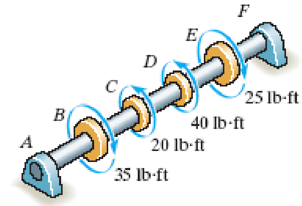
HW# 6 Key Solutions

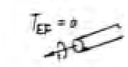
5–7. The solid shaft has a diameter of 0.75 in. If it is subjected to the torques shown, determine the maximum shear stress developed in regions CD and EF of the shaft. The bearings at A and F allow free rotation of the shaft.

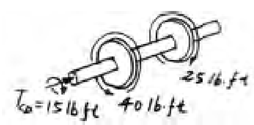


$$(\tau_{EF})_{max} = \frac{T_{EF}\, c}{J} = 0 \qquad \textbf{Ans.}$$

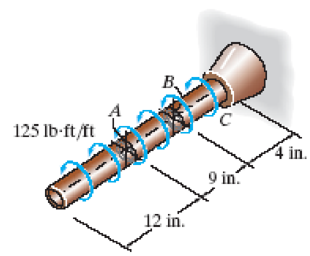
$$(\tau_{CD})_{max} = \frac{T_{CD}\, c}{J} = \frac{15(12)(0.375)}{\frac{\pi}{2}(0.375)^4}$$

$$= 2173 \text{ psi} = 2.17 \text{ ksi} \qquad \textbf{Ans.}$$





•**5–25.** The copper pipe has an outer diameter of 2.50 in. and an inner diameter of 2.30 in. If it is tightly secured to the wall at C and it is subjected to the uniformly distributed torque along its entire length, determine the absolute maximum shear stress in the pipe. Discuss the validity of this result.



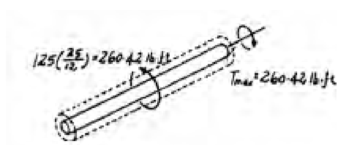
Internal Torque: The maximum torque occurs at the support C.

$$T_{max} = (125 \text{ lb}\cdot\text{ft/ft})\left(\frac{25 \text{ in.}}{12 \text{ in./ft}}\right) = 260.42 \text{ lb}\cdot\text{ft}$$

Maximum Shear Stress: Applying the torsion formula

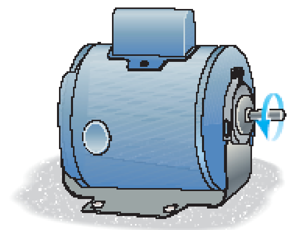
$$\tau^{abs}_{max} = \frac{T_{max}\, c}{J}$$

$$= \frac{260.42(12)(1.25)}{\frac{\pi}{2}(1.25^4 - 1.15^4)} = 3.59 \text{ ksi} \qquad \textbf{Ans.}$$



According to Saint-Venant's principle, application of the torsion formula should be as points sufficiently removed from the supports or points of concentrated loading.

*5–36. The drive shaft of the motor is made of a material having an allowable shear stress of $\tau_{allow} = 75$ MPa. If the outer diameter of the tubular shaft is 20 mm and the wall thickness is 2.5 mm, determine the maximum allowable power that can be supplied to the motor when the shaft is operating at an angular velocity of 1500 rev/min.



Internal Loading: The angular velocity of the shaft is

$$\omega = \left(1500\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 50\pi\ \text{rad/s}$$

We have

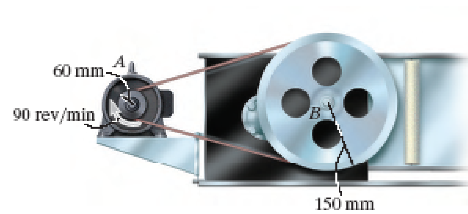
$$T = \frac{P}{\omega} = \frac{P}{50\pi}$$

Allowable Shear Stress: The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(0.01^4 - 0.0075^4\right) = 10.7379(10^{-9})\ \text{m}^4$.

$$\tau_{allow} = \frac{Tc}{J}; \qquad 75(10^6) = \frac{\left(\frac{P}{50\pi}\right)(0.01)}{10.7379(10^{-9})}$$

$$P = 12\,650.25\ \text{W} = 12.7\ \text{kW} \qquad \textbf{Ans.}$$

5–38. The motor A develops a power of 300 W and turns its connected pulley at 90 rev/min. Determine the required diameters of the steel shafts on the pulleys at A and B if the allowable shear stress is $\tau_{\text{allow}} = 85$ MPa.



Internal Torque: For shafts A and B

$$\omega_A = 90\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\frac{1\ \text{min}}{60\ \text{s}} = 3.00\pi\ \text{rad/s}$$

$$P = 300\ \text{W} = 300\ \text{N}\cdot\text{m/s}$$

$$T_A = \frac{P}{\omega_A} = \frac{300}{3.00\pi} = 31.83\ \text{N}\cdot\text{m}$$

$$\omega_B = \omega_A\left(\frac{r_A}{r_B}\right) = 3.00\pi\left(\frac{0.06}{0.15}\right) = 1.20\pi\ \text{rad/s}$$

$$P = 300\ \text{W} = 300\ \text{N}\cdot\text{m/s}$$

$$T_B = \frac{P}{\omega_B} = \frac{300}{1.20\pi} = 79.58\ \text{N}\cdot\text{m}$$

Allowable Shear Stress: For shaft A

$$\tau_{\max} = \tau_{\text{allow}} = \frac{T_A\,c}{J}$$

$$85\left(10^6\right) = \frac{31.83\left(\frac{d_A}{2}\right)}{\frac{\pi}{2}\left(\frac{d_A}{2}\right)^4}$$

$$d_A = 0.01240\ \text{m} = 12.4\ \text{mm}$$ **Ans.**

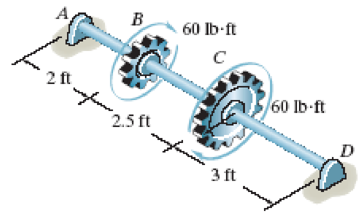
For shaft B

$$\tau_{\max} = \tau_{\text{allow}} = \frac{T_B\,c}{J}$$

$$85\left(10^6\right) = \frac{79.58\left(\frac{d_B}{2}\right)}{\frac{\pi}{2}\left(\frac{d_B}{2}\right)^4}$$

$$d_B = 0.01683\ \text{m} = 16.8\ \text{mm}$$ **Ans.**

***5–60.** The shaft is made of A-36 steel. It has a diameter of 1 in. and is supported by bearings at A and D, which allow free rotation. Determine the angle of twist of gear C with respect to B.



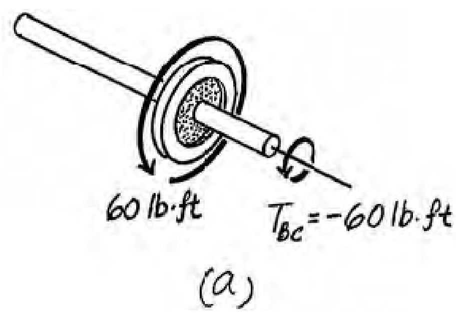
The internal torque developed in segment BC is shown in Fig. a

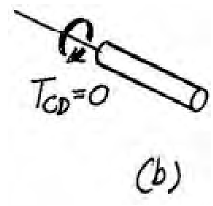
The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.5^4) = 0.03125\pi \text{ in}^4$. Thus,

$$\phi_{C/B} = \frac{T_{BC} L_{BC}}{J G_{st}} = \frac{-60(12)(2.5)(12)}{(0.03125\pi)[11.0(10^6)]}$$

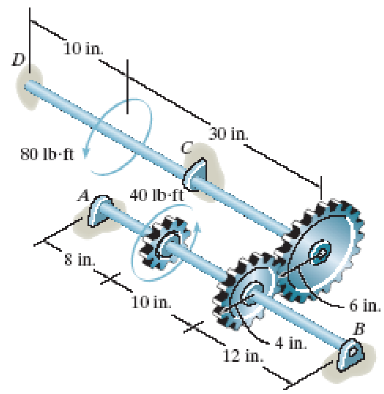
$$= -0.02000 \text{ rad}$$

$$= 1.15° \qquad \textbf{Ans.}$$

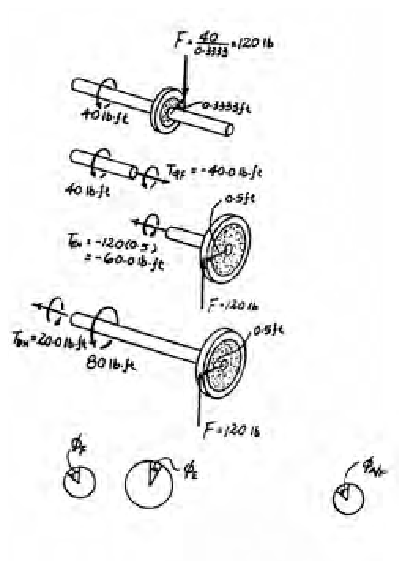




5–62. The two shafts are made of A-36 steel. Each has a diameter of 1 in., and they are supported by bearings at A, B, and C, which allow free rotation. If the support at D is fixed, determine the angle of twist of end A when the torques are applied to the assembly as shown.



Internal Torque: As shown on FBD.



Angle of Twist:

$$\phi_E = \sum \frac{TL}{JG}$$

$$= \frac{1}{\frac{\pi}{2}(0.5^4)(11.0)(10^6)}\left[-60.0(12)(30) + 20.0(12)(10)\right]$$

$$= -0.01778 \text{ rad} = 0.01778 \text{ rad}$$

$$\phi_F = \frac{6}{4}\phi_E = \frac{6}{4}(0.01778) = 0.02667 \text{ rad}$$

$$\phi_{A/F} = \frac{T_{GF}\,L_{GF}}{JG}$$

$$= \frac{-40(12)(10)}{\frac{\pi}{2}(0.5^4)(11.0)(10^6)}$$

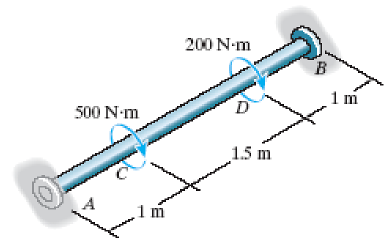
$$= -0.004445 \text{ rad} = 0.004445 \text{ rad}$$

$$\phi_A = \phi_F + \phi_{A/F}$$

$$= 0.02667 + 0.004445$$

$$= 0.03111 \text{ rad} = 1.78^\circ$$ **Ans.**

5–78. The A-36 steel shaft has a diameter of 60 mm and is fixed at its ends A and B. If it is subjected to the torques shown, determine the absolute maximum shear stress in the shaft.



Referring to the FBD of the shaft shown in Fig. *a*,

$$\Sigma M_x = 0; \qquad T_A + T_B - 500 - 200 = 0 \qquad \textbf{(1)}$$

Using the method of superposition, Fig. *b*

$$\phi_A = (\phi_A)_{T_A} - (\phi_A)_T$$

$$0 = \frac{T_A(3.5)}{JG} - \left[\frac{500\,(1.5)}{JG} + \frac{700\,(1)}{JG}\right]$$

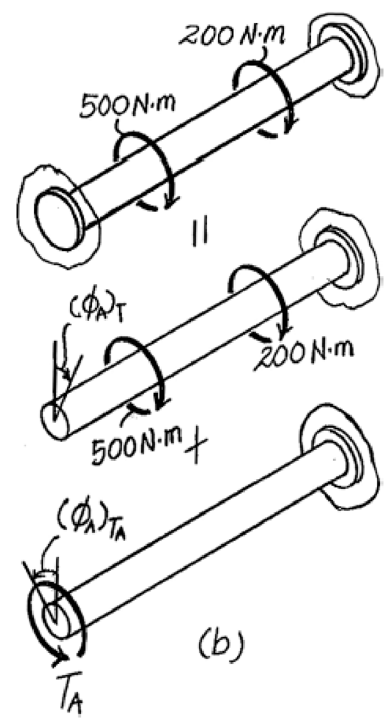
$$T_A = 414.29\ \mathrm{N \cdot m}$$

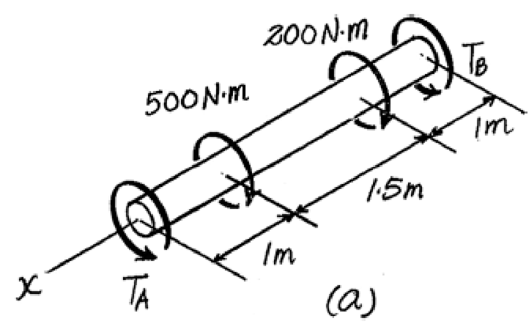
Substitute this result into Eq (1),

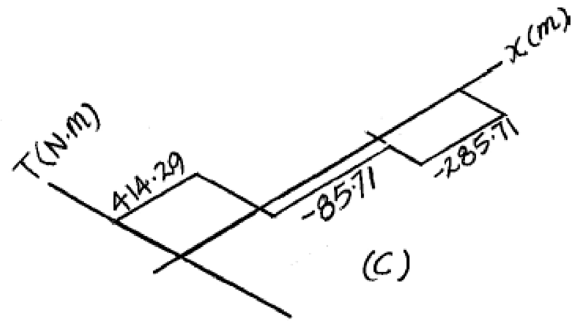
$$T_B = 285.71\ \mathrm{N \cdot m}$$

Referring to the torque diagram shown in Fig. *c*, segment AC is subjected to maximum internal torque. Thus, the absolute maximum shear stress occurs here.

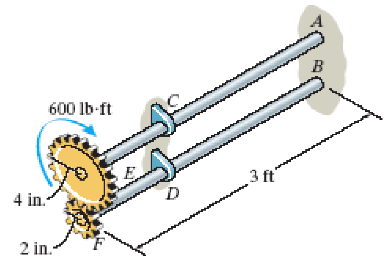
$$\tau_{Abs} = \frac{T_{AC}\,c}{J} = \frac{414.29\,(0.03)}{\frac{\pi}{2}(0.03)^4} = 9.77\ \mathrm{MPa} \qquad \textbf{Ans.}$$

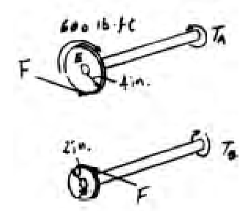






5–90. The two 3-ft-long shafts are made of 2014-T6 aluminum. Each has a diameter of 1.5 in. and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of 600 lb · ft is applied to the top gear as shown, determine the maximum shear stress in each shaft.





$$T_A + F\left(\frac{4}{12}\right) - 600 = 0 \qquad \textbf{(1)}$$

$$T_B - F\left(\frac{2}{12}\right) = 0 \qquad \textbf{(2)}$$

From Eqs. (1) and (2)

$$T_A + 2T_B - 600 = 0 \qquad \textbf{(3)}$$

$$4(\phi_E) = 2(\phi_F); \qquad \phi_E = 0.5\phi_F$$

$$\frac{T_A L}{JG} = 0.5\left(\frac{T_B L}{JG}\right); \qquad T_A = 0.5T_B \qquad \textbf{(4)}$$

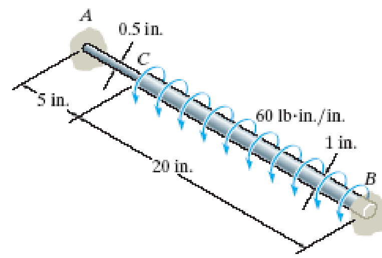
Solving Eqs. (3) and (4) yields:

$$T_B = 240 \text{ lb} \cdot \text{ft}; \qquad T_A = 120 \text{ lb} \cdot \text{ft}$$

$$(\tau_{BD})_{\max} = \frac{T_B\, c}{J} = \frac{240(12)(0.75)}{\frac{\pi}{2}(0.75^4)} = 4.35 \text{ ksi} \qquad \textbf{Ans.}$$

$$(\tau_{AC})_{\max} = \frac{T_A\, c}{J} = \frac{120(12)(0.75)}{\frac{\pi}{2}(0.75^4)} = 2.17 \text{ ksi} \qquad \textbf{Ans.}$$

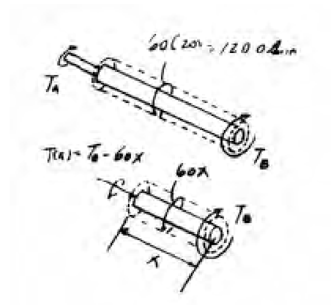
5–91. The A-36 steel shaft is made from two segments: AC has a diameter of 0.5 in. and CB has a diameter of 1 in. If the shaft is fixed at its ends A and B and subjected to a uniform distributed torque of $60 \text{ lb}\cdot\text{in./in.}$ along segment CB, determine the absolute maximum shear stress in the shaft.



Equilibrium:

$$T_A + T_B - 60(20) = 0 \qquad \textbf{(1)}$$

Compatibility condition:



$$\phi_{C/B} = \phi_{C/A}$$

$$\phi_{C/B} = \int \frac{T(x)\,dx}{JG} = \int_0^{20} \frac{(T_B - 60x)\,dx}{\frac{\pi}{2}(0.5^4)(11.0)(10^6)}$$

$$= 18.52(10^{-6})T_B - 0.011112$$

$$18.52(10^{-6})T_B - 0.011112 = \frac{T_A(5)}{\frac{\pi}{2}(0.25^4)(11.0)(10^6)}$$

$$18.52(10^{-6})T_B - 74.08(10^{-6})T_A = 0.011112$$

$$18.52T_B - 74.08T_A = 11112 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

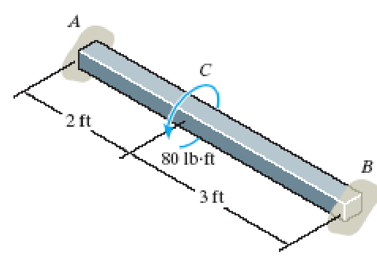
$$T_A = 120.0 \text{ lb}\cdot\text{in.}; \qquad T_B = 1080 \text{ lb}\cdot\text{in.}$$

$$(\tau_{\max})_{BC} = \frac{T_B\,c}{J} = \frac{1080(0.5)}{\frac{\pi}{2}(0.5^4)} = 5.50 \text{ ksi}$$

$$(\tau_{\max})_{AC} = \frac{T_A\,c}{J} = \frac{120.0(0.25)}{\frac{\pi}{2}(0.25^4)} = 4.89 \text{ ksi}$$

$$\tau_{\max}^{\text{abs}} = 5.50 \text{ ksi} \qquad \textbf{Ans.}$$

5–102. The aluminum strut is fixed between the two walls at A and B. If it has a 2 in. by 2 in. square cross section, and it is subjected to the torque of $80 \text{ lb}\cdot\text{ft}$ at C, determine the reactions at the fixed supports. Also, what is the angle of twist at C? $G_{al} = 3.8(10^3)$ ksi.

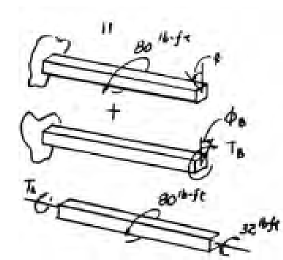


By superposition:

$$0 = \phi - \phi_B$$

$$0 = \frac{7.10(80)(2)}{a^4G} - \frac{7.10(T_B)(5)}{a^4G}$$

$$T_B = 32 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

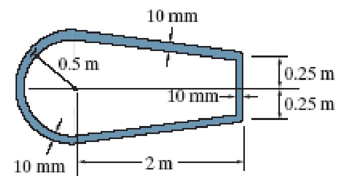


$$T_A + 32 - 80 = 0$$

$$T_A = 48 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

$$\phi_C = \frac{7.10(32)(12)(3)(12)}{(2^4)(3.8)(10^6)} = 0.00161 \text{ rad} = 0.0925^\circ \qquad \textbf{Ans.}$$

•5–117. The mean dimensions of the cross section of the leading edge and torsion box of an airplane wing can be approximated as shown. If the wing is made of 2014-T6 aluminum alloy having an allowable shear stress of $\tau_{allow} = 125$ MPa and the wall thickness is 10 mm, determine the maximum allowable torque and the corresponding angle of twist per meter length of the wing.



Section Properties: Referring to the geometry shown in Fig. *a*,

$$A_m = \frac{\pi}{2}\left(0.5^2\right) + \frac{1}{2}\left(1 + 0.5\right)(2) = 1.8927 \text{ m}^2$$

$$\oint ds = \pi(0.5) + 2\sqrt{2^2 + 0.25^2} + 0.5 = 6.1019 \text{ m}$$

Allowable Average Shear Stress:

$$\left(\tau_{avg}\right)_{allow} = \frac{T}{2tA_m}; \qquad 125(10^6) = \frac{T}{2(0.01)(1.8927)}$$

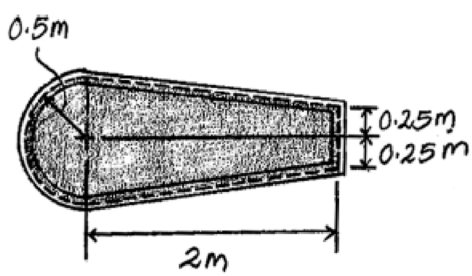
$$T = 4.7317(10^6)\text{N} \cdot \text{m} = 4.73 \text{ MN} \cdot \text{m} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi = \frac{TL}{4A_m{}^2G}\oint\frac{ds}{t}$$

$$= \frac{4.7317(10^6)(1)}{4(1.8927^2)(27)(10^9)}\left(\frac{6.1019}{0.01}\right)$$

$$= 7.463(10^{-3}) \text{ rad} = 0.428°/\text{m} \qquad \textbf{Ans.}$$



(a)