بسم الله الرحمن الرحيم

King Fahd University of Petroleum & Minerals

DEPARTMENT OF CIVIL ENGINEERING

Second Semester 1431-32 / 2010-11 (102)

CE 203 STRUCTURAL MECHANICS I

Major Exam II

Tuesday, May 10, 2011, 7:00-9:15 P.M.

Student Name	Family				First			
ID No. (9 Digits)								

CIRCLE YOUR COURSE--SECTION NO.						
Section #	1 & 2	3	4	5	6 & 7	8
Instructor	Altayyib	Dulaijan	Ghamdi	Suwaiyan	Khathlan	Ahmad

Summary of Scores

Problem	Full Mark	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	**100**	
Remarks		

Notes:

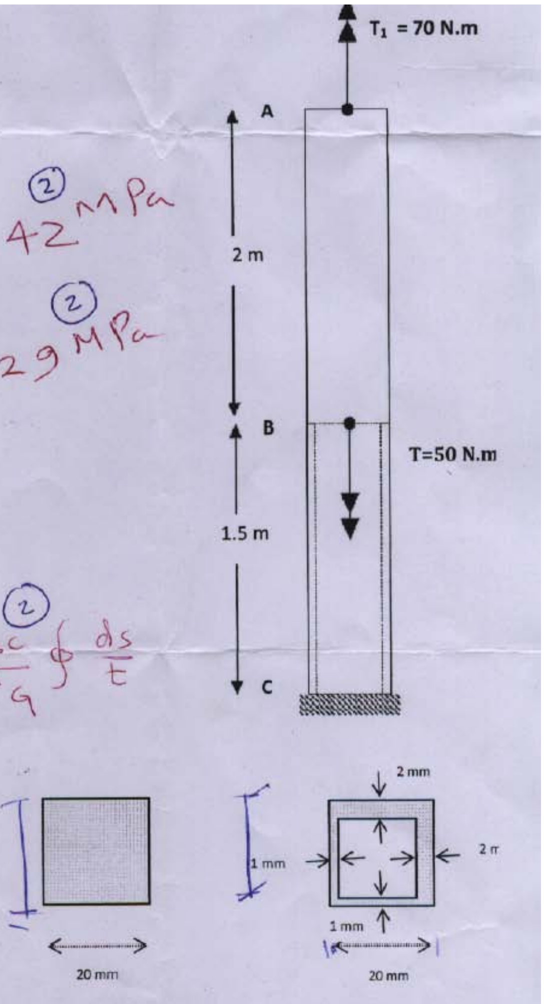
1. A sheet that includes selected Basic Formulae and definitions is provided with this examination.
2. Write clearly and show all calculations, FBDs, and units.

Problem # 1

Shaft ABC is fixed at C and has a solid section between A and B and a thin hollow section between B and C.
Calculate

a) The maximum shear stress in the shaft.
b) The angle of twist at the free end.

G= 80 GPa



Cross-section between A and B

Cross-section between B and C

$T_{AB} = 70$ N.m (1)

$T_{BC} = 20$ N.m (2)

max (1) $\Rightarrow$ $\tau_{max\,AB} = \frac{4.81\,T_{AB}}{a^3}$ (1) $= 42$ MPa (2)

$\tau_{max\,BC} = \frac{T_{BC}}{2 A_m t}$ (1) $= 29$ MPa (2)

$\varphi_A = \varphi_{AB} + \varphi_{BC}$

$= \frac{7.1\,T_{AB} L_{AB}}{a^4 G}$ (2) $+ \frac{T_{BC} L_{BC}}{4 A_m^2 G} \oint \frac{ds}{t}$ (2)

0.078 (1) $+ .044$ (1)

$= 0.122$

close to 7°

$342.25 = A_m = 18.5^2$ (2)

$55.5 = \oint \frac{ds}{t} = 2\left(\frac{18.5}{1} + \frac{18.5}{2}\right)$ (2)

Problem # 2

Shaft, shown below, is fixed at both ends *A* and *C*. The segment *AB* has diameter 90 mm and G_{AB} = 40 GPa. The segment *BC* has diameter 60 mm and G_{BC} = 80 GPa. Calculate the maximum torque *T* that can be applied at *B*, if $\tau_{(allow)AB}$ = 50 MPa and $\tau_{(allow)BC}$ = 70 MPa.

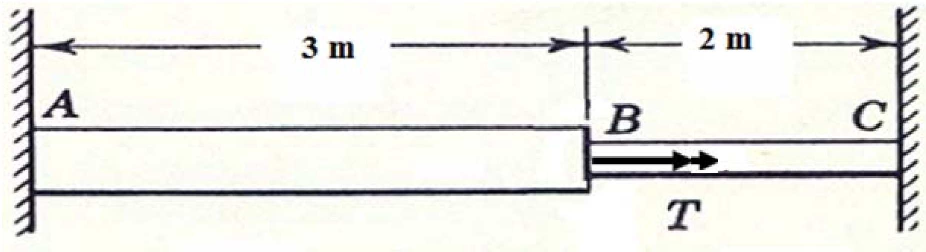


Figure 1

Let T_A and T_C are the torque reactions at ends A and C, respectively.

$$\Sigma M = 0 \Rightarrow -T_A + T - T_C = 0$$

$$\Rightarrow T_A + T_C = T \quad \text{- - - (1)}$$

The compatability condition is as follows:

$$\phi_{A/C} = \phi_{AB} + \phi_{BC} = 0$$

$$\Rightarrow \frac{T_{AB} L_{AB}}{J_{AB} G_{AB}} + \frac{T_{BC} L_{BC}}{J_{BC} G_{BC}} = 0$$

$\underline{\underline{T_{AB}}}$

$\Sigma M = 0$

$T_{AB} = T_A$

$\underline{\underline{T_{BC}}}$

$\Sigma M = 0$

$T_{BC} = -T_C$

Substituting $T_{AB}, T_{BC}, L_{AB}, L_{BC}, J_{AB}, J_{BC}$, G_{AB}, and G_{BC} in the compatability condition :-

$$\frac{T_A \times 3\times10^3}{\frac{\pi}{2}\left(\frac{90}{2}\right)^4 \times 40\times10^3} + \frac{(-T_c)\times 2\times10^3}{\frac{\pi}{2}\left(\frac{60}{2}\right)^4 \times 80\times10^3} = 0$$

$$\Rightarrow T_A = 1.6875\, T_c \quad --- (2)$$

Solving Equations (1) and (2)

$$\underline{\underline{T_A = 0.6279T}} \text{ and } \underline{\underline{T_c = 0.3721T}}$$

T_{AB} and T_{BC} from τ_{allow} values :-

$$(\tau_{max})_{AB} = \frac{T_{AB}C_{AB}}{J_{AB}} = (\tau_{allow})_{AB}$$

$$\Rightarrow T_{AB} = \frac{50 \times \frac{\pi}{2}\left(\frac{90}{2}\right)^4}{(90/2)} = 7156940 \text{ N-mm} = 7.157 \text{ KN-m}$$

Similarly, $T_{BC} = \dfrac{70\times\frac{\pi}{2}\left(\frac{60}{2}\right)^4}{(60/2)} = 2968805$ N-mm $= 2.969$ KN-m

Finally,

$$T_{AB} = T_A = 0.6279T = 7.157$$

$$\Rightarrow T = 11.398 \text{ KN-m}$$

and $T_{BC} = |T_c| = 0.3721T = 2.969$

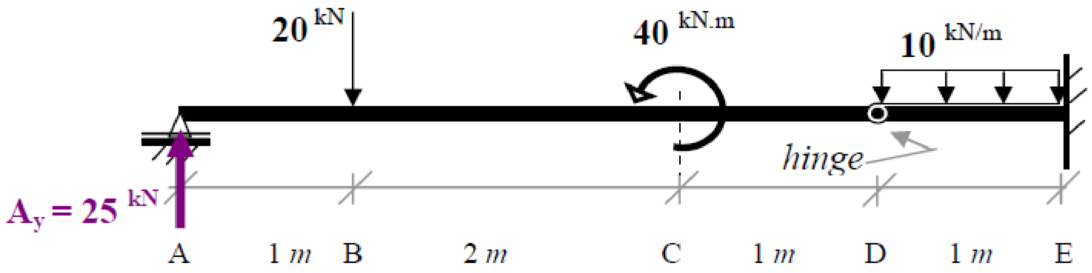
$$\Rightarrow T = 7.979 \text{ KN-m}$$

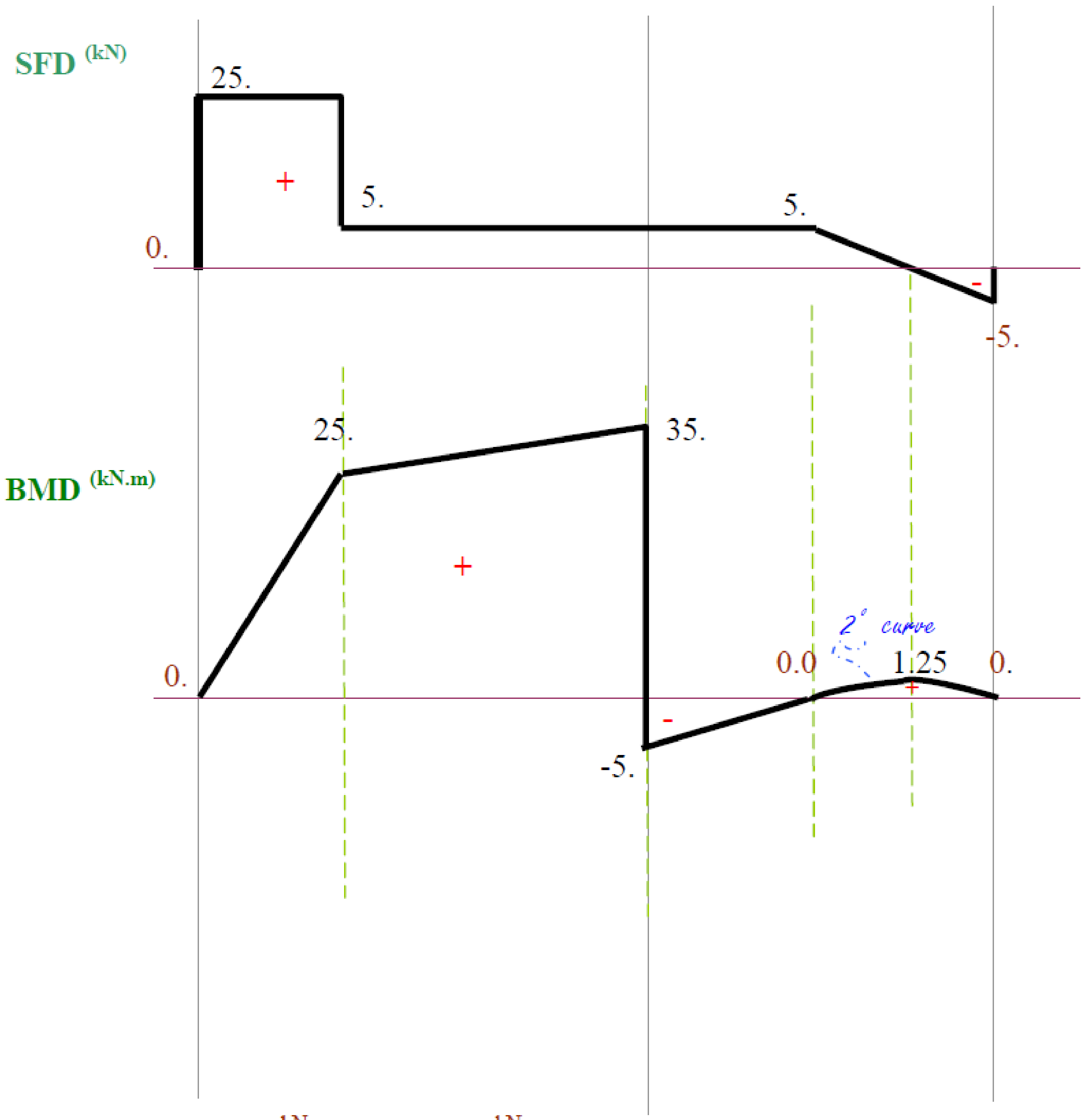
Therefore, maximum torque, T, that can be applied = $\underline{\underline{7.977 \text{ KN-m}}}$ Ans.

Problem # 3

Use the graphical method (*i.e.* areas summation) to draw shear force diagram (SFD) and bending moment diagram (BMD) for beam ABCDE having a *hinge* at D.

Note: Reaction $A_y = 25$ kN (↑).





Note: $E_y = 5.0$ kN and $M_E = 0.0$ kN.m.

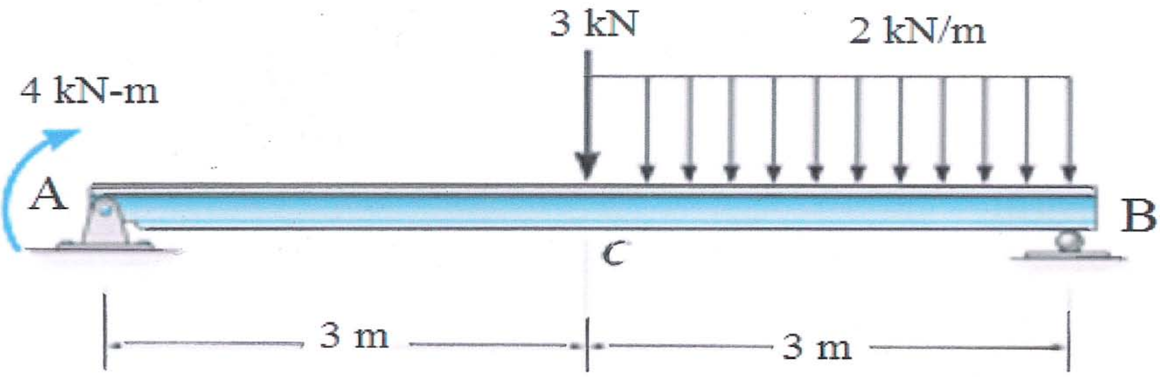
Problem # 4

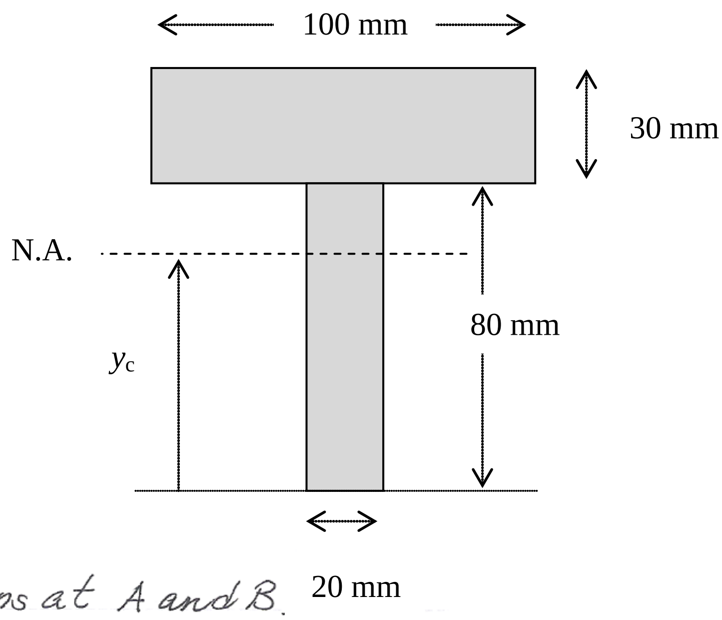
The simply supported T-beam is subjected to the loading shown:

a) Verify that $\boldsymbol{y_c = 75.87}$ **mm** from the bottom of the T- cross-section and that $\boldsymbol{I_{NA} = 4.235\text{x}10^{-6}}$ $\mathbf{m^4}$.

b) Determine the maximum tensile and the maximum compressive bending stress in the T-cross-section at point *C* in the beam.

c) Plot the bending stress distribution along the height of the T-cross-section at point *C* in the beam.





Solution:

a) First determine reactions at A and B.

$\Sigma F_y^{+\uparrow} = 0;$ $\quad R_A + R_B = 3 + 2\times3 = 9$ kN

$\Sigma M_A = 0;$ $\quad -4 - 3\times3 - 2\times3(3 + 3/2) + R_B(6) = 0$

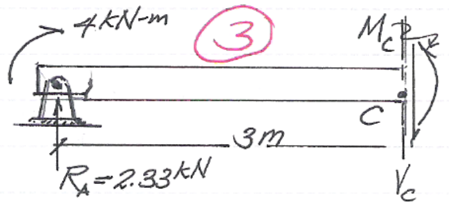
$\therefore$ $R_B = \underline{6.67}$ kN↑

and $R_A = \underline{2.33}$ kN↑

2

$\Sigma M_C = 0; \; M_C - 4 - 2.33 \times 3 = 0$

$\therefore M_C = 10.99 \cong \underline{11.0 \text{ kN-m}}$



b) $y_{c \text{ from bottom}} = \dfrac{\Sigma \bar{y}' A}{\Sigma A}$

2

$$= \frac{(40)(20 \times 80) + (95)(30 \times 100)}{20 \times 80 + 30 \times 100} = \underline{75.87 \text{ mm}}$$

$$I_{NA} = \Sigma (I_{x'} + Ad^2)$$

$$= \left[\tfrac{1}{12}(0.02)(0.08)^3 + (0.02 \times 0.08)(0.07587 - 0.04)^2\right]$$

$$+ \left[\tfrac{1}{12}(0.1)(0.03)^3 + (0.03 \times 0.1)(0.11 - 0.07587 - 0.015)^2\right]$$

$$= 0.853 \times 10^{-6} + 2.059 \times 10^{-6} + 0.225 \times 10^{-6} + 1.098 \times 10^{-6}$$

$$= \underline{4.235 \times 10^{-6} \text{ m}^4}$$

3

c) Since the bending moment is positive, then the max tensile bending stress is in the bottom fiber of the cross-section at $C_{bottom} = 75.87$ mm,

1

or

$$(\sigma_{max})_{Ten} = \frac{M_C C_{bottom}}{I_{NA}} = \frac{(11 \times 10^3 \text{ N-m})(0.07587 \text{ m})}{4.235 \times 10^{-6} \text{ m}^4}$$

$$= \underline{197.065 \text{ MPa}}$$

2

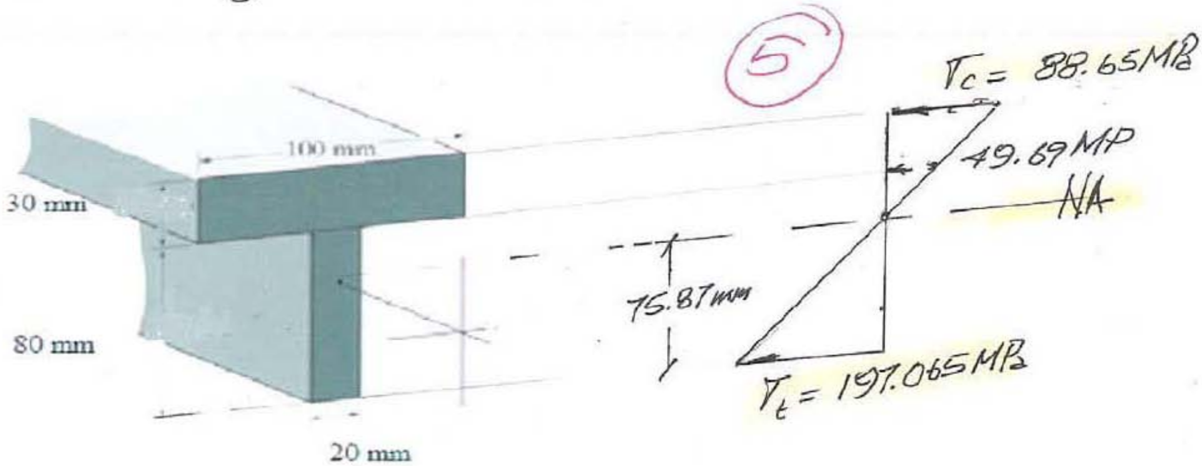
and

$$(\sigma_{max})_{Comp} = \frac{M_C C_{Top}}{I_{NA}} = \frac{(11 \times 10^3 \text{ N-m})(0.03413 \text{ m})}{4.235 \times 10^{-6} \text{ m}^4}$$

$$= \underline{88.65 \text{ MPa}}$$

2

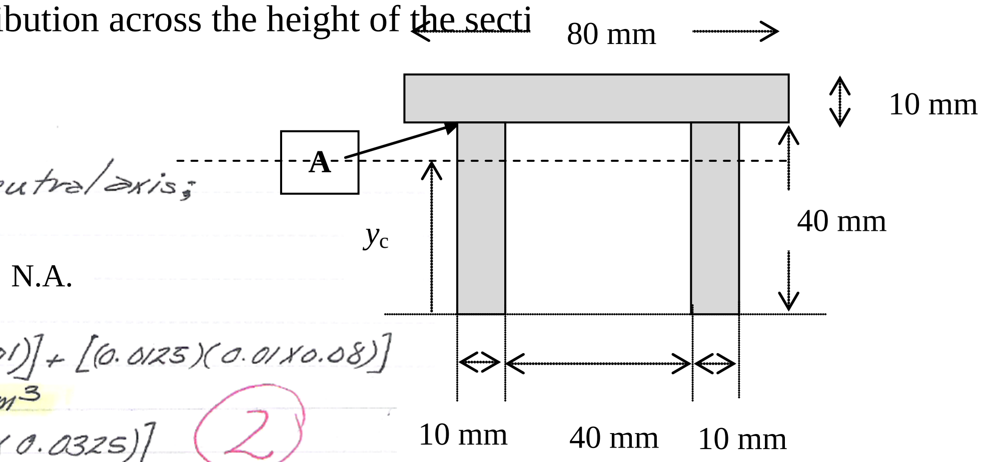
d) Bending Stress distribution:



Problem # 5

If the beam cross-sectional area shown below is subjected to a shear of $V = 115$ kN, and it is calculated that its $\boldsymbol{y_c = 32.5}$ **mm** from the bottom, and its $\boldsymbol{I_{NA} = 0.3633\text{x}10^{-6}}$ $\mathbf{m^4}$, then determine:

a) the maximum shear stress τ_{max} in the cross section,
b) the shear stress just above point A,
c) the shear stress just below point A,
d) the shear stress distribution across the height of the secti



Solution:

a) τ_{max} occurs at the neutral axis;

$$\tau_{max} = \frac{VQ}{I_{NA}t}$$

$$Q_{Top} = \Sigma y'A = 2\left[\frac{0.0075}{2}(0.0075\times0.01)\right] + [(0.0125)(0.01\times0.08)]$$
$$= 10.5625\times10^{-6}\ m^3$$

$$Q_{Bottom} = \Sigma y'A] = 2\left[\left(\frac{0.0325}{2}\right)(0.01\times0.0325)\right]$$
$$= 10.5625\times10^{-6}\ m^3$$

2

$$\therefore\ \tau_{max} = \frac{VQ_{max}}{I_{NA}t} = \frac{(115\times10^3\ N)(10.5625\times10^{-6}\ m^3)}{(0.3633\times10^{-6}\ m^4)(2\times0.01\ m)}$$
$$= 167.17\ MPa$$

3

b) $$\tau_{Above\ c} = \frac{VQ_c}{I_{NA}\,t_{Above\ C}}$$

$$Q_c = (0.0125)(0.01\times0.08) = 10\times10^{-6}\ m^3$$ 2

$$\therefore\ \tau_{Above\ C} = \frac{(115\times10^3\ N)(10\times10^{-6}\ m^3)}{(0.3633\times10^{-6})(0.08\ m)}$$
$$= 39.57\ MPa$$

3

c) $$\tau_{Below\ C} = \frac{VQ_c}{I_{NA}\,t_{Below\ C}}$$
$$= \frac{(115\times10^3\ N)(10\times10^{-6}\ m^3)}{(0.3633\times10^{-6}\ m^4)(2\times0.01\ m)}$$ 2
$$= 158.27\ MPa$$ 3

d) Shear Stress distribution;

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