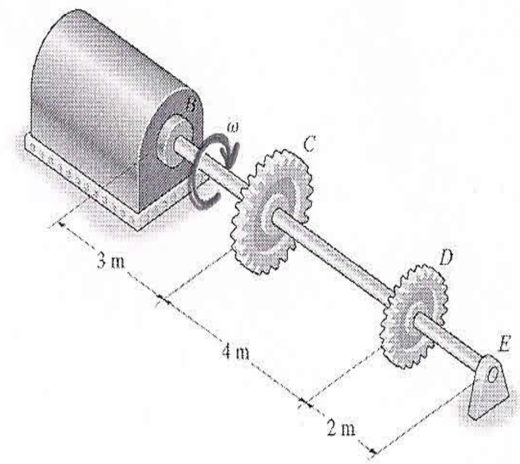
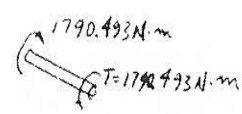
5–53. The turbine develops 150 kW of power, which is transmitted to the gears such that C receives 70% and D receives 30%. If the rotation of the 100-mm-diameter A-36 steel shaft is $\omega = 800$ rev/min., determine the absolute maximum shear stress in the shaft and the angle of twist of end E of the shaft relative to B. The journal bearing at E allows the shaft to turn freely about its axis.

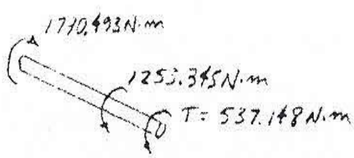


$$P = T\omega; \qquad 150(10^3)W = T(800\frac{\text{rev}}{\text{min}})(\frac{1 \text{ min}}{60 \text{ sec}})(\frac{2\pi \text{ rad}}{1 \text{ rev}})$$



$$T = 1790.493 \text{ N}\cdot\text{m}$$

$$T_C = 1790.493(0.7) = 1253.345 \text{ N}\cdot\text{m}$$
$$T_D = 1790.493(0.3) = 537.148 \text{ N}\cdot\text{m}$$



Maximum torque is in region BC.

$$\tau_{max} = \frac{Tc}{J} = \frac{1790.493(0.05)}{\frac{\pi}{2}(0.05)^4} = 9.12 \text{ MPa} \qquad \textbf{Ans}$$

$$\phi_{E/B} = \Sigma(\frac{TL}{JG}) = \frac{1}{JG}[1790.493(3) + 537.148(4) + 0]$$

Problem # 2

Determine the required thickness, t for the shaft shown below to carry the applied load (P = 12 kN) safely. The shaft is made from a material for which the allowable shear stress, τ_{all} =350 MPa and the allowable angle of twist, $\emptyset_{all}$ is 2.

Given G= 80 GPa

Key equations:

$$\tau_{all} = \tau_{cal}^{max} \qquad \boxed{1}$$

$$\phi_{all} = \phi_{cal}^{max} \qquad \boxed{1}$$

(A) For thin-walled tube,

$$\tau_{all} = \frac{T}{2 A_m t_{min}} \qquad \boxed{2}$$

$\tau_{all} = 350 \times 10^6$ Pa.

$T = 12 \times 10^3 (0.6) = 7.2 \times 10^3$ N-m $\boxed{2}$

$A_m = (0.1)(0.05) = 5 \times 10^{-3}$ m^2 $\boxed{2}$

$$\therefore \quad 350 \times 10^6 = \frac{(7.2 \times 10^3)}{2(5 \times 10^{-3})t} \Rightarrow t \simeq 2 \text{ mm} \quad \boxed{1} \quad \boxed{1}$$

(B) For thin-walled tube,

$$\phi_{all} = \frac{TL}{4A_m^2 G} \sum_{i=1}^{4} \frac{S_i}{t_i} \qquad \boxed{2}$$

$\phi_{all} = 2\left(\frac{\pi}{180}\right)$ rad. $\boxed{2}$

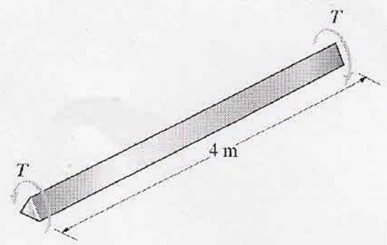
$$\therefore \quad 2\left(\frac{\pi}{180}\right) = \frac{(12 \times 10^3)(.6)(2)}{4(5 \times 10^{-3})^2 (80 \times 10^9)} \underbrace{\left[2\left(\frac{.1}{2t} + \frac{0.05}{t}\right)\right]}_{\boxed{3}}$$

$\therefore \quad t \simeq 10$ mm $\boxed{1}$

$\therefore$ Use the largest thickness

$$\therefore \quad t_{req.} \simeq 10 \text{ mm} \qquad \boxed{2}$$

5–95. The brass wire has a triangular cross section, 2 mm on a side. If the yield stress for brass is $\tau_Y = 205$ MPa, determine the maximum torque T to which it can be subjected so that the wire will not yield. If this torque is applied to a segment 4 m long, determine the greatest angle of twist of one end of the wire relative to the other end that will not cause permanent damage to the wire. $G_{br} = 37$ GPa.



Allowable Shear Stress :

$$\tau_{max} = \tau_Y = \frac{20T}{a^3}$$

$$205(10^6) = \frac{20T}{0.002^3}$$

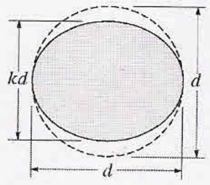
$$T = 0.0820 \text{ N}\cdot\text{m} \qquad \textbf{Ans}$$

Angle of Twist :

$$\phi = \frac{46TL}{a^4 G} = \frac{46(0.0820)(4)}{(0.002^4)(37)(10^9)}$$

$$= 25.5 \text{ rad} \qquad \textbf{Ans}$$

***5–96.** It is intended to manufacture a circular bar to resist torque; however, the bar is made elliptical in the process of manufacturing, with one dimension smaller than the other by a factor k as shown. Determine the factor by which the maximum shear stress is increased.



For the circular shaft :

$$(\tau_{max})_c = \frac{Tc}{J} = \frac{T(\frac{d}{2})}{\frac{\pi}{2}(\frac{d}{2})^4} = \frac{16T}{\pi d^3}$$

For the elliptical shaft :

$$(\tau_{max})_e = \frac{2T}{\pi a b^2} = \frac{2T}{\pi(\frac{d}{2})(\frac{kd}{2})^2} = \frac{16T}{\pi k^2 d^3}$$

$$\text{Factor of increase in shear stress} = \frac{(\tau_{max})_e}{(\tau_{max})_c} = \frac{\frac{16T}{\pi k^2 d^3}}{\frac{16T}{\pi d^3}}$$

$$= \frac{1}{k^2} \qquad \textbf{Ans}$$

Problem # 4:

Given:

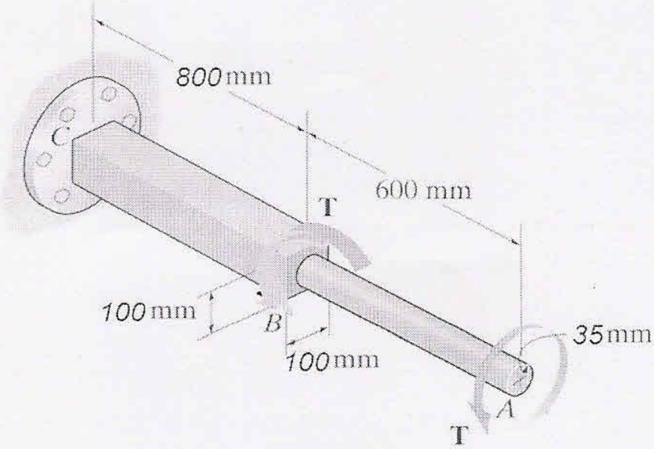
The figure shown

$\tau_{allow} = 120\ MPa;\quad \varphi^{A}_{max} = 1^{o}\ ;$

$\varphi^{B}_{max} = 0.5^{o};\quad G = 100\ GPa$

Required:

Maximum allowable ***T***

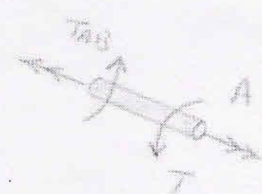


Solution:

Here, we have four criteria (how?!) we need to satisfy. We calculate $\mathbf{T}_{\substack{\mathbf{max}\\ \mathbf{allow}}}$ *for each, and then we choose the smallest value for the answer* $\mathbf{T}_{\substack{\mathbf{max}\\ \mathbf{allow}}}$. *(Why?! see previous HW!).*

The problem is statically determinate so that we can find the <u>*internal*</u> ***T*** *directly. We have* <u>*two segments*</u> *AB and BC. (Why?!)*

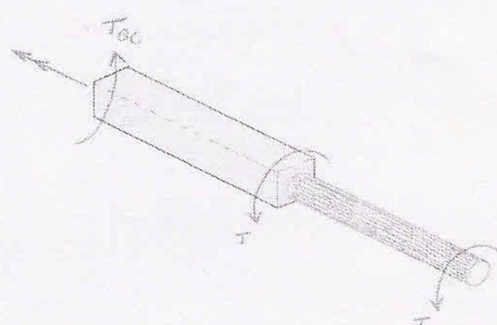
From the FBD's,



$T_{AB} = T$

$T_{BC} = 2T$

For segment AB, set $\tau_{max} \equiv 120\ MPa$.



$$\tau_{max} = \frac{T_{AB}\,r_{max}}{J_{AB}} = \frac{T_{max}\ (0.035)}{\frac{\pi}{2}\ (0.035)^4} \equiv 120\ (10)^6$$

$$\Rightarrow\ T^{1}_{max} = 8.082\ kN.m$$

For segment BC, set $\tau_{max} \equiv 120\ MPa$.

$$\tau_{max} = \frac{4.81\ T_{BC}}{a^3} = \frac{4.81\ (2T_{max})}{(0.1)^3} \equiv 120\ (10)^6$$

$$\Rightarrow\ T^{2}_{max} = 12.47\ kN.m$$

Now consider φ^{B}, *then* φ^{A}. *(Why?!)*

$$\varphi^B = \frac{7.10\, T_{BC} L_{BC}}{a^4 G} = \frac{7.10\,(2T_{max})(0.8)}{(0.1)^4(100)(10)^9} \equiv 0.5\left(\frac{\pi}{180}\right)$$

$$\Rightarrow T^3_{max} = 7.682\ kN.m$$

$$\varphi^A = \sum \varphi = \left(\frac{TL}{JG}\right)_{AB} + \left(\frac{7.10\,TL}{aG}\right)_{BC} = \frac{T_{max}(0.6)}{\frac{\pi}{2}(0.035)^4(100)(10)^9} + \frac{7.10\,(2T_{max})(0.8)}{(0.1)^4(100)(10)^9}$$

$$2.54542\,(10)^{-6}\,T_{max} + 1.136\,(10)^{-6}\,T_{max} \equiv 1\left(\frac{\pi}{180}\right)$$

$$\Rightarrow T^4_{max} = 4.741\ kN.m$$

T^4_{max} *due to* φ^A_{max} *controls.* (*Why?!*)

$$\Rightarrow \quad \boxed{T_{\substack{max \\ allow}} = 4.741\ kN.m}$$

Problem # 1:

Given:

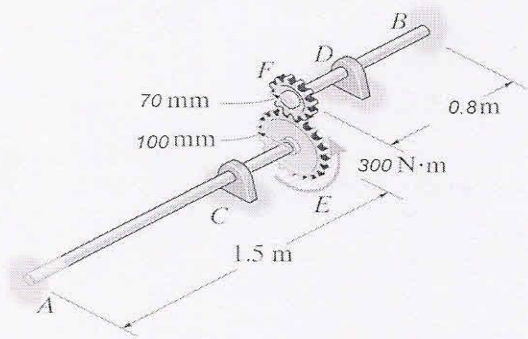
The figure shown

r = 30 mm ; G = 100 GPa

Required:

Value and location of $\tau_{max,}$

φ_F



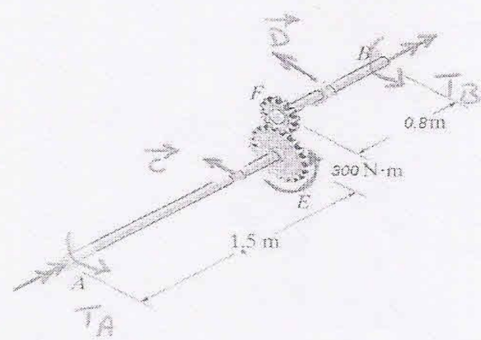
Solution:

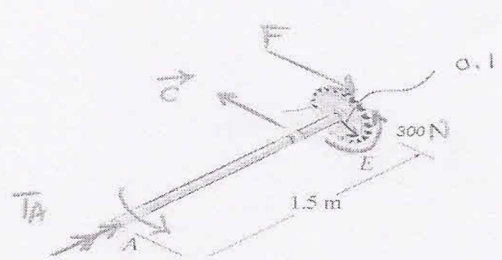
The system is statically indeterminate as it is fixed at A and B; thus, there are ***two unknowns*** *T_A and T_B (reactions) and only* ***one*** *equilibrium equation ($\Sigma T = 0$), as shown below in the FBD.*

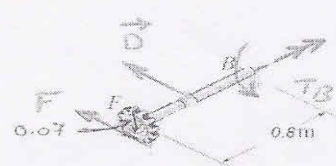
(1) *Equilibrium*

$$\Sigma T_{axis} = 0$$

Since the two shafts are not on the same line, when we take Σ T about the axis of one of them, the reaction on the bearing of the other one will produce "T", and thus the reaction appears on the equation. Therefore, it is better to separate the two shafts from the gear and take each one separately as shown below.







$$\sum T_{\substack{AE \\ axis}} = 0 \Rightarrow T_A - 300 + F(0.1) = 0 \qquad (1)$$

$$\sum T_{\substack{FB \\ axis}} = 0 \Rightarrow T_B + F(0.07) = 0 \qquad (2)$$

$From\ (2),\qquad F = -\dfrac{T_B}{0.07} \Rightarrow$

$T_A - 300 + \left(-\dfrac{T_B}{0.07}\right)(0.1) = 0 \Rightarrow$

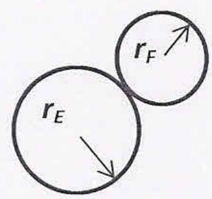
$\boldsymbol{T_A - \frac{10}{7}T_B - 300 = 0}$ **(3)**

(2) *Geometric Compatibility:*

The geometric compatibility of the gears will be used (as discussed in the previous HW).

$r_E\theta_E = r_F\theta_F$

$\theta = \varphi \qquad in\ our\ case\ \Rightarrow$



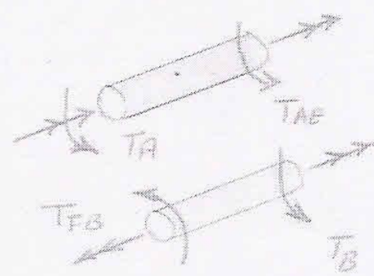
$\boldsymbol{0.1\varphi_E = 0.07\varphi_F \Rightarrow \varphi_E = 0.7\varphi_F}$ **(4)**

$From\ the\ boundary\ conditions,$

$\varphi_E = \varphi_{E/A} = \varphi_{AE} \quad as\ A\ is\ fixed$

$\varphi_F = \varphi_{F/B} = \varphi_{FB} \quad as\ B\ is\ fixed$

$\boldsymbol{Then, eq.\ (4) becomes}\ \left(\frac{TL}{JG}\right)_{AE} = 0.7\left(\frac{TL}{JG}\right)_{FB}$ **(5)**



$From\ the\ FBD\ shown$

$T_{AE} = -T_A$

$T_{FB} = T_B$

Note that both <u>*internal*</u> *torques are assumed positive. (Why & How?!)*

***J* & *G* are common in the two shafts, thus eq. (5) becomes** $-1.5T_A = 0.7(0.8T_B) \Rightarrow$

$\boldsymbol{T_B = -2.67857T_B}$ **(6)**

$From\ eq.\ (6) into\ eq.\ (3),\ T_A - \frac{10}{7}(-2.67857T_A) - 300 = 0 \Rightarrow$

$\boldsymbol{T_A = 62.156\ N.m} \Rightarrow$

$\boldsymbol{T_B = -166.49\ N.m = 166.49\ N.m}$

Since **r** is the same for the two shafts, $\boldsymbol{\tau_{max}}$ *will be @* $\boldsymbol{T_{max} = T_{FB} = T_B} \Rightarrow$

$$\tau_{max} = \frac{T_{max}r_{max}}{J} = \frac{T_B r_{out}}{J} \Rightarrow$$

$$\tau_{max} = \frac{166.49\ (0.03)}{\frac{\pi}{2}(0.03)^4} \Rightarrow$$

$$\boxed{\tau_{max} = 3.926\ MPa\ @\ outer\ raduis\ in\ shaft\ FB}$$

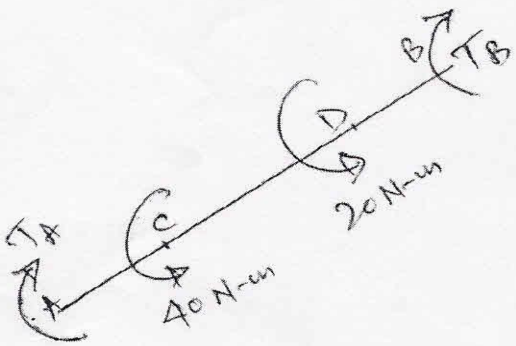
((Do not worry about the sign!!))

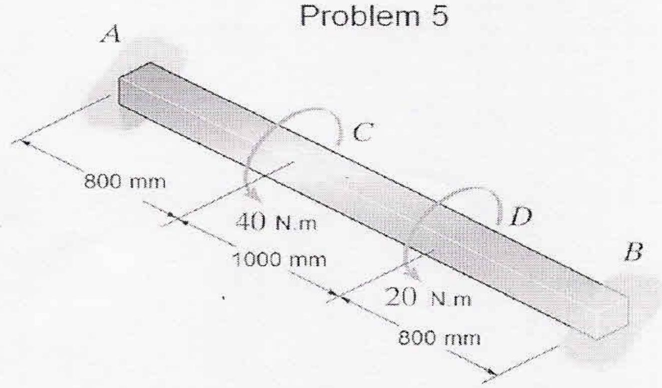
$$\varphi_F = \varphi_{F/B} = \varphi_{FB} \qquad as\ F\ is\ fixed\ \Rightarrow$$

$$\varphi_F = \left(\frac{TL}{JG}\right)_{FB} = \left[\frac{166.49\ (0.8)}{\frac{\pi}{2}(0.03)^4(100)(10)^9}\right] \Rightarrow$$

$\boldsymbol{\varphi_F = 1.0468\ (10)^{-3}\ rad = 0.05998\ ^o}$	CW

5. The given shaft has a solid square cross section ($a = 60$ mm.) and is fixed to rigid walls at A and B. Determine the magnitude and location of the largest shear stress in the whole shaft. Also, determine the angle of twist of point D with respect to C. Use $G = 100$ GPa.





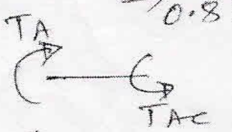
$\Sigma M = 0 \Rightarrow -T_A + 40 + 20 - T_B = 0$

$\Rightarrow T_A + T_B = 60 \quad --- \quad (1)$

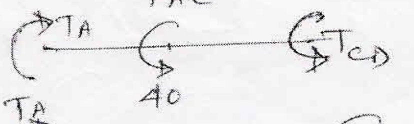
we have, $\phi_{A/B} = 0$

$$\Rightarrow \frac{7.1\, T_{AC} L_{AC}}{a^4 G} + \frac{7.1\, T_{CD} L_{CD}}{a^4 G} + \frac{7.1\, T_{DB} \cdot L_{DB}}{a^4 G} = 0$$

$\Rightarrow 0.8 T_{AC} + T_{CD} + 0.8 T_{DB} = 0 \quad --- \quad (2)$

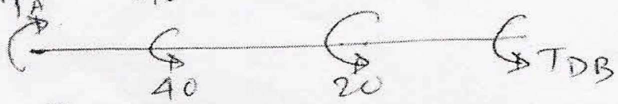


$\Sigma M = 0 \Rightarrow -T_A + T_{AC} = 0 \Rightarrow \underline{T_{AC} = T_A}$



$\Sigma M = 0 \Rightarrow -T_A + 40 + T_{CD} = 0$

$\Rightarrow \underline{T_{CD} = T_A - 40}$

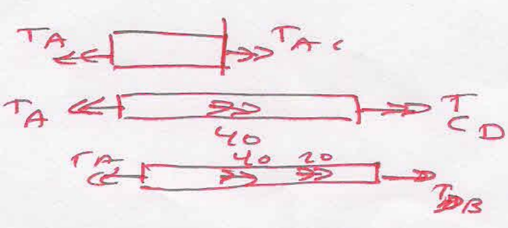


$\Sigma M = 0 \Rightarrow -T_A + 40 + 20 + T_{DB} = 0 \Rightarrow \underline{T_{DB} = T_A - 60}$

Substituting T_{AC}, T_{CD}, and T_{DB} in Eq. 2

we get, $T_A = 33.85$ N-m

From Eq.(1), $T_B = 26.15$ N-m



$T_{AC} - T_A = 0$

$\underline{T_{CD} = T_A}$

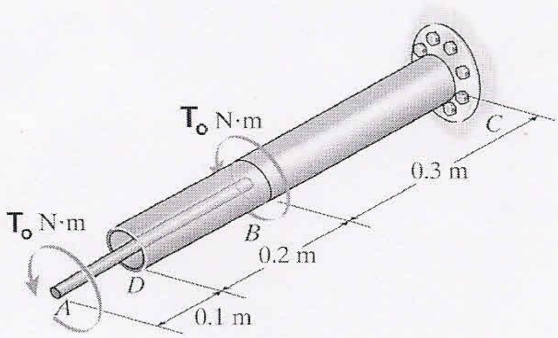
$-T_A + 40 + T_{CD} = 0, \quad T_{CD} = (T_{AC} - 40)$

$-T_A + 40 + 20 + T_{DB} = 0, \quad T_{DB} = T_A - 60$

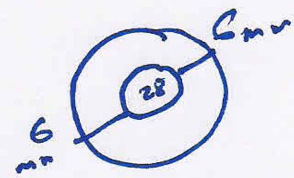
Problem # 2:-

Given Data:

- The shown figure.
- Solid rod AB (d=12 mm).
- Rigid disk at B.
- Tube DC outer diameter is 40 mm and a thickness of 6 mm.
- The max allowable stress is 50 MPa.



40

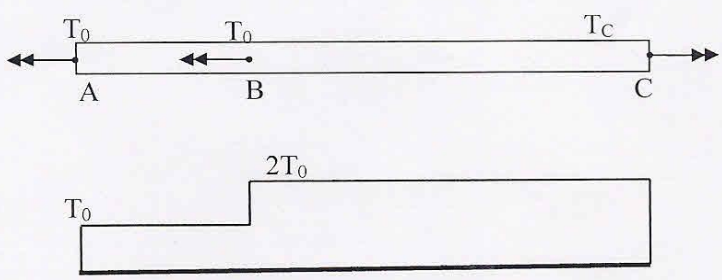


Required:

❖ **The largest value of torque T_0 that can be safely applied.**

Solution:

As it is shown in the following free body diagram and torque diagram,



Free body diagram and torque diagram

$$J_{BC} = \frac{\pi}{2} * (0.02^4 - 0.014^4) = 60.792 * 10^{-9}\ \pi\ m^4$$

$$J_{DA} = \frac{\pi}{2} * (0.006^4) = 0.648 * 10^{-9}\ \pi\ m^4$$

From Equilibrium, $T_{BC} = 2T_0$; *and* $T_{AB} = T_0$

$$\tau_{BC} = \frac{2T_0 * 0.02}{60.792 * 10^{-9}\ \pi} = 50 * 10^6\ \frac{N}{m^2} \quad \Rightarrow \quad (T_0)_{BC} = 238.730\ N.m$$

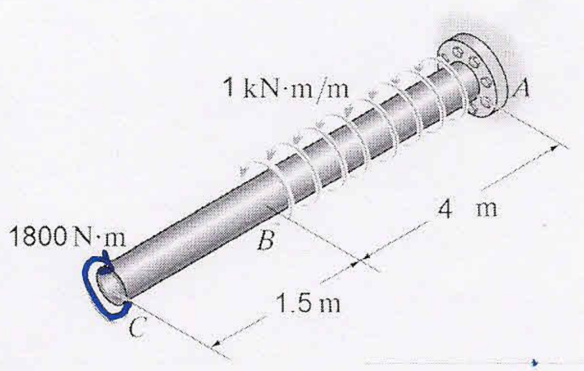
$$\tau_{AB} = \frac{T_0 * 0.006}{0.648 * 10^{-9}\ \pi} = 50 * 10^6\ \frac{N}{m^2} \quad \Rightarrow \quad (T_0)_{AB} = 16.965\ N.m$$

$\therefore$ *The max Torque T_0 that can be safely applied is* $16.965\ N.m$ *Ans*

Problem # 3:-

Given Data:

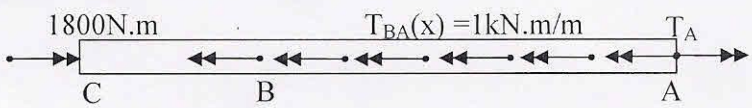
- The shown figure (hollow circular shaft).
- Outer radius = 50 mm ,
- Inner radius = 30 mm



Required:

- **The absolute maximum shear stresses in BC**
- **The absolute maximum shear stresses in BA.**
- **A diagram sketch for the value of the internal torque T_R along axis CA**

Solution:



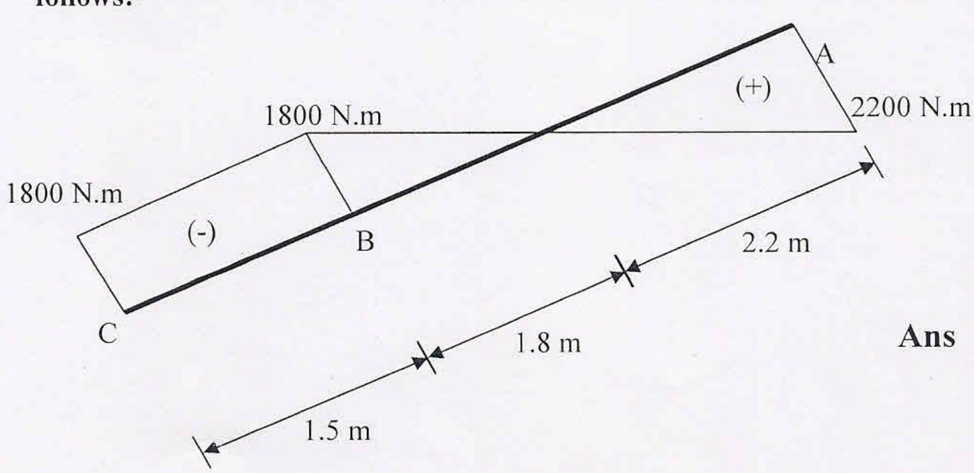
Free body diagram

$$T_A + 1800 - 1 * 10^3 * 4 = 0; \Rightarrow T_A = 2200 \text{ N.m}$$

$$\therefore \max T_{BA} = 2200 \text{ N.m} \quad ; \quad \text{and} \quad \max T_{BC} = 1800 \text{ N.m}$$

$$J_{BC} = J_{BA} = \frac{\pi}{2} * (0.05^4 - 0.03^4) = 2.72 * 10^{-6}\ \pi\ m^4$$

- $(\tau_{max})_{BC} = \frac{1800 * 0.05}{2.72 * 10^{-6}\ \pi} = 10.532$ MPa. Ans
- $(\tau_{max})_{BA} = \frac{2200 * 0.05}{2.72 * 10^{-6}\ \pi} = 12.873$ MPa. Ans
- **The diagram sketch for the value of the internal torque T_R along axis CA is as follows:-**



Ans

Option 1:

Two solid steel circular shafts are connected by the gear shown.

a) If the allowable shear stress in the shafts is 50 MPa, determine the largest value of T_A that can be safely applied.

b) Using $T_A = 100$ N.m, determine the angle of twist of point A ($G = 80$ GPa)

a) shaft AB $\quad T_R = +T_A$

$$\therefore +50 = \frac{+T_A(21)}{\pi(42)^4/32} \qquad T_A = 0.727\times10^6 \text{ N.mm}$$

shaft CD $\quad T_R = 3T_A$

$$50 = \frac{3T_A(30)}{\pi(60)^4/32} \qquad T_A = 0.707\times10^6 \text{ N.mm}$$

$$\therefore T_{max} = 0.707\times10^6 \text{ N.m}$$

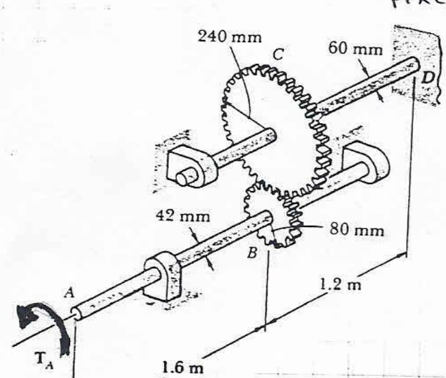
b)

$$\phi_A = \frac{T_A(l)}{GJ} + (3)\frac{3T_A(l)}{GJ}$$

use $T_A = 100\times10^3$ N.mm

$\phi_A =$ ______

fixed



$r_c \phi_c = r_B \phi_B$

$\phi_B = \frac{r_c}{r_B}\phi_c$

ϕ_B

Problem # 1

Shaft ABC has a solid circular cross section with diameter d = 4 cm. and is subjected to a torque **T** applied at B. The shaft is held fixed at end A while end C allows a rotation angle ϕ of *not* more than 0.02 radians applied at B.

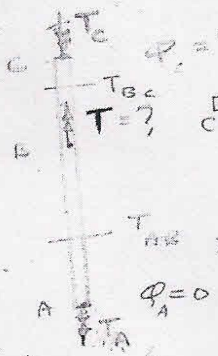
a) Determine the maximum allowable *torque* **T** *that may* safely be applied.
b) Determine the relative angle of twist $\phi_{A/B}$ corresponding to T.

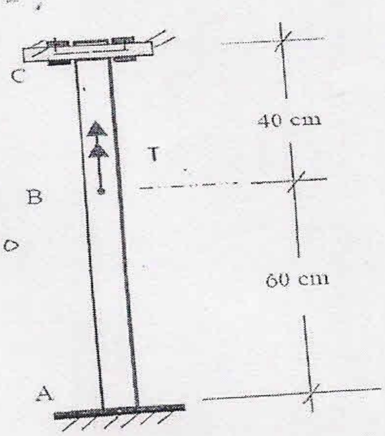
Given: Allowable shear stress τ = 50 MPa; G = 70 GPa.

$J = \frac{\pi}{2} r^4 = 2.5133\times10^{-7}\ m^4$

$GJ = 1.7593\times10^{4}\ N\cdot m^2$

FBD





Since the allowable T must be such that

$\varphi_C > 0.02$ rad and

$\tau_{max} > \tau_{all} = 50\ MPa$,

for $\varphi_C = \varphi_B = 0.02$ rad

$T_C = 0 \Rightarrow 0.02 = \frac{T_{AB}(0.6)}{GJ}$

$\therefore T_{AB} = T = \frac{0.02\ GJ}{0.60}$

$= \frac{0.02 \times \pi/2\ (0.02)^4 \times 70\times10^6}{0.60}$

$= 0.5864$ kN.m

$\tau_{max} = T_{AB}\ r/J = 0.5864 \times 0.02/J$

$= 4.67\times10^4$ kPa $= 46.7$ MPa $< \tau_{all}$ ②

$\therefore \tau_{all}$ controls $\Rightarrow \therefore T_C \neq 0 \Rightarrow T - T_A - T_C = 0 \quad \dots (1)$ ③

$0.02 = \frac{T_{AB}(0.6)}{GJ} + \frac{T_{BC}(0.4)}{GJ} \quad \dots (2)$ ③

$0.02\ GJ = 0.6\,T_A + (T_A - T)\,0.4$

$= T_A - 0.4\,T \Rightarrow T = \frac{T_A - 0.02\,GJ}{0.40} \quad \dots (3)$ ③

and in terms of $T_C \Rightarrow T = \frac{T_C + 0.02\,GJ}{0.6} \quad \dots (4)$

$\because T_{all} = \tau_{all}\ J/r = 50 \times J/0.02 = 628.3$ N.m

Using Eqns. (3) and (4) $\Rightarrow T = \min[691.1\ ;\ 1633.6]$ N.m ③

$\therefore$ max. safe value of torque T = 691.1 N.m

b) $\varphi_B = \varphi_A + \varphi_{B/A} = 0 - \varphi_{A/B} \Rightarrow \varphi_{A/B} = -\varphi_B = -\frac{T_{AB}(0.6)}{GJ}$

$\therefore \varphi_{A/B} = -\frac{T_A(0.6)}{GJ} = -\frac{(0.02\,GJ + 0.4T)\,0.6}{GJ}$ ③

$= -(0.012 + 0.24T/GJ) \Rightarrow \varphi_{A/B} = -0.02143$ rad

Problem # 6:

Given:

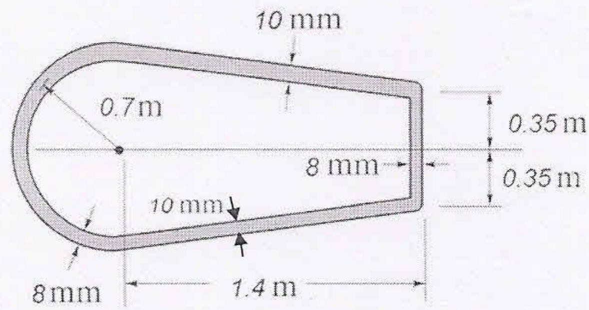
The cross-section of a shaft shown

$T = 300\ N.m; \quad G = 100\ GPa$

Required:

Value and location of $\boldsymbol{\tau_{max}}$

$d\varphi/_{dz}$

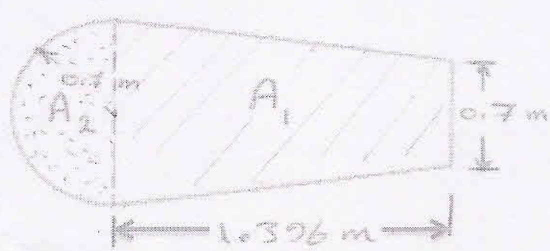


Solution:

The section is "thin-walled closed". $\Rightarrow$

$$\tau = \frac{T}{2tA_m}$$

A_m *is the area contained within the* *mean perimeter (not material area)* *as shown in the below figure.*



$$A_m = A_1 + A_2$$

$$= \frac{0.7 + 2\,(0.7)}{2}(1.396) + \frac{\pi}{2}\,(0.7)^2$$

$$= 2.23549\ m^2$$

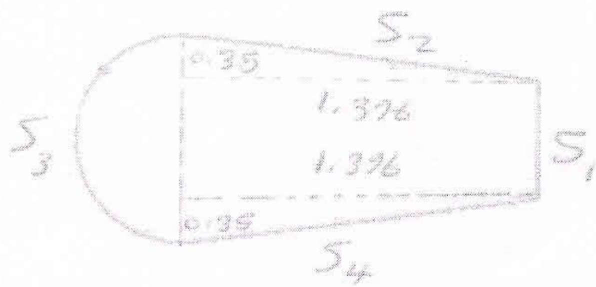
Since t is in the denominator of the $\boldsymbol{\tau}$ *formula,* $\boldsymbol{\tau_{max}}$ *will be at* $t_{min} = 8\ mm$ $\Rightarrow$

$$\tau_{max} = \frac{300\,(10)^3}{2\,(0.008)(2.23549)} \Rightarrow$$

$$\boxed{\boldsymbol{\tau_{max} = 8.387\ MPa\ @\ the\ 8 - mm\ thicknes}}$$

$$\frac{d\varphi}{dz} = \frac{T}{4A_m^2 G}\oint \frac{ds}{t}$$

$$= \frac{T}{4A_m^2 G}\sum_{i=1}^{4}\frac{s_i}{t_i}$$ *in our case (Why?!)*



$S_1 = 0.7\,m \quad ; \quad t_1 = 0.008\,m$

$S_2 = \sqrt{(1.396)^2 + (0.35)^2}$

$= 1.43921\,m = S_4$

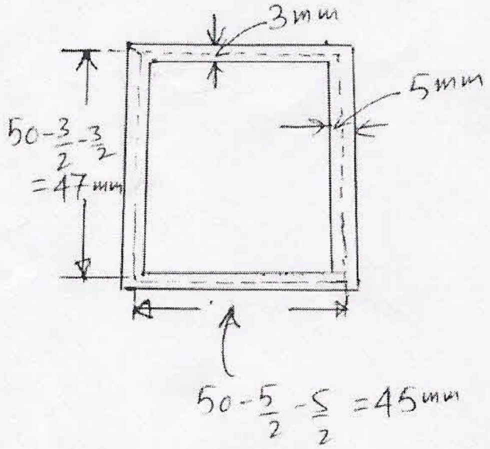
$t_2 = t_4 = 0.01\,m$

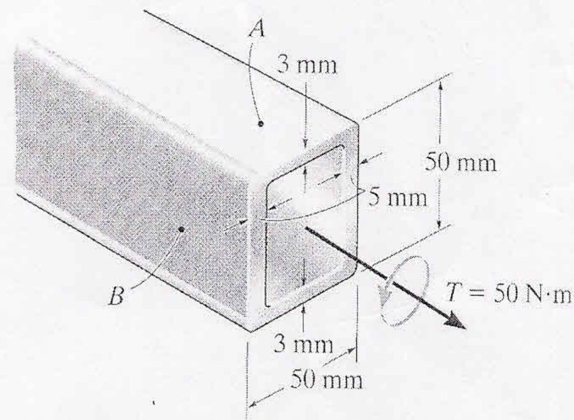
$S_3 = \pi r = \pi(0.7) = 2.19911\,m \quad ; \quad t_3 = 0.008\,m$

$$\frac{d\varphi}{dz} = \frac{300\,(10)^3}{4\,(2.23549)^2(100)(10)^9}\left(\frac{0.7}{0.008} + 2\frac{1.43921}{0.01} + \frac{2.19911}{0.008}\right)$$

$$\boxed{\frac{d\varphi}{dz} = 9.759\,(10)^{-5}\ {}^{rad}/_{m} = 5.591\,(10)^{-3}\ {}^{o}/_{m}}$$

3. Use the figure for problem 5-102 in the textbook. Assuming G = 100 GPa, and the shaft length is 1 meter, determine the angle of twist of one end relative to the other, and the maximum shear stress in the whole shaft (indicate its location).





Problem 3

$$A_m = 47 \times 45 = 2115 \text{ mm}^2$$

$$\oint \frac{ds}{t} = \frac{45}{3} + \frac{45}{3} + \frac{47}{5} + \frac{47}{5} = 48.8 \text{ mm/mm}$$

$$\phi = \frac{TL}{4A_m^2 G} \oint \frac{ds}{t}$$

$$= \frac{50 \times 10^3 \times 1 \times 10^3}{4 \times (2115)^2 \times 100 \times 10^3} \times 48.8 = 1.3636 \times 10^{-3} \text{ rad}$$

$$= 0.078 \text{ deg}$$ Ans

$$(\tau_{avg})_{max} = \frac{T}{2A_m t_{min}} = \frac{50 \times 10^3}{2 \times 2115 \times 3}$$

$$= 3.94 \text{ MPa (at A)}$$ Answer

Problem # 1

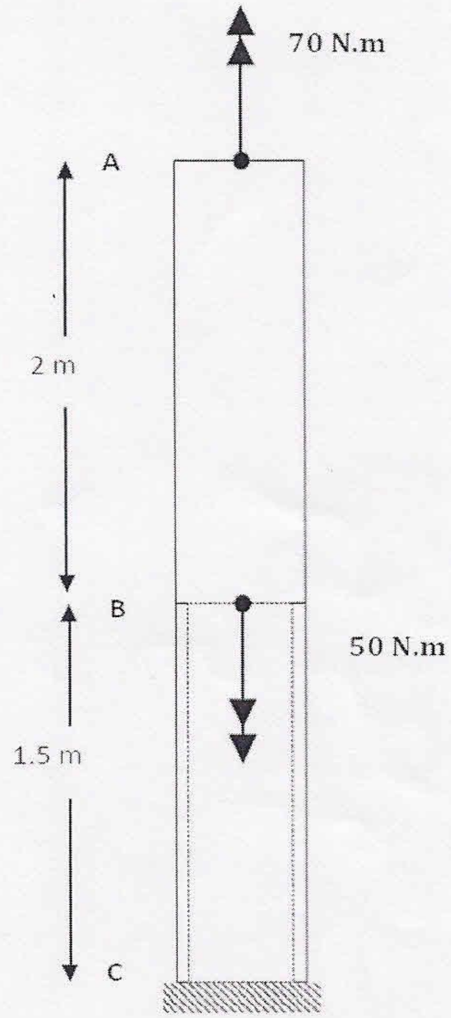
Shaft ABC is fixed at C and has a solid section between A and B and a thin hollow section between B and C.

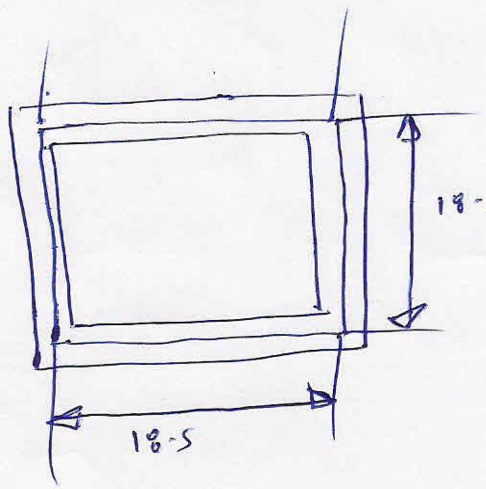
Calculate

a) The maximum shear stress in the shaft.

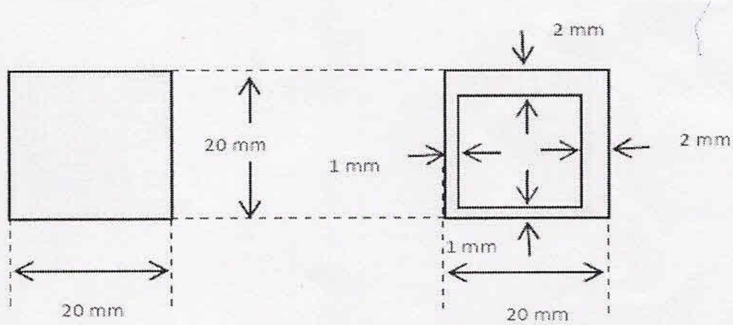
b) The angle of twist at the free end.

G= 80 GPa





$A_m = 18.5 \times 18.5$



Cross-section between A and B　　Cross-section between B and C

Problem # 1

Shaft ABC is fixed at C and has a solid section between A and B and a thin hollow section between B and C.
Calculate

a) The maximum shear stress in the shaft.
b) The angle of twist at the free end.

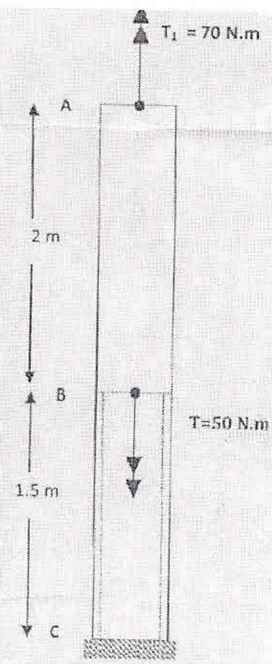
G= 80 GPa

$T_{AB} = 70$ N.m (1)

$T_{BC} = 80$ N.m (2)

max (1) $\Rightarrow \tau_{max\,AB} = \frac{4.81\, T_{AB}}{a^3}$ (1) $= 42$ MPa (2)

$\tau_{max\,BC} = \frac{T_{BC}}{2 A_m t}$ (1) $= 29$ MPa (2)



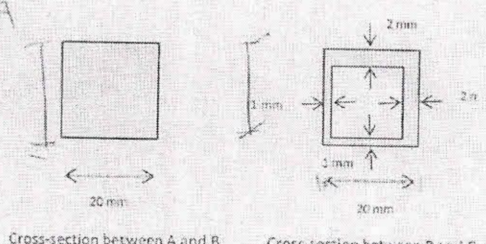
$\varphi_A = \varphi_{AB} + \varphi_{BC}$

$= \frac{7.1\, T_{AB} L_{AB}}{a^4 G}$ (2) $+ \frac{T_{BC} L_{BC}}{4 A_m^2 G} \oint \frac{ds}{t}$ (2)

0.078 (1) $+ .044$ (1)

$= 0.122$

close to 7°



$342.25 = A_m = 18.5^2$ (2)

$55.5 = \oint \frac{ds}{t} = 2\left(\frac{18.5}{1} + \frac{18.5}{2}\right)$ (2)

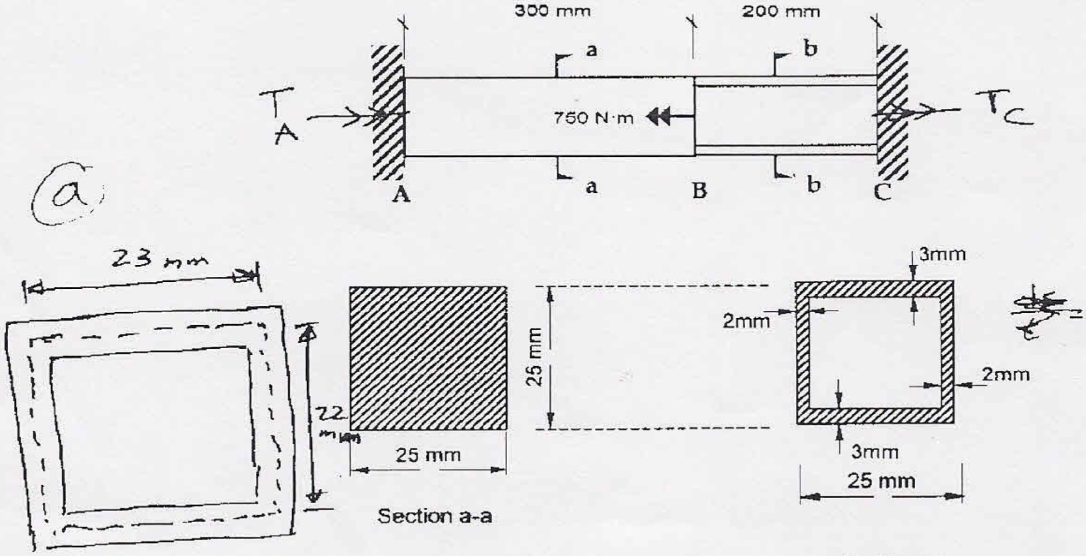
Problem # 2

The shaft is made from two segments: AB is a solid section, and BC is a thin tube.

a) Determine the maximum shear stress in the whole shaft and indicate its location.

b) Determine the angle of twist of B.

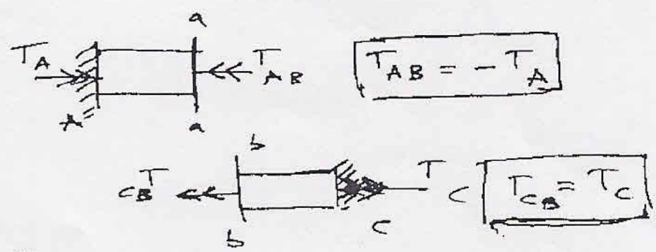
$G_{steel} = 75\ GPa$



Section a-a

Section b-b

This problem is statically indeterminate.



$\boxed{T_{AB} = -T_A}$

$\boxed{T_{CB} = T_C}$

From Equilibrium

$$\boxed{T_A + T_C - 750 = 0} \quad (1)$$

$A_m = 22 \times 23 = 506\ mm^2$
$= 5.06 \times 10^{-4}\ m^2$

From Compatibility

$$\phi_{A/B} + \phi_{B/C} = 0$$

$$\frac{7.1\,(-T_A)\,(0.3)}{(0.025)^4(75 \times 10^9)} + \frac{T_C\,(0.2)}{4\,(5.06 \times 10^{-4})^2(75 \times 10^9)}\left[2\left(\frac{22}{2}\right) + 2\left(\frac{23}{2}\right)\right] = 0$$

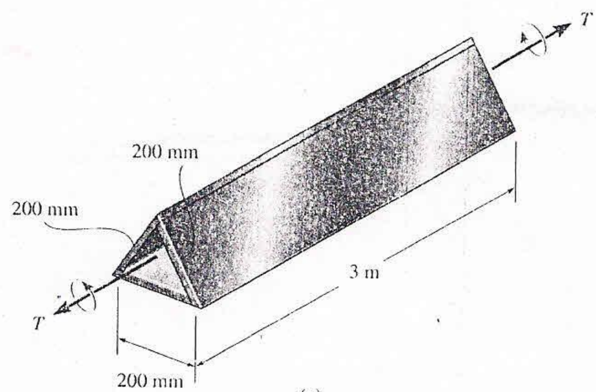
$$-7.2704 \times 10^{-5}\, T_A + 9.7199 \times 10^{-5}\, T_C = 0 \quad (2)$$

solving (1) & (2)

$$\boxed{\begin{aligned} T_C &= 320.94\ N.m \\ T_A &= 429.06\ N.m \end{aligned}}$$

Example

A thin tube is made from three 5-mm-thick A-36 steel plates such that it has a cross section that is triangular as shown in Fig. 1a. Determine the maximum torque T to which it can be subjected, if the allowable shear stress is $\tau_{allow} = 90$ MPa and the tube is restricted to twist no more than $\phi = 2(10^{-3})$ rad.

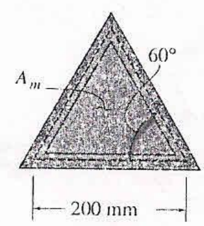


(a)

SOLUTION

The area A_m is shown shaded in Fig. 1b. It is

$$A_m = \frac{1}{2}(200\text{ mm})(200\text{ mm}\sin 60^\circ)$$
$$= 17.32(10^3)\text{ mm}^2(10^{-6}\text{ m}^2/\text{mm}^2) = 17.32(10^{-3})\text{ m}^2$$



(b)

(Fig 1)

The greatest average shear stress occurs at points where the tube's thickness is smallest, which is along the sides and not at the corners. Applying Eq. 5–18, with $t = 0.005$ m, yields

$$\tau_{avg} = \frac{T}{2tA_m}; \qquad 90(10^6)\text{ N/m}^2 = \frac{T}{2(0.005\text{ m})(17.32(10^{-3})\text{ m}^2)}$$
$$T = 15.6\text{ kN}\cdot\text{m}$$

Also, from Eq. 5–20, we have

$$\phi = \frac{TL}{4A_m^2 G}\oint\frac{ds}{t}$$

$$0.002\text{ rad} = \frac{T(3\text{ m})}{4(17.32(10^{-3})\text{ m})^2[75(10^9)\text{ N/m}^2]}\oint\frac{ds}{(0.005\text{ m})}$$

$$300.0 = T\oint ds$$

The integral represents the sum of the dimensions along the three sides of the center-line boundary. Thus,

$$300.0 = T[3(0.20\text{ m})]$$
$$T = 500\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

By comparison, the application of torque is restricted due to the angle of twist.