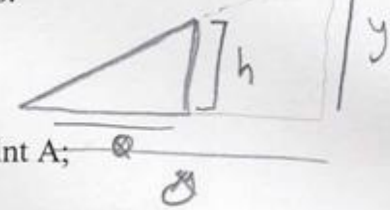
shown in Fig. P-5. Using **discontinuity (singularity)** functions:



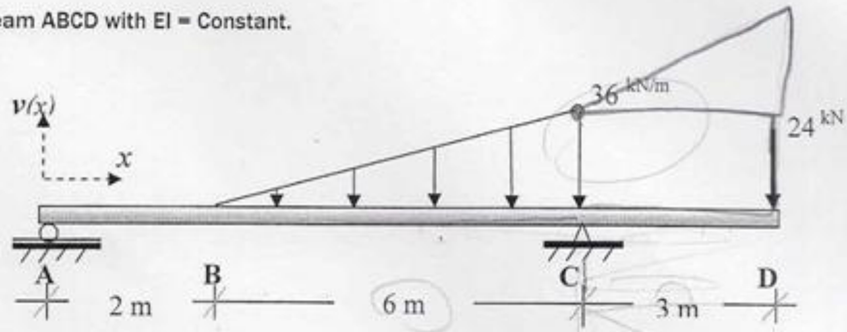
a) determine equation of the elastic curve $v(x)$;

b) determine magnitude and direction of the *slope* at point A;

c) determine magnitude and direction of the *deflection* at point B;

$$\frac{y}{x} = \frac{h}{x}$$

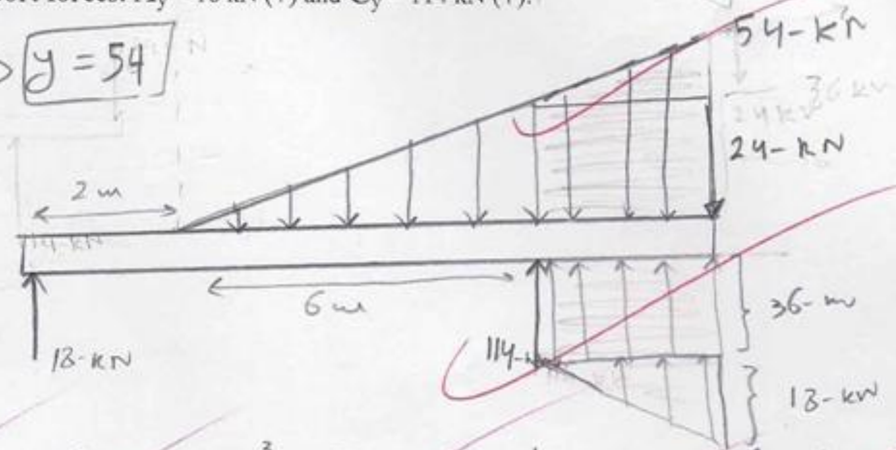
Fig. P-5: Beam ABCD with EI = Constant.



Support forces: $\mathbf{A}_y = 18$ kN (↑) and $\mathbf{C}_y = 114$ kN (↑).

1] $\dfrac{36}{6} = \dfrac{y}{9} \Rightarrow \boxed{y = 54}$

$$\boxed{m = \frac{54}{9} = 6}$$



$$EI\frac{\partial^2 v}{\partial x^2} = 18\langle x-0\rangle^1 - \frac{6}{6}\langle x-2\rangle^3 + 114\langle x-8\rangle^1 + \frac{36}{2}\langle x-8\rangle^2 + \frac{6}{6}\langle x-8\rangle^3$$

$$EI\frac{\partial^2 v}{\partial x^2} = 18x + 114\langle x-8\rangle^1 + 18\langle x-8\rangle^2 - \langle x-2\rangle^3 + \langle x-8\rangle^3$$

$\boxed{C_1 = ?}$

$$EI\frac{\partial v}{\partial x} = \frac{18}{2}x^2 + \frac{114}{2}\langle x-8\rangle^2 + \frac{18}{3}\langle x-8\rangle^3 - \frac{\langle x-2\rangle^4}{4} + \frac{\langle x-8\rangle^4}{4} + C_1$$

① $$EI\frac{\partial v}{\partial x} = 9x^2 + 57\langle x-8\rangle^2 + 6\langle x-8\rangle^3 - \frac{\langle x-2\rangle^4}{4} + \frac{\langle x-8\rangle^4}{4} + C_1$$

② $$EI\,V = \frac{9}{3}x^3 + \frac{57}{3}\langle x-8\rangle^3 + \frac{6}{4}\langle x-8\rangle^4 - \frac{\langle x-2\rangle^5}{20} + \frac{\langle x-8\rangle^5}{20} + C_1x + C_2$$

$$EI\,v = 3x^3 + 19\langle x-8\rangle^3 + 1.5\langle x-8\rangle^4 - 0.05\langle x-2\rangle^5 + 0.05\langle x-8\rangle^5$$

at $x=0$ — $v=0$

sub in (2)

$0 = 0 + 0 + 0 - 0 + 0 + c \longrightarrow \boxed{C_2 = 0}$

so

$$\boxed{EIv = 3x^3 + 19\langle x-8\rangle^3 + 1.5\langle x-8\rangle^4 - 0.05\langle x-2\rangle^5 + 0.05\langle x-8\rangle^5 + C_1 x}$$

at $\underline{x=8}$ ~ $\underline{v=0}$

$0 = 3(8)^3 + 0 + 0 - 0.05(6)^5 + 0 + C_1(8)$

$$\boxed{C_1 = -143.4}$$

1]

$$V = \frac{1}{EI}\left(3x^3 + 19\langle x-8\rangle^3 + 1.5\langle x-8\rangle^4 - 0.05\langle x-2\rangle^5 + 0.05\langle x-8\rangle^5 - 143.4x\right) \quad \boxed{1}$$

at A $\leadsto \boxed{x=0}$

2] $EI\frac{\partial v}{\partial x} = 9x^2 + 57\langle x-8\rangle^2 + 6\langle x-8\rangle^3 - \frac{\langle x-2\rangle^4}{4} + \frac{\langle x-8\rangle^4}{4} - 143.4$

at $\boxed{x=0}$

$$\frac{\partial V}{\partial x} = -\frac{143.4}{EI}$$ — clockwise $\boxed{2}$

3] at (B) $\boxed{x=2}$

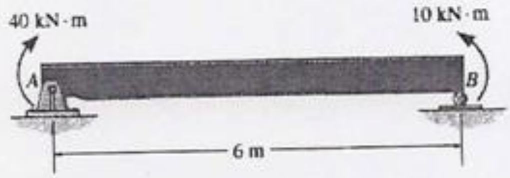
$v(2) = \frac{1}{EI}\left(3(2)^3 + 19(0) + 1.5(0) - 0.05(0) + 0 - 143.4(2)\right)$

$$v_{(2)} = -\frac{262.8}{EI}$$ — downward $\boxed{3}$

Solution of HW # 15

Problem # 1:

Given:
The beam shown
$E = 200$ GPa
$I = 39.9 \times 10^{-6}\ m^4$

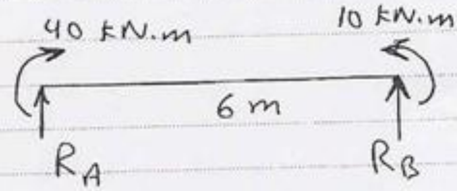


Required:
Maximum deflection

Solution:

We will start with the moment equation.
We may start with the load equation.

We need to find R_A. (Why ?!)

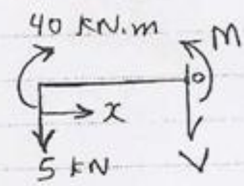


In FBD ①,

$\circlearrowleft + \ \Sigma M_B = 0 \Rightarrow -40 + 10 - 6R_A = 0$ FBD ①

$\Rightarrow R_A = -5\ kN = 5\ kN \downarrow$

To get the moment equation, we draw FBD ② by taking a section "between A and B". Note that there is only one moment equation. (How ?!)



FBD ②

$\circlearrowleft + \ \Sigma M_o = 0 \Rightarrow M = 40 - 5x = EI \dfrac{d^2v}{dx^2}$ (kN.m)

$\text{Slope} = \theta = \dfrac{dv}{dx} = \int \dfrac{M}{EI}\,dx \Rightarrow$

$EI \dfrac{dv}{dx} = \int (40 - 5x)\,dx = 40x - \frac{5}{2}x^2 + C_1$

$EIv = \int EI \frac{dv}{dx}\,dx = \int EI\,(40x - \frac{5}{2}x^2 + C_1)\,dx \Rightarrow$

$EIv = 20x^2 - \frac{5}{6}x^3 + C_1 x + C_2$

Boundary Conditions (B.C's):

We have 2 B.C's. and 2 unknows (C_1 and C_2). Thus, we can find C_1 & C_2.

$v(0) = 0$ [That is the deflection is zero at $x = 0$ (A).] $\Rightarrow$

$0 = 0 + 0 + 0 + C_2 \Rightarrow C_2 = 0$

$v(6) = 0 \Rightarrow 0 = 20(6)^2 - \frac{5}{6}(6)^3 + C_1(6) \Rightarrow$

$C_1 = -90$

"with appropriate units"!

Thus,

$EIv = 20x^2 - \frac{5}{6}x^3 - 90x$

To get v_{max}, we set $\frac{dv}{dx} = 0 \Rightarrow$ get x. (Why?)

$\Rightarrow \frac{dv}{dx}EI = 40x - \frac{5}{2}x^2 - 90 \equiv 0 \Rightarrow$

$x^2 - 16x + 36 = 0 \Rightarrow x = 13.292$ or 2.7085 m

X (outside the range)

$\Rightarrow v_{max} = \left[20(2.7085)^2 - \frac{5}{6}(2.7085)^3 - 90(2.7085) / 200(10)^9(39.9)(10)^{-6}\right](10)^3$

kN → N

$\Rightarrow v = -0.014236$ m

$\Rightarrow \boxed{v = 14.24 \text{ mm} \downarrow @ x = 2.709 \text{ m}}$

"Very small as in most applications"

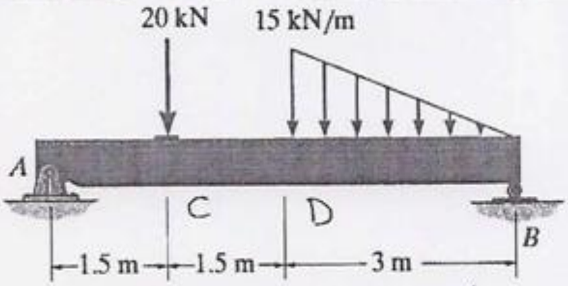
Problem # 3:

Given:

The beam shown

$E = 200$ GPa

$I = 65(10)^6$ mm^4

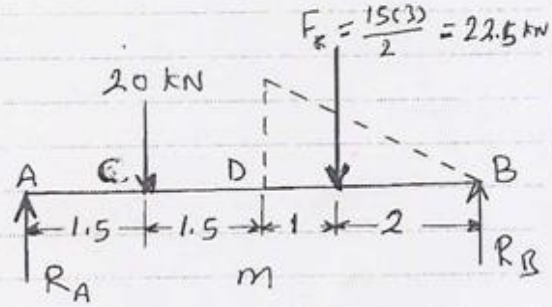


Required:

Maximum deflection

Solution:

Starting with the moment equation, we need R_A in FBD ①. ⇒



FBD ①

$\circlearrowleft \Sigma M_B = 0 \Rightarrow$

$-6R_A + 20(4.5) + 22.5(2) = 0 \Rightarrow$

$R_A = 22.5$ kN

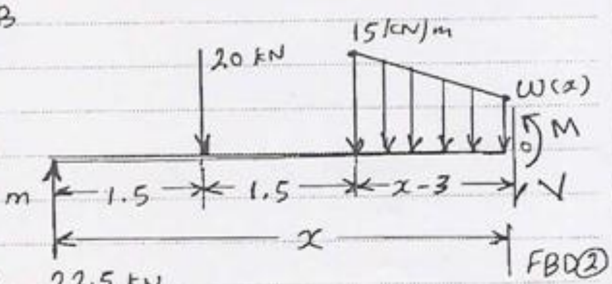
To get the moment equation, we make a section in the last segment ($\Downarrow$ DB) and draw FBD ② for the left part.

$\frac{w(x)}{15} = \frac{6-x}{3} \Rightarrow$

$w(x) = 5(6-x)$

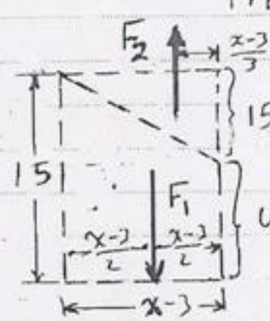
$F_1 = 15(x-3)$ ↓

$F_2 = (15-w)\left(\frac{x-3}{2}\right)$ ↑



FBD②

The locations of F_1 and F_2 are as shown.



$F_2 = (15-w)\left(\frac{x-3}{2}\right)$

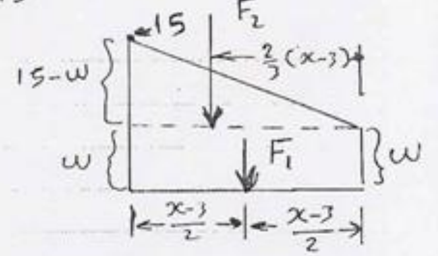
$= [15 - 5(6-x)](x-3)/2$

$= 5[(3-6+x)](x-3)/2$

$= \frac{5}{2}(x-3)^2$

Solution of HW # 15

Try taking F_1 and F_2 as shown! $\Longrightarrow$



What did you find?!

$\overset{\curvearrowleft}{+}\ \Sigma M_o = 0 \Rightarrow$

$$M - 22.5x + 20(x-1.5) + 15(x-3)\left(\frac{x-3}{2}\right) - \frac{5}{2}(x-3)^2\left(\frac{x-3}{3}\right) = 0$$

$F_1 = w(x-3) \downarrow$

$F_2 = (15-w)(x-3)/2 \downarrow$

Moving the terms to the other side and using the singularity function form, we get:

$$M = 22.5\langle x-0\rangle^1 - 20\langle x-1.5\rangle^1 - \frac{15}{2}\langle x-3\rangle^2 + \frac{5}{6}\langle x-3\rangle^3$$

$$EI\frac{d^2v}{dx^2} = M \Rightarrow EI\frac{dv}{dx} = \int M\,dx \Rightarrow$$

$$EI\frac{dv}{dx} = 11.25\langle x-0\rangle^2 - 10\langle x-1.5\rangle^2 - \frac{15}{6}\langle x-3\rangle^3 + \frac{5}{24}\langle x-3\rangle^4 + C_1$$

$$EIv = 3.75\langle x-0\rangle^3 - \frac{10}{3}\langle x-1.5\rangle^3 - \frac{15}{24}\langle x-3\rangle^4 + \frac{1}{24}\langle x-3\rangle^5 + C_1x + C_2$$

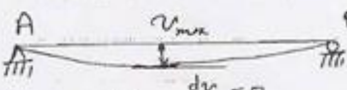
<u>B.C.s.</u>: $v(0) = 0 \Rightarrow 0 - 0 - 0 + 0 + 0 + C_2 = 0 \Rightarrow C_2 = 0$

Note that the value within $\langle x-a\rangle$ can never be negative. If so, it is ZERO! [why and how?!]

$$v(6) = v_B = 0 \Rightarrow 3.75(6)^3 - \frac{10}{3}(4.5)^3 - \frac{15}{24}(3)^4 + \frac{1}{24}(3)^5 + 6C_1 = 0$$

$\Rightarrow C_1 = -77.625$ [units in kN, m]

v_{max} is @ $\frac{dv}{dx} = 0$



[OR at the free end if there is:]

$\Rightarrow$ Set $\frac{dv}{dx} = 0 \Rightarrow 11.25x^2 - 10\langle x-1.5\rangle^2 - \frac{15}{6}\langle x-3\rangle^3 + \frac{5}{24}\langle x-3\rangle^4 - 77.625 = 0$

We can either find the four roots of this 4th degree equation, or <u>assume</u> the root within our range $(0 \le x \le 6)$

is between 0 and 3 m (as there is only one possible root within this range, as seen in the figure above). In this case, we have to check our assumption ($x \overset{?}{\le} 3$). This will simplify our solution as the 3rd and 4th order terms are dropped. (How?!) ⇒

$$11.25\,x^2 - 10(x-1.5)^2 - 77.625 = 0 \Rightarrow$$

$$x = 2.96997 \quad \text{or} \quad -26.96997$$

«ok» «not possible»

⇒ $x_m = 2.970$ m «within our range 0–3» ⇒ ok

Thus v_{max} @ $x_m = 2.970$ m ⇒

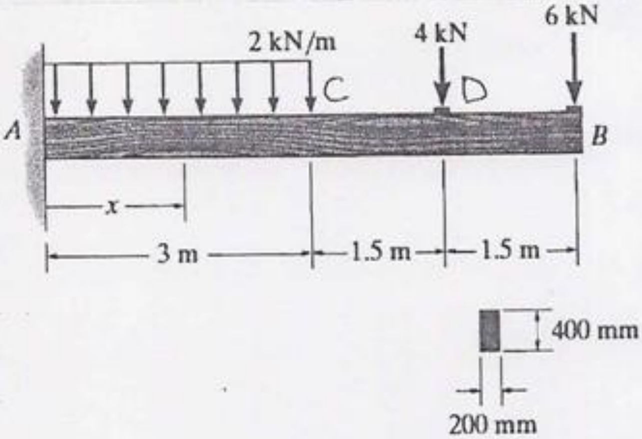
$$v_{max} = \left[3.75\,x_m^3 - \frac{10}{3}(x_m - 1.5)^3 - 77.625\,x_m\right](10)^3 \Big/ 200(10)^9\,65(10)^{-6}$$

⇒ $\boxed{v_{max} = -0.010992 \text{ m} = 10.99 \text{ mm} \downarrow}$

Again, very small compared with $L = 6$ m

Problem # 4:

Given:
The beam shown
$E = 12$ GPa



Required:
The eq. of the elastic curve (deflection)
Deflection & slope @ B

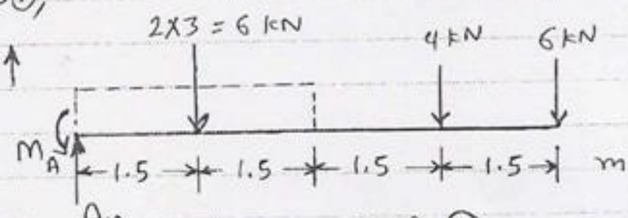
Solution:
First, we need to find the reactions. From FBD①,

$+\uparrow \Sigma F_y = 0 \Rightarrow A_y = 16$ kN $\uparrow$

$\circlearrowleft + \Sigma M_A = 0 \Rightarrow$

$M_A - 6(1.5) - 4(4.5) - 6(6) = 0$

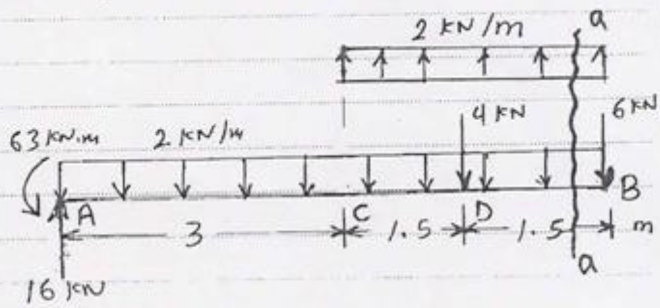
$\Rightarrow M_A = 63$ kN.m $\circlearrowleft$



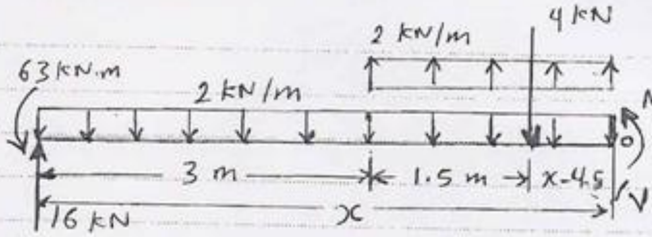
FBD ①

Using the singularity function, you need to remember that once the distributed L. starts (from anywhere/x), it has to go all the way up to the end. (Why ?!) Thus, the 2 kN/m - load has to go from A to B. Therefore, we have to add an upward load equals to 2 kN/m from C to B, thus making the total loads equal to the original loading. This is shown in the figure below.

Now, we make a section (a-a) in the last segment (DB), and draw a FBD by taking



the lef part, as shown FBD②. Note that any concentrated load/moment (whether applied or reaction) does



FBD②

not appear! Does it mean "it does not affect the beam"?! NO!! HOW??!!

$\overset{+}{\circlearrowleft}\ \Sigma M_o = 0 \Rightarrow$ In a singularity function form,

$$M = 16\langle x-0\rangle^1 - 63\langle x-0\rangle^0 - \frac{2}{2}\langle x-0\rangle^2 + \frac{2}{2}\langle x-3\rangle^2 - 4\langle x-4.5\rangle^1$$

$$EI\frac{dv}{dx} = 8\langle x-0\rangle^2 - 63\langle x-0\rangle^1 - \frac{1}{3}\langle x-0\rangle^3 + \frac{1}{3}\langle x-3\rangle^3 - 2\langle x-4.5\rangle^2 + C_1$$

$$EI\,v = \frac{8}{3}\langle x-0\rangle^3 - \frac{63}{2}\langle x-0\rangle^2 - \frac{1}{12}\langle x-0\rangle^4 + \frac{1}{12}\langle x-3\rangle^4 - \frac{2}{3}\langle x-4.5\rangle^3 + C_1x + C_2$$

B.C.s.: dv/dx and $v = 0$ @ $x = 0$ (A) $\Rightarrow$

$dv/dx\,(0) = 0 - 0 - 0 + 0 - 0 + C_1 = 0 \Rightarrow C_1 = 0$

$v(0) = 0 = 0 - 0 - 0 + 0 - 0 + 0 + C_2 \Rightarrow C_2 = 0$

Thus,

$$\boxed{v = \frac{1}{EI}\left[\frac{8}{3}\langle x-0\rangle^3 - \frac{63}{2}\langle x-0\rangle^2 - \frac{1}{12}\langle x-0\rangle^4 + \frac{1}{12}\langle x-3\rangle^4 - \frac{2}{3}\langle x-4.5\rangle^3\right.}$$

At B, $x = 6\,m$; $E = 12(10)^9\ N/m^2$

$I = bh^3/12 \Rightarrow I = (0.2)(0.4)^3/12 = \frac{3.2}{3}(10)^{-3}\ m^4$

slope $= \theta_B = dv/dx = [8(6)^2 - 63(6) - \frac{1}{3}(6)^3 + \frac{1}{3}(3)^3 - 2(1.5)^2](10)^3/EI$

$\Rightarrow \boxed{\theta_B = -0.01230\ rad = -0.7050°}$ "cw"

$v_B = [\frac{8}{3}(6)^3 - \frac{63}{2}(6)^2 - \frac{1}{12}(6)^4 + \frac{1}{12}(3)^4 - \frac{2}{3}(1.5)^3]/EI \Rightarrow \boxed{v_B = -0.05168\ m\downarrow}$

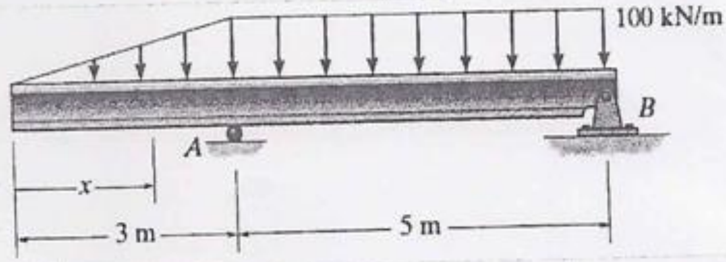
not to scale

Problem # 5:

Given:

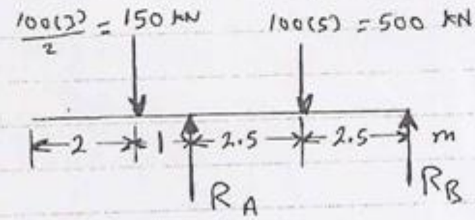
The beam shown

EI = Constant



Required:

Eq. of elastic curve (deflection)



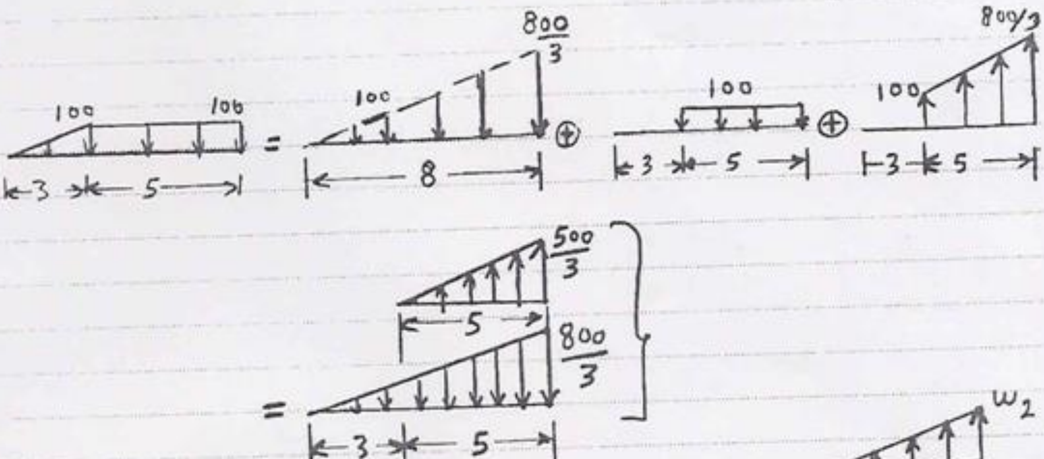
FBD ①

Solution:

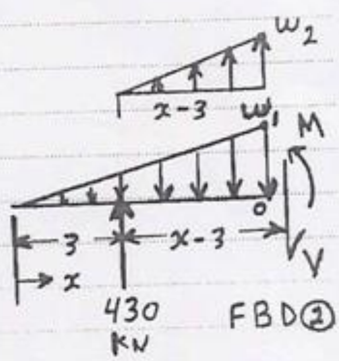
First, we need to determine R_A in FBD ①.

$\curvearrowleft + \; \Sigma M_B = 0 \Rightarrow 150(6) - 5R_A + 500(2.5) = 0 \Rightarrow R_A = 430$ kN $\uparrow$

Since the distributed load has to continue to the end once it starts, we need to make equivalent load as shown below.



Now, a section is made in the last segment (between A and B), and FBD ② is drawn.



Solution of HW # 15

$$\frac{w_1}{x} = \frac{800/3}{8} \Rightarrow w_1 = \frac{100}{3}x$$

$$\frac{w_2}{x-3} = \frac{500/3}{5} \Rightarrow w_2 = \frac{100}{3}(x-3)$$

$$+\circlearrowleft \Sigma M_o = 0 \Rightarrow -430(x-3) + \frac{100}{3}x\left(\frac{x}{2}\right)\left(\frac{x}{3}\right) - \frac{100}{3}(x-3)\left(\frac{x-3}{2}\right)\left(\frac{x-3}{3}\right) + M = 0$$

Moving terms to the other side and expressing them in a singularity function form:

$$M = -\frac{100}{18}\langle x-0\rangle^3 + 430\langle x-3\rangle^1 + \frac{100}{18}\langle x-3\rangle^3$$

$$EI\frac{dv}{dx} = \int M\,dx = -\frac{25}{18}\langle x-0\rangle^4 + 215\langle x-3\rangle^2 + \frac{25}{18}\langle x-3\rangle^4 + C_1$$

$$EI\,v = -\frac{5}{18}\langle x-0\rangle^5 + \frac{215}{3}\langle x-3\rangle^3 + \frac{5}{18}\langle x-3\rangle^5 + C_1x + C_2$$

B.C's.: $v_A = v(3) = 0$ and $v_B = v(8) = 0$

Two equations and two constants: We can solve.

$$v(3) = 0 = -\frac{5}{18}(3)^5 + 3C_1 + C_2 \quad (1)$$

$$v(8) = 0 = -\frac{5}{18}(8)^5 + \frac{215}{3}(5)^3 + \frac{5}{18}(5)^5 + 8C_1 + C_2 \quad (2)$$

Solving eqs. (1) and (2) yields

$$C_1 = -\frac{475}{3} \quad \& \quad C_2 = \frac{1085}{2} \Rightarrow$$

$$\boxed{v = \left[-\frac{5}{18}\langle x-0\rangle^5 + \frac{215}{3}\langle x-3\rangle^3 + \frac{5}{18}\langle x-3\rangle^5 - \frac{475}{3}x + \frac{1085}{2}\right]\Big/EI}$$

units in kN, m