline{\underline{49.7 \text{ MPa}}}$

King Fahd University of Petroleum & Minerals

CIVIL ENGINEERING DEPARTMENT

CE 203 STRUCTURAL MECHANICS I

Second Semester 1433 / 2012 (**112**)

HOMEWORK NO. 7

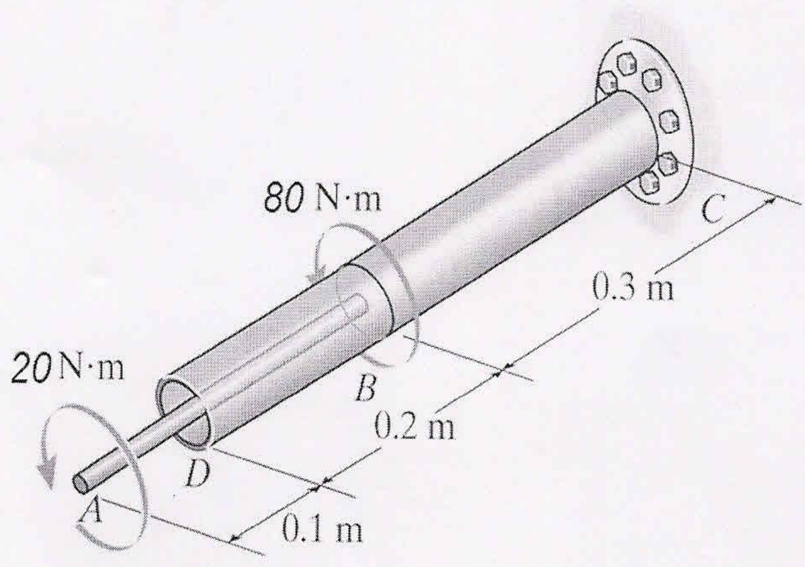
- **Textbook Sections Covered:** 5.1 – 5.4, Torsion : stress and angle of twist
- **DUE DATE**: **Monday 19-March-2012**

---

1 - Solve problem **5- 40** in the textbook. Use the shaft diameter as 30 mm (instead of 25 mm).

2- Use the figure and data for problem **5-70** in the textbook. Determine the absolute maximum shear stress in the shaft and the angle of twist of E with respect to B , and the angle of twist of E with respect to A.

3- The assembly (shown below) consists of a solid rod AB (d=20 mm) connected to the inside of a tube DC using a rigid disk at B. The tube DC has an outer diameter of 55 mm and a thickness of 5 mm. Determine the absolute maximum shear stress in the whole shaft and the angle of twist of D , and the angle of twist of A. **G = 100 GPa** .



King Fahd University of Petroleum & Minerals

CIVIL ENGINEERING DEPARTMENT

CE 203 STRUCTURAL MECHANICS I

First Semester 2012 / 2013 (**121**)

HOMEWORK NO. 7 (Key Solution)

- **Textbook Sections Covered: 5.1 & 5.3** , Torsion of circular shafts

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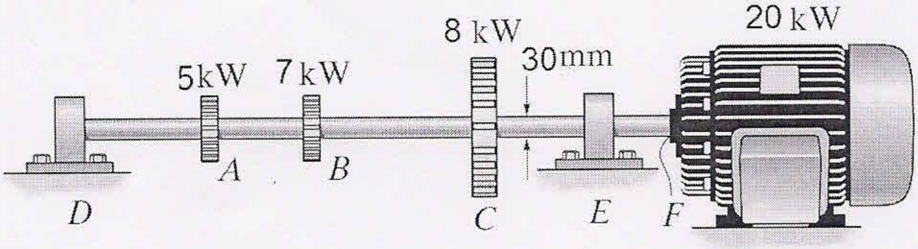
Problem # 1:-

Given Data:

- The shown figure.
- Smooth bearings at *D* and *E (induce zero resistance torque).*
- Motor delivers 20 KW of Power to the shaft.
- Turning at 60 rev/sec.
- Gears *A*, *B*, and *C* remove 5 kw, 7 kw, and 8 kw respectively

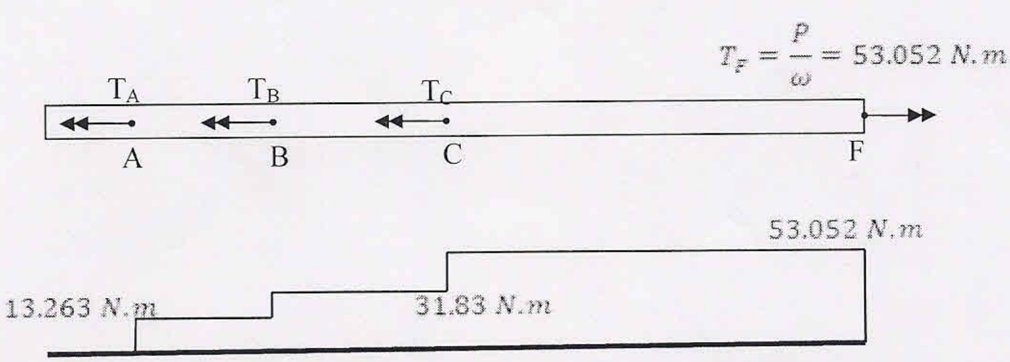


Required:

- **The maximum shear stress developed in the shaft within regions *CF* and *BC*.**

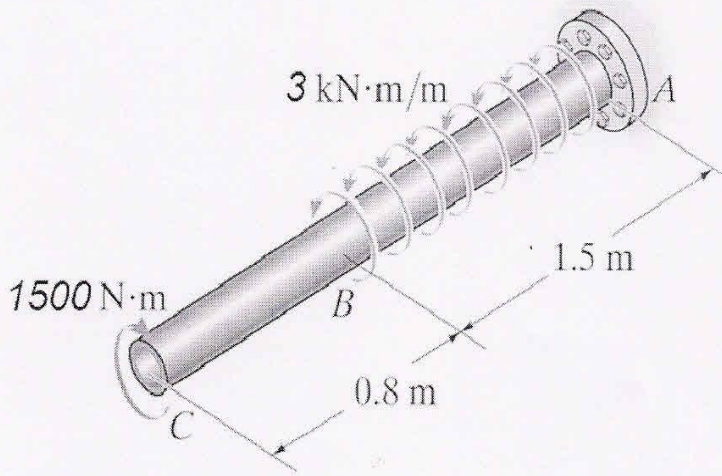
Solution:

$\omega = 60\frac{rev}{se}\left[2\pi\frac{rad}{rev}\right] = 120\pi\frac{rad}{s}$ As it is shown in the following free body diagram and torque diagram,

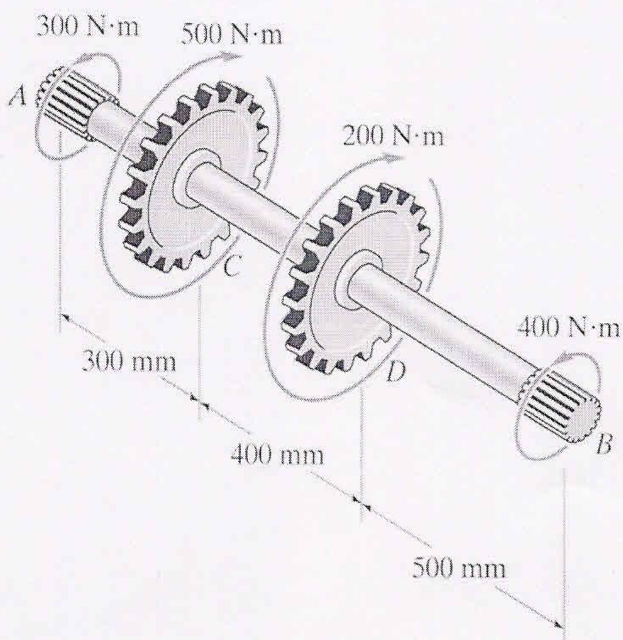


Free body diagram and torque diagram

4 – The solid circular shaft is subjected to the shown torques. If the angle of twist of end C is not to exceed 1 degree , and the allowable shear stress is 60 MPa, determine the smallest (required) diameter of the shaft that may be used. **G = 80 GPa.**



5- The given shaft has an outer diameter = 50 mm , and an inner diameter = 30 mm. Determine the shear stress at the inner and outer surfaces ***in segment CD only***. Plot the shear stress distribution along the radius in that segment. Also, calculate the angle of twist of B with respect to A. **G = 100 GPa**



CE 203 – 112

Solution of HW # 7

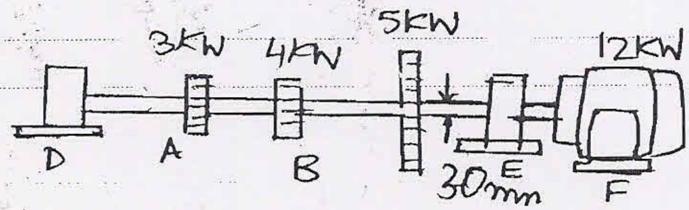
Problem #1

Given:-

The figure shown

$D_{shaft} = 30mm$

$Speed = 50\ rev/s$



Required:

τ_{max}

Solution:-

To find τ_{max}, we need to locate Tmax as we have only one D. We have "power" ⇒ we need to get T

$\Rightarrow P = T\omega$

$$\omega = \text{angular velocity} = 50\frac{rev}{s}\left(\frac{2\pi\ rad}{1\ rev.}\right)$$

$$= 1000\pi\ rad/s$$

Clearly, Tmax is @ Pmax which is between C and F

$\Rightarrow P_{max} = 12\ KW$

$\Rightarrow 12(10^3) = T_{max}(100\pi)$

$\Rightarrow T_{max} = 38.1972\ N\cdot m$

$$\Rightarrow \tau_{max} = \frac{T_{max}\, r_{max}}{J} = \frac{T_{max}\, r_{out}}{J} = \frac{T_{max}\, C}{J}$$

$$\tau_{max} = \frac{39.1972(15)(10)^{-3}}{\pi/2\left[(15)(10)^{-3}\right]^4}$$

$$\Rightarrow \boxed{\tau_{max} = 7.205\ MPa}$$

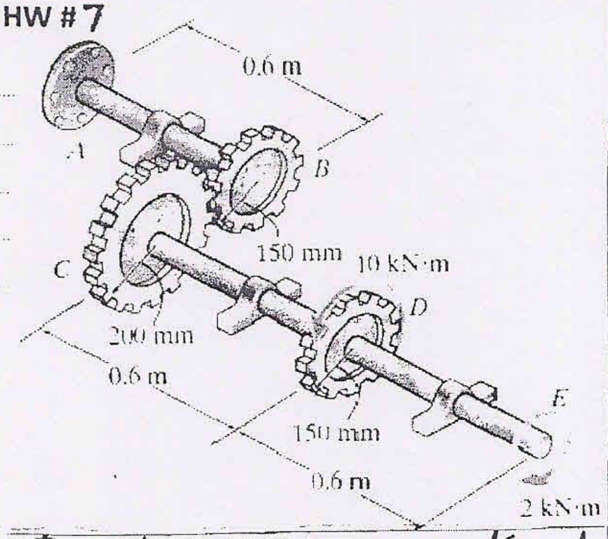
Solution of HW # 7

Problem # 2:-

Given:-

The figure shown

A-36 steel shaft; D = 80 mm



Required:-

τ_{max} ; $\phi_{E/B}$; $\phi_{E/A}$

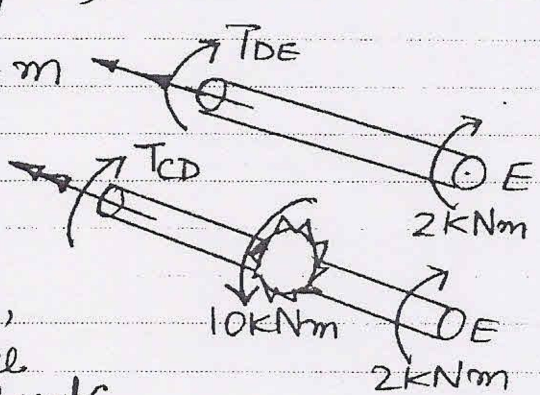
Solution:

Since all shafts in the system have the same diameter, τ_{max} will be at T_{max}. T is the internal torque in each shaft. Thus, we need to draw FBD's for all "segments" to determine $T_{internal}$ in each, as shown below. Note that all internal T's are assumed ⊕ (i.e. on the "right" part).

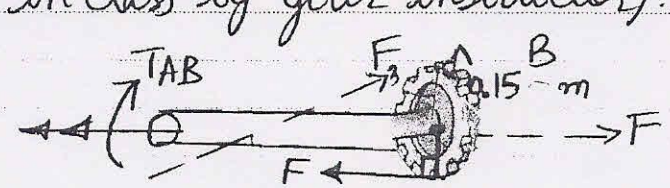
$\Sigma T = 0 \Rightarrow T_{DE} = -2 \text{ KN.m}$

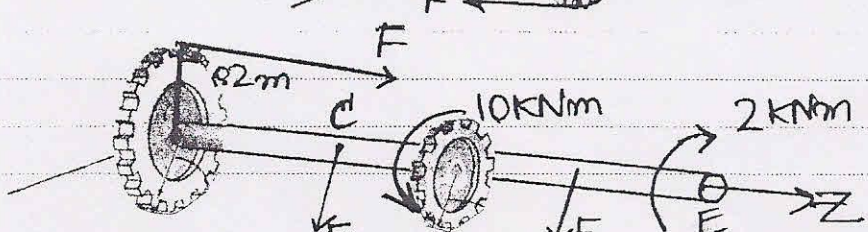
$-T_{CD} + 10 - 2 = 0$

$\Rightarrow T_{CD} = 8 \text{ KN.m}$



To find the torque in AB, we have to find the force between the two gears B and C (as explained in class by your instructor). Thus,





$\Sigma M_z = 0$ in CDE $\Rightarrow 10 - 2 - 0.2(F) = 0 \Rightarrow F = 40\,KN$

$\Sigma M_z = 0$ in AB $\Rightarrow -T_{AB} - 40(0.15) = 0 \Rightarrow T_{AB} = -6\,KNm$

From the values of $|T_{AB}|$, $|T_{CD}|$ and $|T_{DE}|$,

$T_{max} = |T_{CD}| = 8\,KNm$

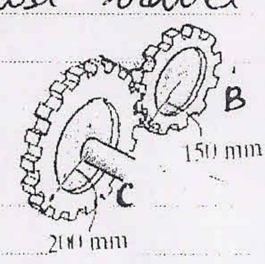
$$\Rightarrow \tau_{max} = \frac{T_{max}\, r_{max}}{J} = \frac{T_{CD}\, r_{out}}{J} = \frac{8(10)^3\,(0.08/2)}{\pi/2\,(0.08/2)^4}$$

$$\Rightarrow \boxed{\tau_{max} = 79.58\text{ Mpa @ } r_{out} \text{ in segment CD}}$$

$\phi_{E/B} = \Sigma\, \phi_{E \text{ to } B}$

$\phi_{E/A} = \Sigma\, \phi_{E \text{ to } A}$

Since we have gears, we need to calculate the rotation of gear C relative to B.
As they are connected together, they must "travel" the same distance; thus,



$$\theta_B\, r_B = \theta_C\, r_C$$

In our case, θ = angle of twist = ϕ

$$\Rightarrow \phi_B\, r_B = \phi_C\, r_C$$

$$\Rightarrow \phi_C = \frac{r_B}{r_C}\phi_B = \frac{150}{200}\phi_B = 0.75\,\phi_B$$

Therefore, $\phi_{E/A} = \phi_{E/C} + \phi_C$

$\phi_{E/A} = \phi_{CD} + \phi_{DE} + \phi_C = \phi_E$ as A is fixed

$$\phi_{CD} = \left(\frac{TL}{JG}\right)_{CD} = \frac{8(10)^3(0.6)}{\pi/2(0.04)^4\underbrace{(75)(10)^9}_{\text{from the table at the end of the text book.}}} = 0.0159155 \text{ rad}$$

$$\phi_{DE} = \left(\frac{TL}{JG}\right)_{ED} = \frac{-2(1)^3(0.6)}{\pi/2(0.04)^4(75)(10)^9} = -0.00397887 \text{ rad}$$

To find ϕ_C, we first need to determine ϕ_B

$\phi_{B/A} = \phi_B = \phi_{AB}$ as A is fixed

$$\Rightarrow \phi_B = \left(\frac{TL}{JG}\right)_{AB} = \frac{-6(10)^3(0.6)}{\pi/2(0.04)^4(75)(10)^9} = -0.0119366 \text{ rad}$$

$\Rightarrow \phi_C = -0.75(-0.0119366) = 0.0089524 7 \text{ rad}$ [Be careful about signs!]

Thus, $\phi_{E/A} = 0.0159155 - 0.00397887 + 0.0089524 7$

$$\boxed{\phi_{E/A} = 0.02088 \uparrow \text{rad} = 1.197^\circ}$$

$\phi_{E/B} = \phi_{E/A} - \phi_B$ (Be careful about signs!)

$\Rightarrow \phi_{E/B} = 0.020889 - (-0.0119366)$

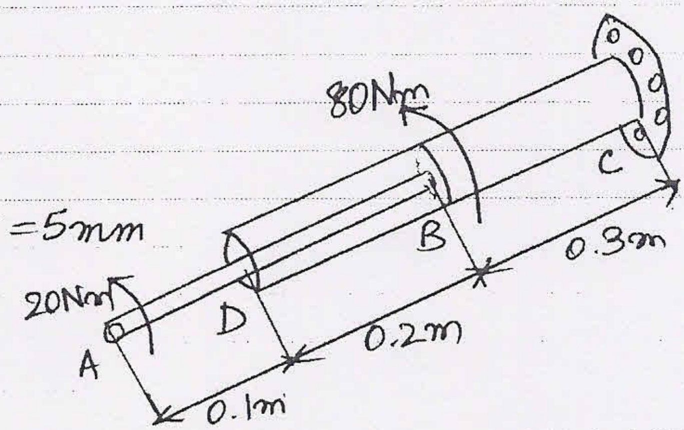
$$\Rightarrow \boxed{\phi_{E/B} = 0.03283 \text{ rad} = 1.881^\circ}$$

Problem # 3:

Given:

The figure shown

$D_{AB} = 20\ mm$



DC: $D_{out} = 55\ mm;\ t = 5\ mm$

$G = 100\ GPa$

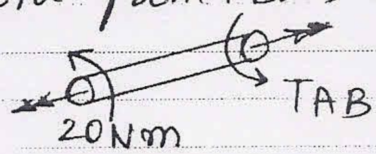
Required:

$\tau_{max};\ \phi_A;\ \phi_D$

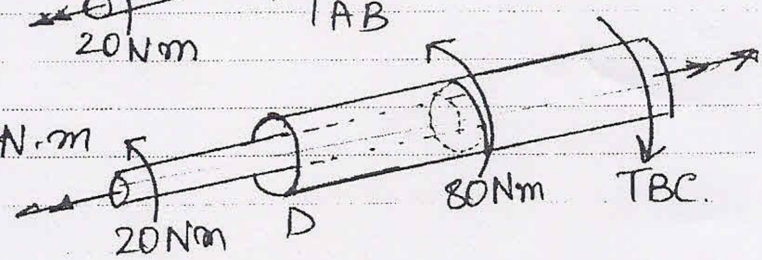
Solution:

We know that $\tau = \frac{Tr}{J}$ in each shaft; r is given for each, and we need to determine T (which is internal) for each segment from FBD's as shown below

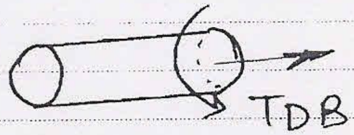
$T_{AB} = 20\ N.m$



$T_{BC} = 20 + 80 = 100\ N.m$



$T_{DB} = 0$



$$\tau_{max_{AB}} = \frac{T_{AB}\, r_{max}}{J} = \frac{20(10)(10)^{-3}}{\pi/2\left[(10)(10)^{-3}\right]^4} = 12.732\ MPa.$$

$$\tau_{BC}^{max} = \frac{100\ (55/2)(10)^{-3}}{\pi/2\left[\left\{\frac{55}{2}(10)^{-3}\right\}^4 - \left\{\frac{55-10}{2}(10)^{-3}\right\}^4\right]} = 5.5468\ MPa$$

$\tau_{DB} = 0$

$\tau_{max} = \tau_{AB}^{max} \Rightarrow \boxed{\tau_{max} = 12.73\text{ Mpa at rout in AB}}$

$\phi_A = \phi_{AC}$ since C is fixed

$= \phi_{AB} + \phi_{BC}$

$= \left(\frac{TL}{JG}\right)_{AB} + \left(\frac{TL}{JG}\right)_{BC}$

$= \frac{0.3}{100(10)^9 \pi/2}\left[\frac{20}{\{10(10)^{-3}\}^4} + \frac{100}{\{\frac{55}{2}(10)^{-3}\}^4 - \{\frac{55-10}{2}(10)^{-3}\}^4}\right]$

$\boxed{\phi_A = 9.8708(10)^{-4}_{rad} = 0.05656^\circ}$

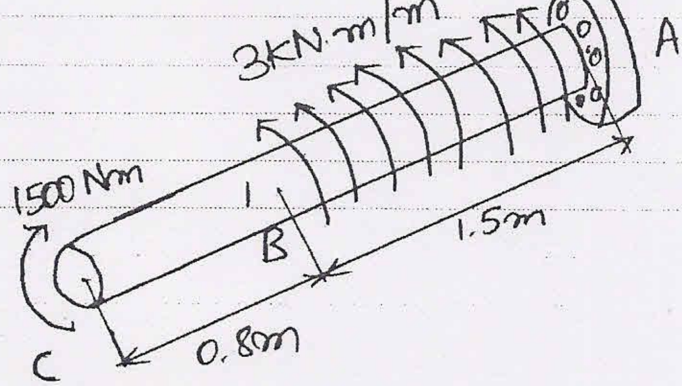
$\phi_D = \phi_{DB} + \phi_{BC} = \underset{as\, T_{DB}=0}{0} + \frac{0.3(100)}{\frac{\pi}{2}\left[\left\{\frac{55}{2}(10)^{-3}\right\}^4 - \left\{\frac{55-10}{2}(10)^{-3}\right\}^4\right] 100 \times 10^9}$

$\boxed{\phi_D = 6.051(10)^{-4}\text{rad} = 0.03467^\circ}$

Problem # 4:-

Given:
The figure shown
$\phi_C^{max} = 1^\circ$
$\tau_{allow} = 60\,MPa$

Required:-
Dmin



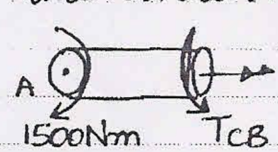
Solution :-
Here, we have two criteria we need to satisfy: the angle of twist and the shear stress. There are two ways to solve the problem. The first one is to determine Dmin for each case, then take the bigger one. Or, assume one criterion controls (i.e. its max/allow) will be reached before the other one, and from that determine Dmin. After that we need to check our assumption by calculating the other criterion using Dmin found. We will follow the first method, as it is easier for the student to comprehend.

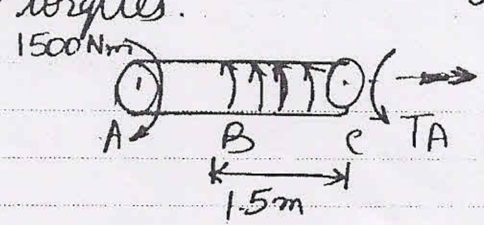
Start with τ_{max} criterion:

$$\tau = \frac{Tr}{J}$$

Since the shaft has one diameter, and the two applied forces are in different directions, τ_{max} will be in segment CB or at end A, depending on the values of the internal torques.

From the FBD's

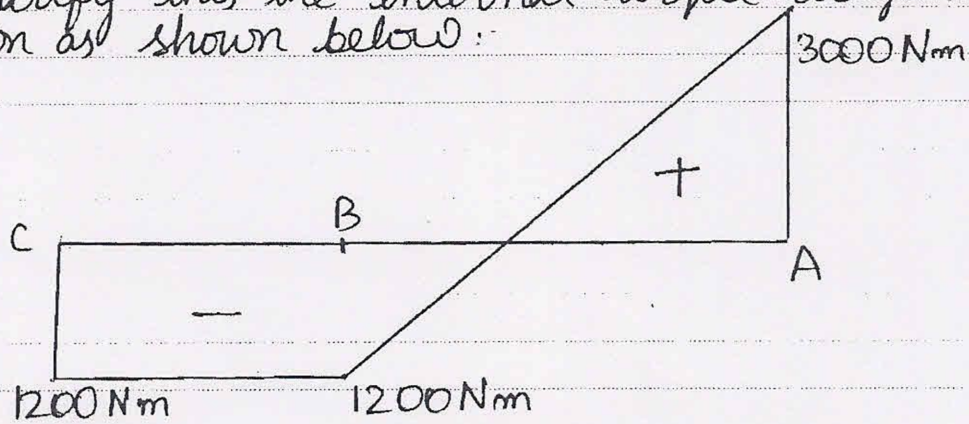




$T_{CB} = -1500\ Nm$

$T_A = 3000(1.5) - 1500 = 3000\,Nm$

To clarify this the internal torque diagram can be drawn as shown below:



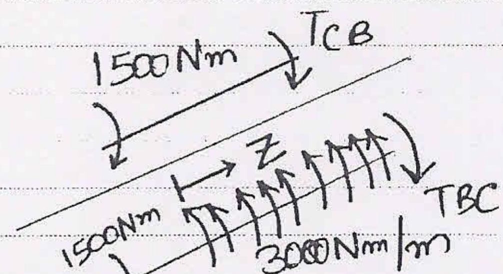
$\tau_{max} = \dfrac{T_{max}\, r_{max}}{J}$

Set $\tau_{max} = 60(10)^6 = \dfrac{3000\,(r_{out})}{\pi/2\,(r_{out})^4} \Rightarrow$

$\underline{r_{min} = 0.031692\ m}$

Next, ϕ Criterion:-

$\phi_C = \phi_{CB} + \phi_{BA}$



For CB, T, r and G are constant. Thus, we can use the formula $\phi = \dfrac{TL}{JG}$; but for BA, T is <u>not</u> constant.

So we must integrate as $\phi = \int \dfrac{T}{JG}\,dz$.

$\phi_C = \left(\dfrac{TL}{JG}\right)_{CB} + \int_0^{1.5} \left(\dfrac{T}{JG}\right)_{BA} dz$

$T_{CB} = -1500\,Nm$ $\qquad T_{BC} = -1500 + 3000z$

$$= \frac{1}{JG}\left[-1500(0.8) + \int_0^{1.5}(-1500+3000z)\,dz\right] \text{ (As JG are common \& constant)}$$

$$= \frac{1}{\pi/2\, r^4 (80)(10)^9}\left[-1500(0.8) - 1500(1.5) + \frac{3000}{2}(1.5)^2\right]$$

$$= \frac{-1.875(10)^{-9}}{\pi r^4}$$ (Do not worry about the − sign! why?)

Now, set $\phi = \phi_{min} = 1^\circ = \frac{\pi}{180^\circ} = \frac{1.875(10)^{-9}}{\pi r^4_{min}}$

$\Rightarrow \underline{r^{\phi}_{min} = 0.013599\ m}$

From r^{τ}_{min} & r^{ϕ}_{min}, we pick the bigger one for the min r (why?).

$\Rightarrow r_{min} = 0.031692\,m$

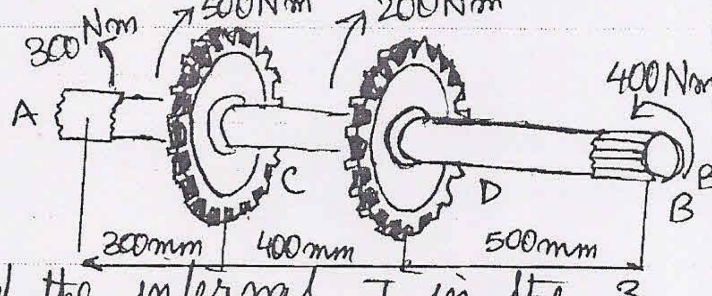
[Note: for "typical" values and materials, the stress usually controls; i.e. the diameter needed to satisfy the shear stress condition is usually bigger than that required for the angle of twist, as the case here].

Thus, $\boxed{D_{min} = 0.063384\,m = 63.4\,mm}$

Solution of HW # 7

Problem #5

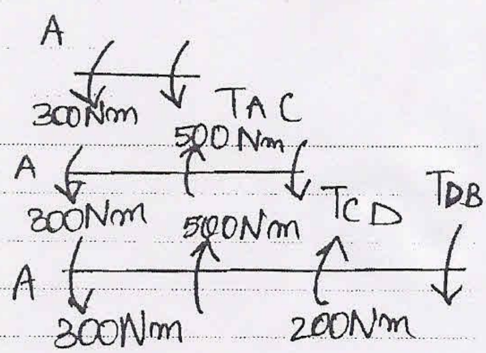
Given:-
The figure shown,
$D_{out} = 50mm$; $D_{in} = 30mm$; $G = 100\,GPa$



Required:
τ in CD
$\phi_{B/A}$

Solution:-
First, we need to find the internal T in the 3 segments AC, CD and DB. (why 3 segments?)

From the FBD's



$T_{AC} = -300\,Nm$

$T_{CD} = 500 - 300 = 200\,Nm$

$T_{DB} = -300 + 500 + 200 = 400\,Nm$

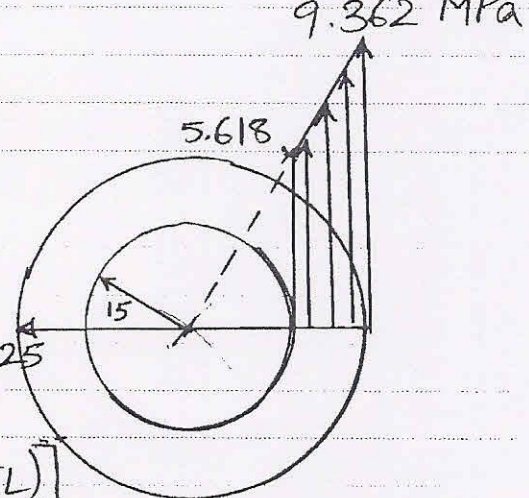
$$\tau = \frac{Tr}{J}$$

$$J = \pi/2 \left[\left(\frac{50}{2}\right)^4 - \left(\frac{30}{2}\right)^4 \right] \left[(10)^{-3} \right]^4 = 1.7(10)^{-7}\pi\ m^4$$

$$\tau_{in}^{CD} = \frac{T_{CD} r_{in}}{J} = \frac{200(15)(10)^{-3}}{1.7(10)^{-7}\pi} = \boxed{\tau_{in}^{CD} = 5.617\,MPa}$$

$$\tau_{out}^{CD} = \frac{T_{CD} r_{out}}{J} = \frac{200(25)}{1.7(10)^{-7}\pi} \Rightarrow \boxed{\tau_{out}^{CD} = 9.362\,MPa}$$

It has a linear distribution as shown in the figure.



$$\phi_{B/A} = \phi_{B/D} + \phi_{D/C} + \phi_{C/A}$$

Since T, J and G are constant in all 3 segments, we can use the formula $\phi = \frac{TL}{JG}$ directly.

$$\phi_{B/A} = \frac{1}{JG}\left[(TL)_{BD} + (TL)_{DC} + (TL)_{CA}\right]$$

$$= \frac{1}{1.7(10)^{-7}\pi(100)(10)^9}\left[400(0.5) + 200(0.4) + (-3000)(0.3)\right]$$

$$\boxed{\phi_{B/A} = 3.558(10)^{-3}\,\text{rad} = 0.2038^\circ}$$